

◆ IMPLICATIONS ◆

$I_1$	$P \wedge Q \Rightarrow P$	}	(simplification)
$I_2$	$P \wedge Q \Rightarrow Q$		
$I_3$	$P \Rightarrow P \vee Q$	}	(addition)
$I_4$	$Q \Rightarrow P \vee Q$		
$I_5$	$\neg P \Rightarrow P \rightarrow Q$		
$I_6$	$Q \Rightarrow P \rightarrow Q$		
$I_7$	$\neg(P \rightarrow Q) \Rightarrow P$		
$I_8$	$\neg(P \rightarrow Q) \Rightarrow \neg Q$		
$I_9$	$P, Q \Rightarrow P \wedge Q$		
$I_{10}$	$\neg P, P \vee Q \Rightarrow Q$		(disjunctive syllogism)
$I_{11}$	$P, P \rightarrow Q \Rightarrow Q$		(modus ponens)
$I_{12}$	$\neg Q, P \rightarrow Q \Rightarrow \neg P$		(modus tollens)
$I_{13}$	$P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R$		(hypothetical syllogism)
$I_{14}$	$P \vee Q, P \rightarrow R, Q \rightarrow R \Rightarrow R$		(dilemma)

◆ EQUIVALENCES ◆

$E_1$	$\neg\neg P \Leftrightarrow P$		(double negation)
$E_2$	$P \wedge Q \Leftrightarrow Q \wedge P$	}	(commutative laws)
$E_3$	$P \vee Q \Leftrightarrow Q \vee P$		
$E_4$	$(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$	}	(associative laws)
$E_5$	$(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$		
$E_6$	$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$	}	(distributive laws)
$E_7$	$P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$		
$E_8$	$\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$	}	(De Morgan's laws)
$E_9$	$\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$		
$E_{10}$	$P \vee P \Leftrightarrow P$		
$E_{11}$	$P \wedge P \Leftrightarrow P$		
$E_{12}$	$R \vee (P \wedge \neg P) \Leftrightarrow R$		
$E_{13}$	$R \wedge (P \vee \neg P) \Leftrightarrow R$		
$E_{14}$	$R \vee (P \vee \neg P) \Leftrightarrow T$		
$E_{15}$	$R \wedge (P \wedge \neg P) \Leftrightarrow F$	$E_{16}$	$P \rightarrow Q \Leftrightarrow \neg P \vee Q$
$E_{17}$	$\neg(P \rightarrow Q) \Leftrightarrow P \wedge \neg Q$	$E_{18}$	$P \rightarrow Q \Leftrightarrow \neg Q \rightarrow \neg P$
$E_{19}$	$P \rightarrow (Q \rightarrow R) \Leftrightarrow (P \wedge Q) \rightarrow R$	$E_{20}$	$\neg(P \leftrightarrow Q) \Leftrightarrow P \leftrightarrow \neg Q$
$E_{21}$	$P \leftrightarrow Q \Leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)$	$E_{22}$	$P \leftrightarrow Q \Leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$

$(\exists x)(A(x) \vee B(x)) \Leftrightarrow (\exists x)A(x) \vee (\exists x)B(x)$	$E_{23}$
$(\forall x)(A(x) \wedge B(x)) \Leftrightarrow (\forall x)A(x) \wedge (\forall x)B(x)$	$E_{24}$
$\neg(\exists x)A(x) \Leftrightarrow (\forall x)\neg A(x)$	$E_{25}$
$\neg(\forall x)A(x) \Leftrightarrow (\exists x)\neg A(x)$	$E_{26}$
$(\forall x)A(x) \vee (\forall x)B(x) \Rightarrow (\forall x)(A(x) \vee B(x))$	$I_{15}$
$(\exists x)(A(x) \wedge B(x)) \Rightarrow (\exists x)A(x) \wedge (\exists x)B(x)$	$I_{16}$

$(\forall x)(A \vee B(x)) \Leftrightarrow A \vee (\forall x)B(x)$	$E_{27}$
$(\exists x)(A \wedge B(x)) \Leftrightarrow A \wedge (\exists x)B(x)$	$E_{28}$
$(\forall x)A(x) \rightarrow B \Leftrightarrow (\exists x)(A(x) \rightarrow B)$	$E_{29}$
$(\exists x)A(x) \rightarrow B \Leftrightarrow (\forall x)(A(x) \rightarrow B)$	$E_{30}$
$A \rightarrow (\forall x)B(x) \Leftrightarrow (\forall x)(A \rightarrow B(x))$	$E_{31}$
$A \rightarrow (\exists x)B(x) \Leftrightarrow (\exists x)(A \rightarrow B(x))$	$E_{32}$

$(\forall x)(\forall y)P(x,y) \Leftrightarrow (\forall y)(\forall x)P(x,y)$	(1)
$(\forall x)(\forall y)P(x,y) \Rightarrow (\exists y)(\forall x)P(x,y)$	(2)
$(\forall y)(\forall x)P(x,y) \Rightarrow (\exists x)(\forall y)P(x,y)$	(3)
$(\exists y)(\forall x)P(x,y) \Rightarrow (\forall x)(\exists y)P(x,y)$	(4)
$(\exists x)(\forall y)P(x,y) \Rightarrow (\forall y)(\exists x)P(x,y)$	(5)
$(\forall x)(\exists y)P(x,y) \Rightarrow (\exists y)(\exists x)P(x,y)$	(6)
$(\forall y)(\exists x)P(x,y) \Rightarrow (\exists x)(\exists y)P(x,y)$	(7)
$(\exists x)(\exists y)P(x,y) \Leftrightarrow (\exists y)(\exists x)P(x,y)$	(8)