Frequency-Domain Analysis of Control Systems

Frequency Response

- Output for Sinusoidal Input $r(t) = M_i \cos(wt + A_i)$

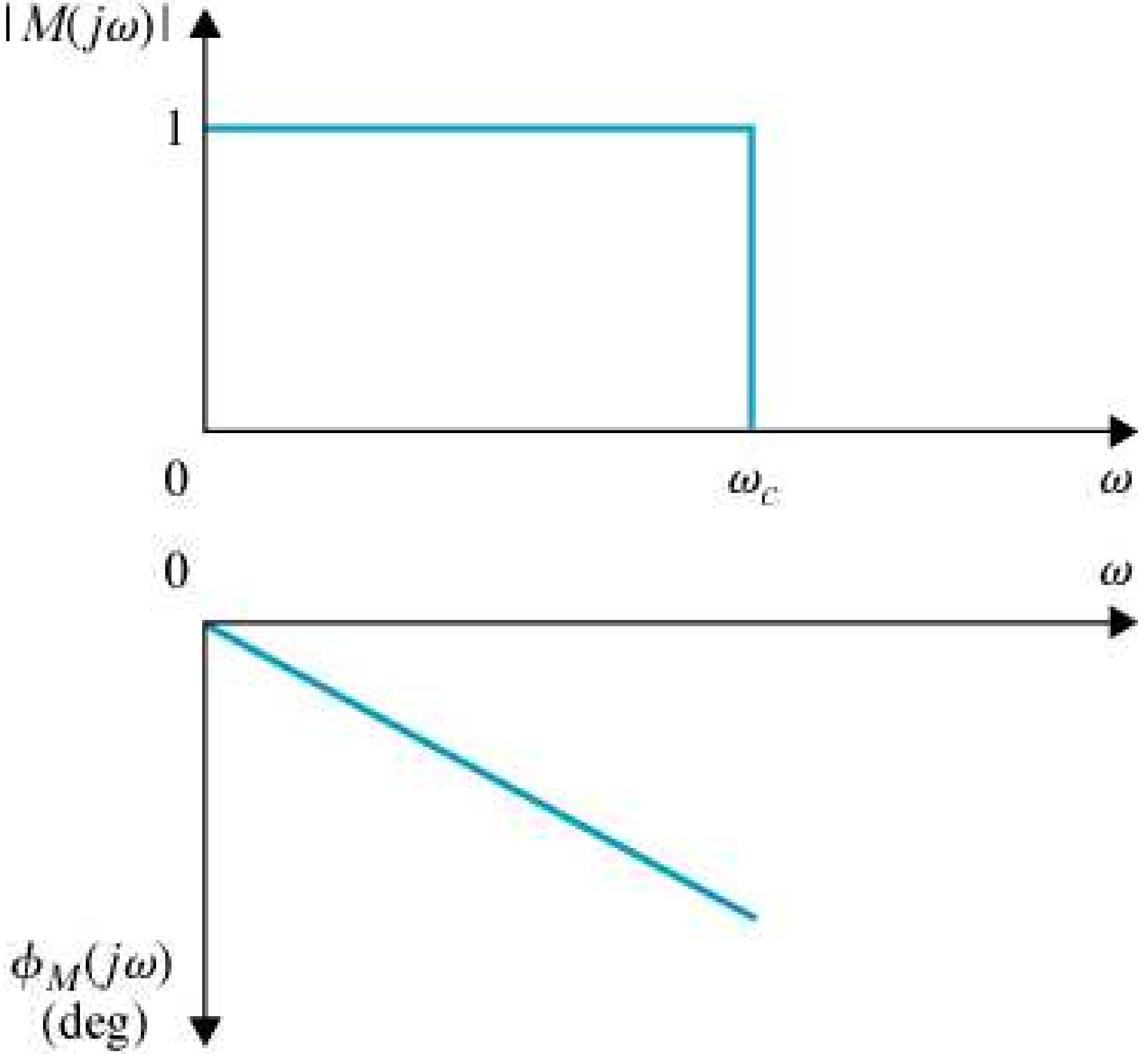
$$c(t) = M_i M_G(w) \cos(wt + A_i + A_G(w))$$

- Frequency Response

$$G(jw) = M_G(w) \angle A_G(w)$$

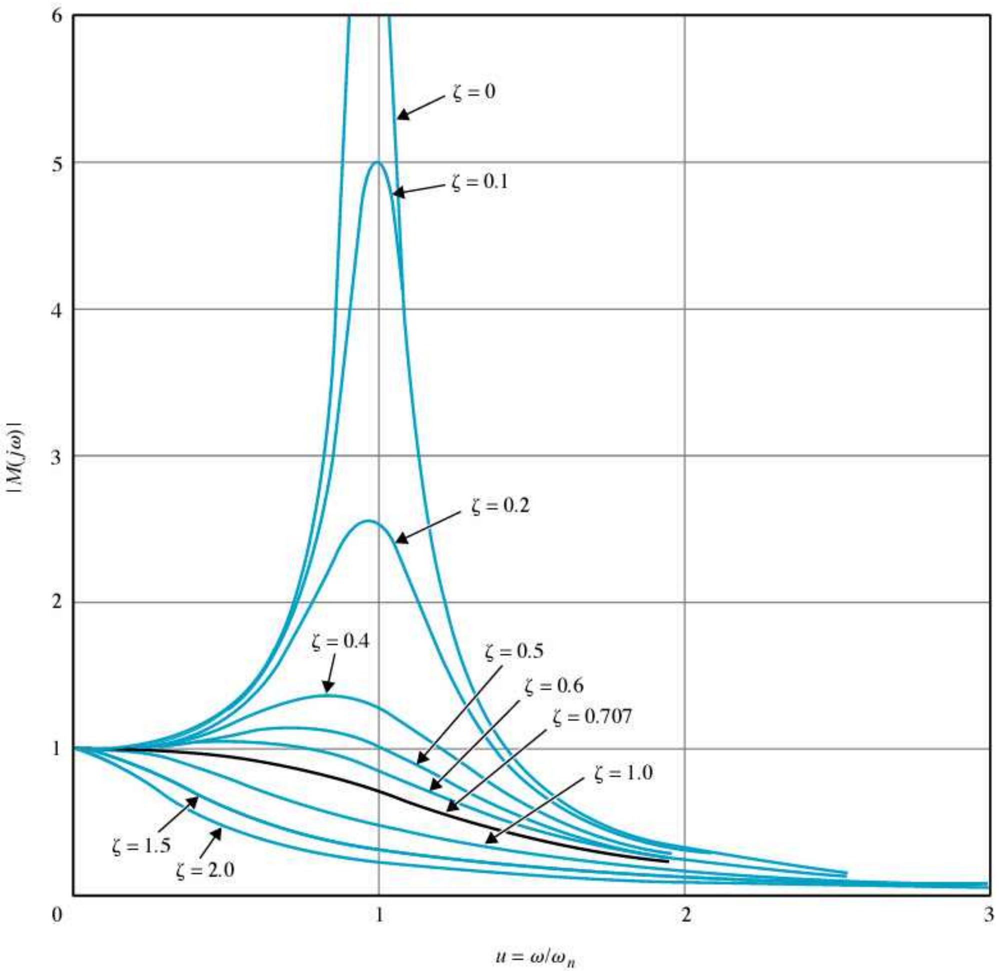
1. Magnitude : $M_G(w)$
2. Phase : $A_G(w)$
   - Bandwidth : -3 dB
   - Resonant Peak : Maximum magnitude, frequency

Ideal Low Pass Filter



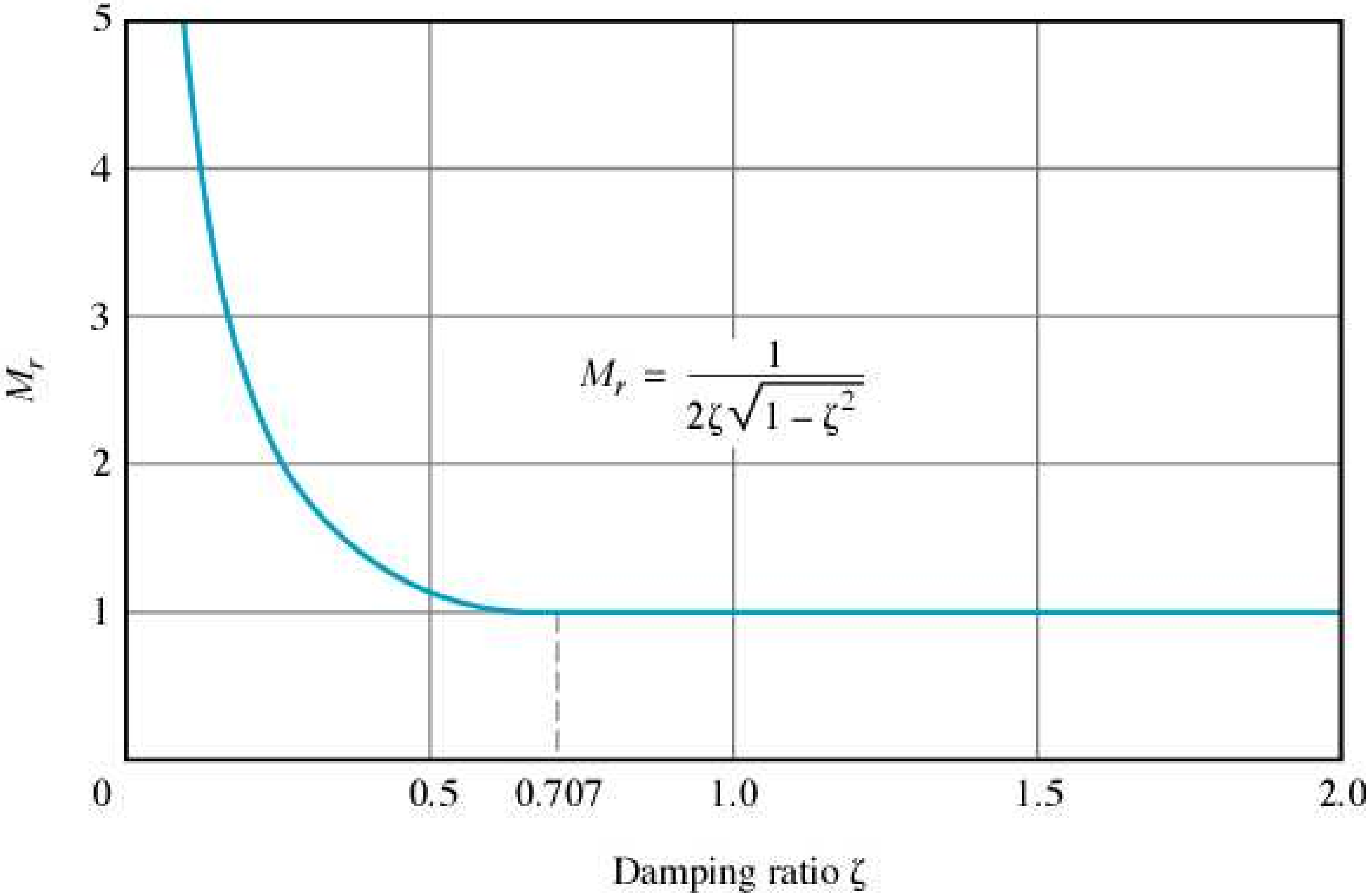
Resonant Peak :

- Damping ratio 가 크면, Resonant Peak 도 작아진다.



M_r versus Damping Ratio

- M_r versus damping ratio for the prototype second-order system



Normalized resonant frequency versus damping ratio for the prototype second-order system

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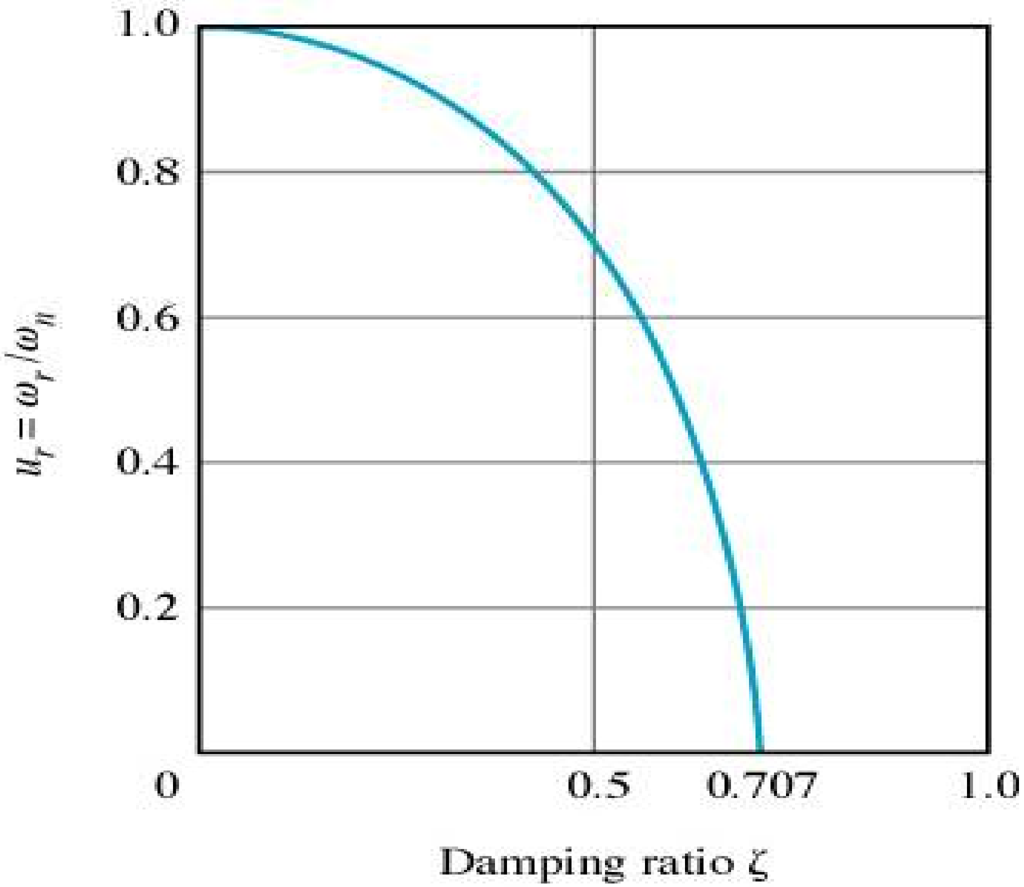
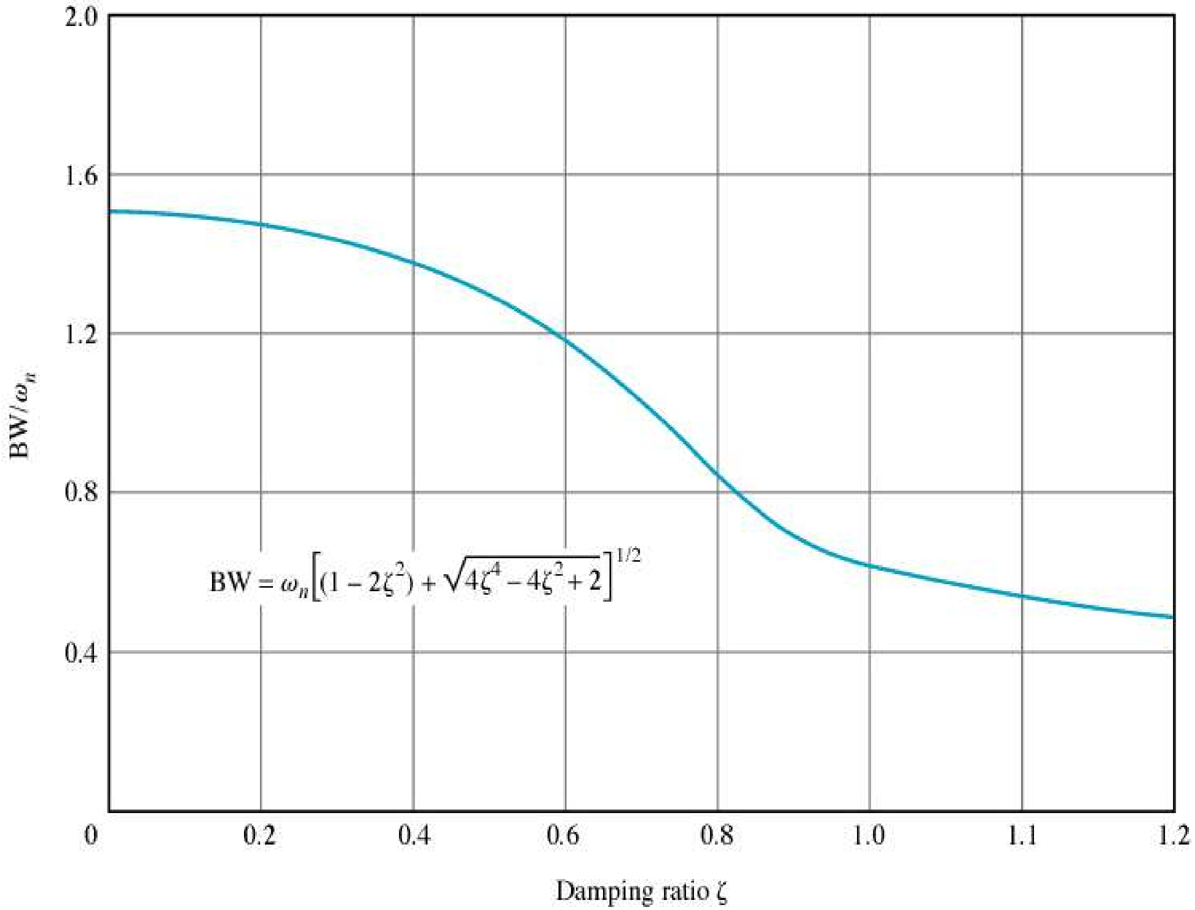
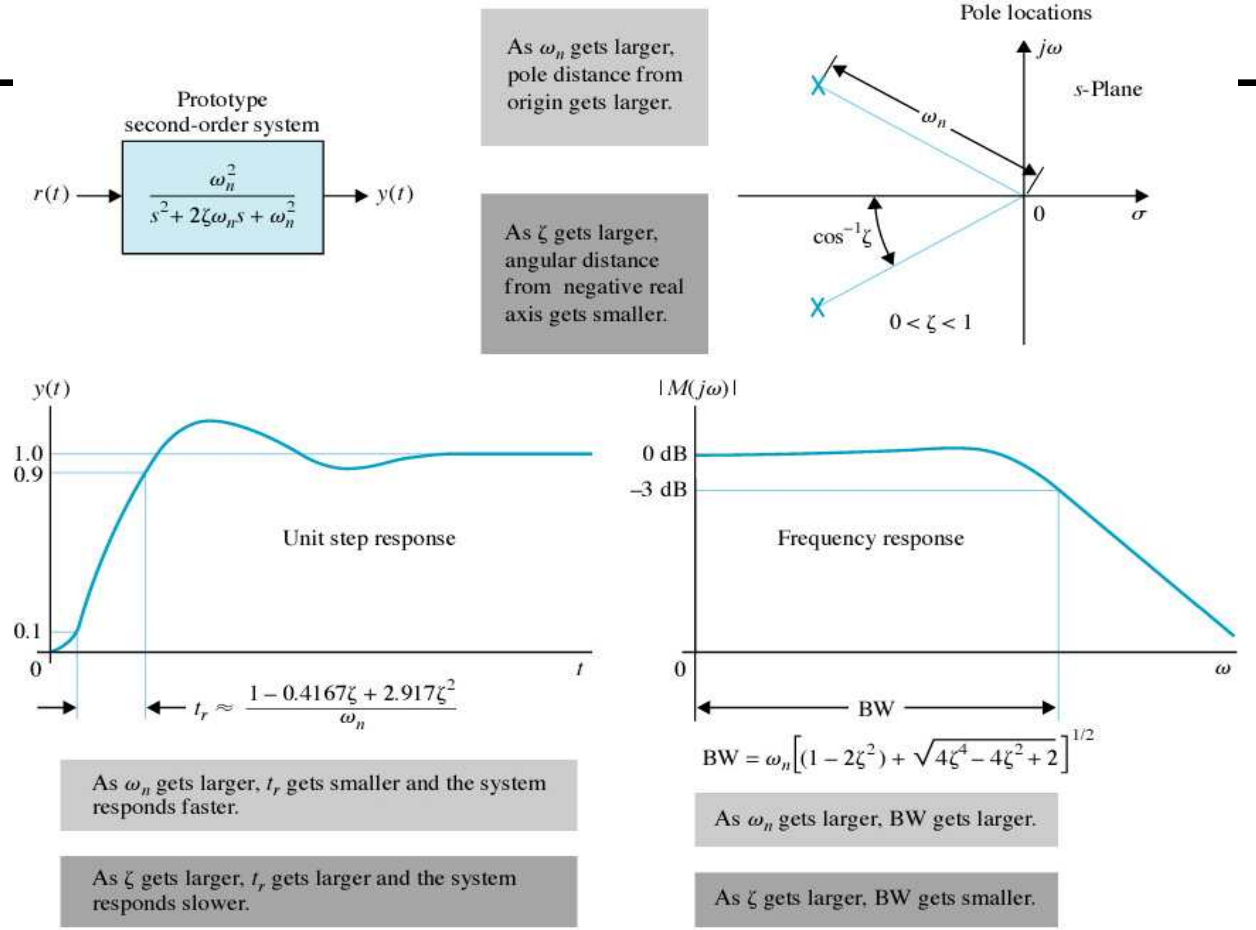


Bandwidth/ω_n versus damping ratio

- Bandwidth/ω_n versus damping ratio for the prototype second-order system.





Bandwidth and rise time are inversely proportional.

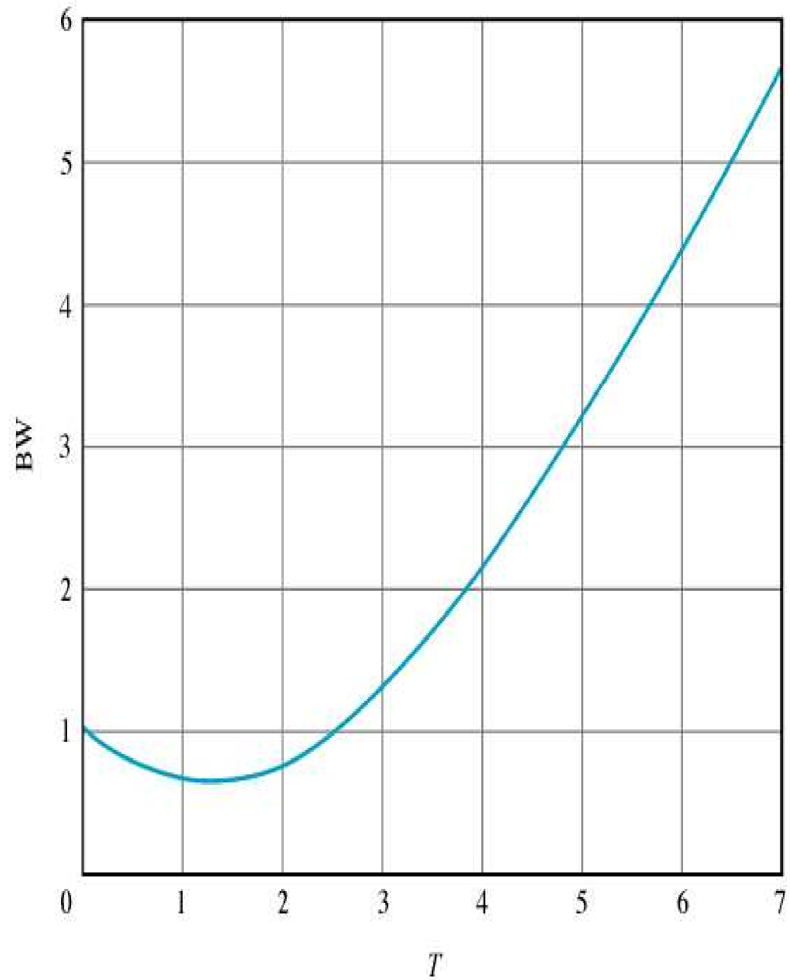
Therefore, the larger the bandwidth is, the faster the system will respond.

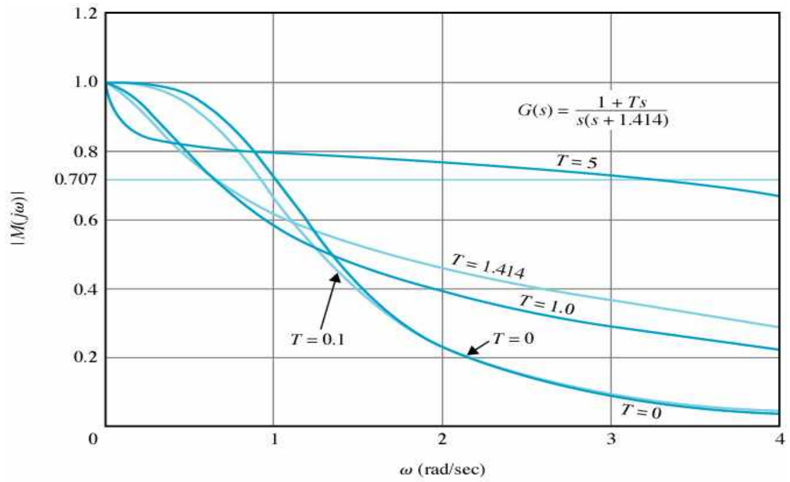
Increasing ω_n increases BW and decreases t_r.

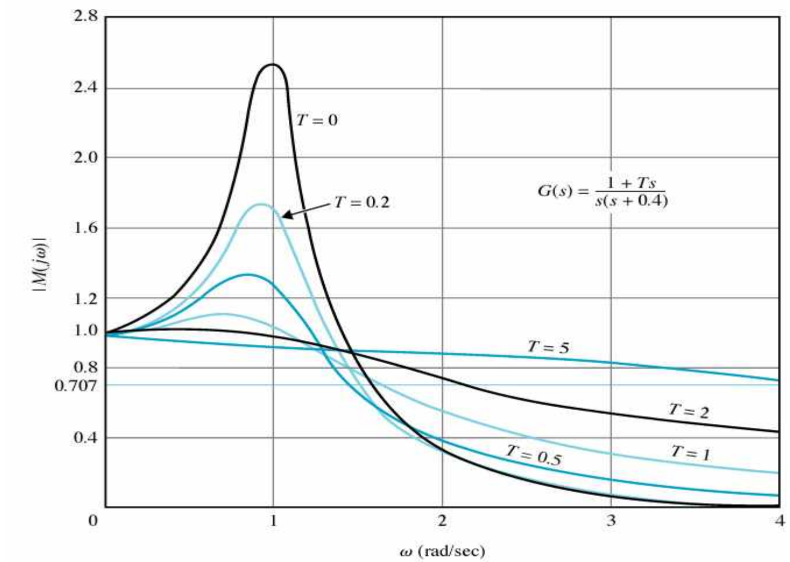
Increasing ζ decreases BW and increases t_r.

Adding zeros

- Bandwidth of a second-order system with open-loop transfer function $G(s) = (1 + Ts)/[s(s + 1.414)]$

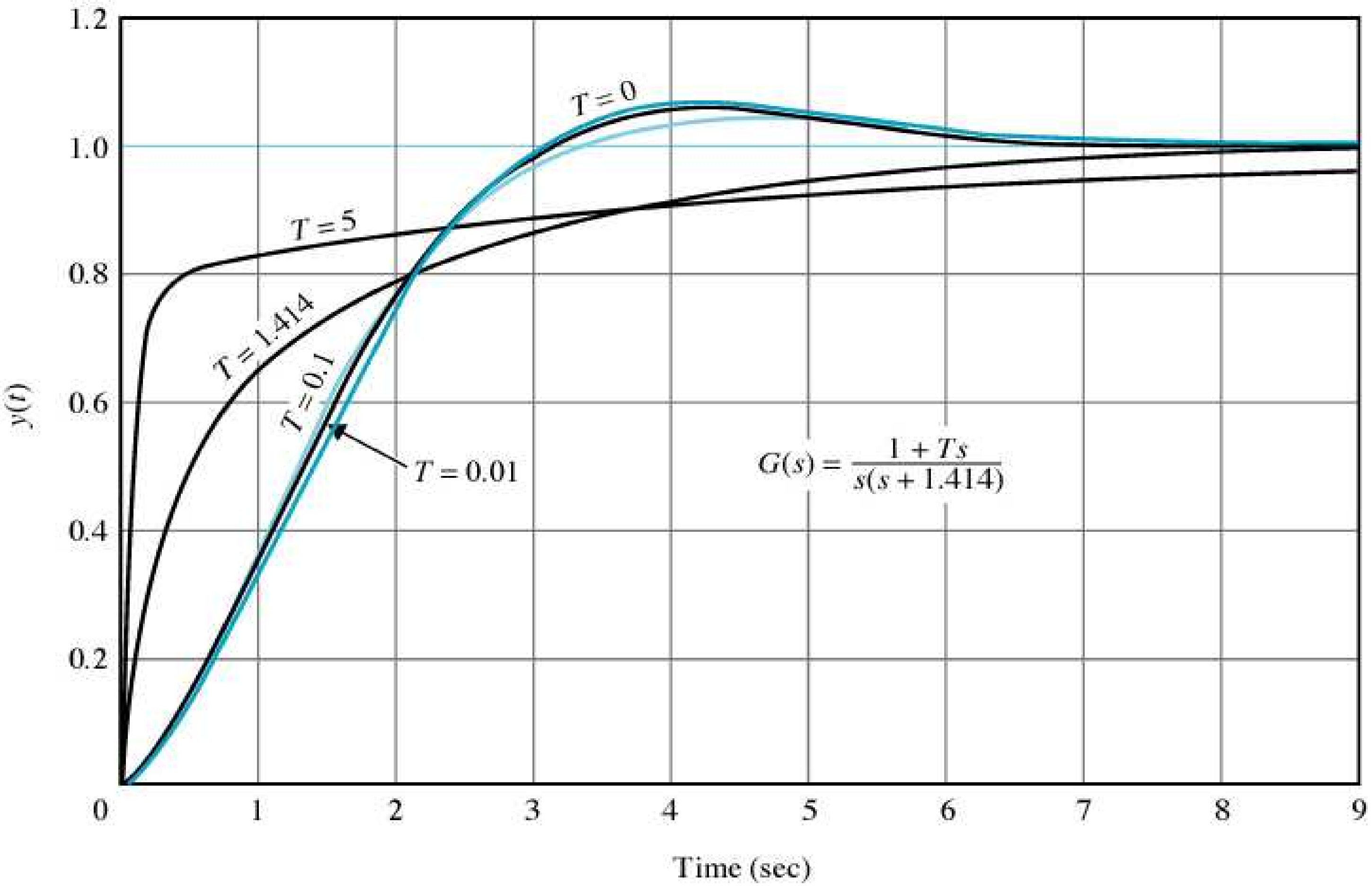




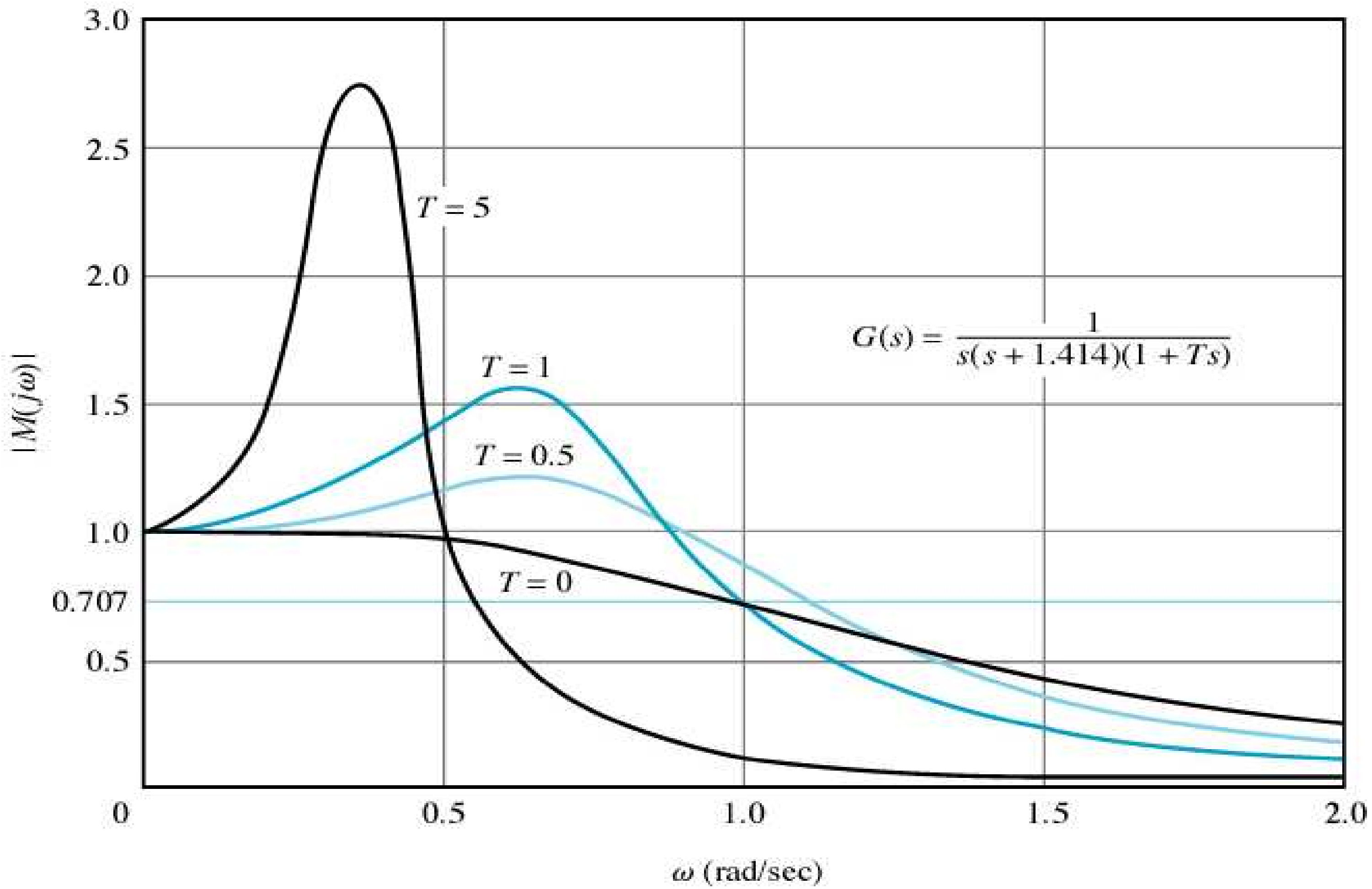


Unit-step responses with Added zero

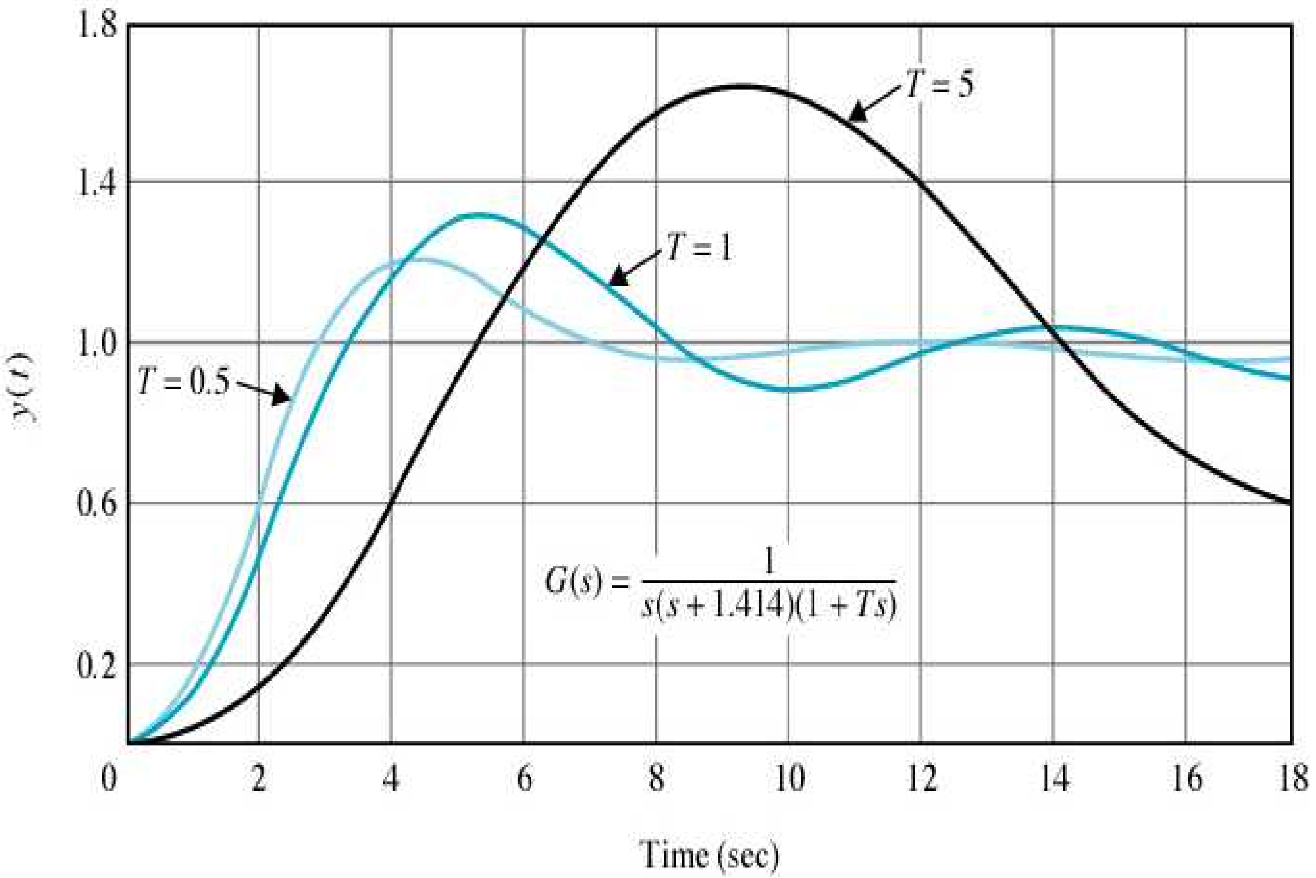
- Unit-step responses of a second-order system with a forward-path transfer function $G(s)$.



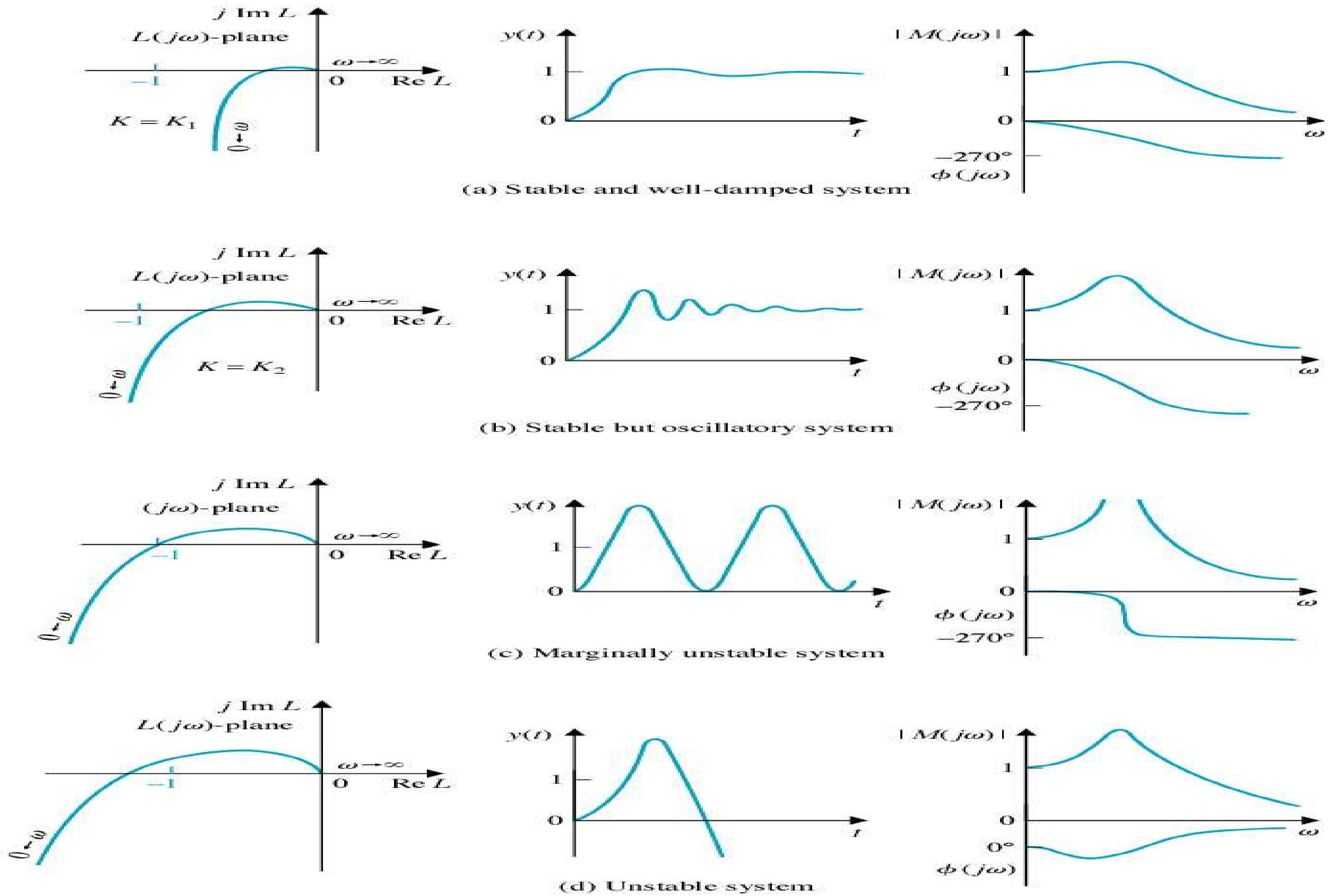
Adding pole ; Magnification curves



Adding a Pole : Unit-step responses

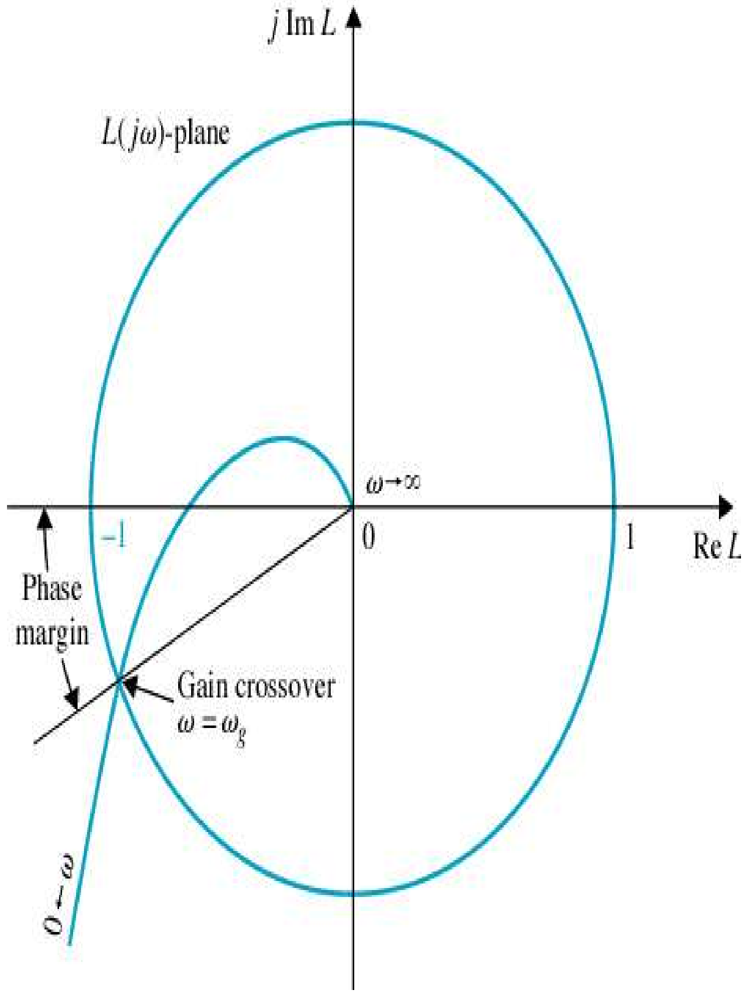


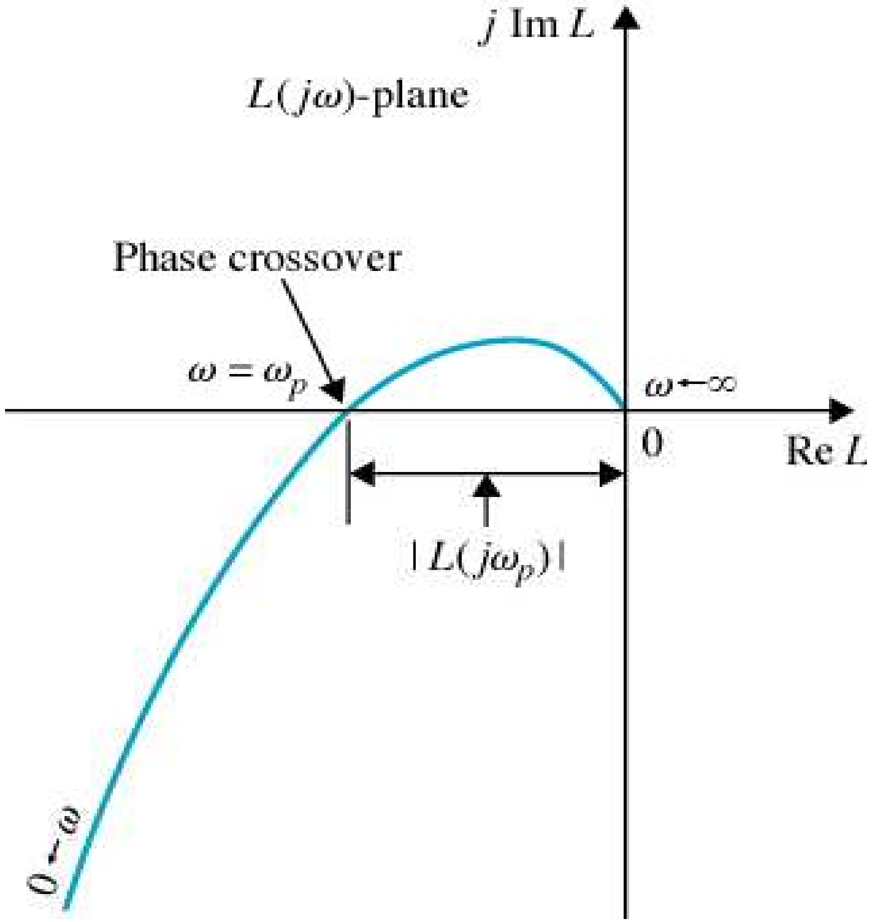
Nyquist Plot of L(s)



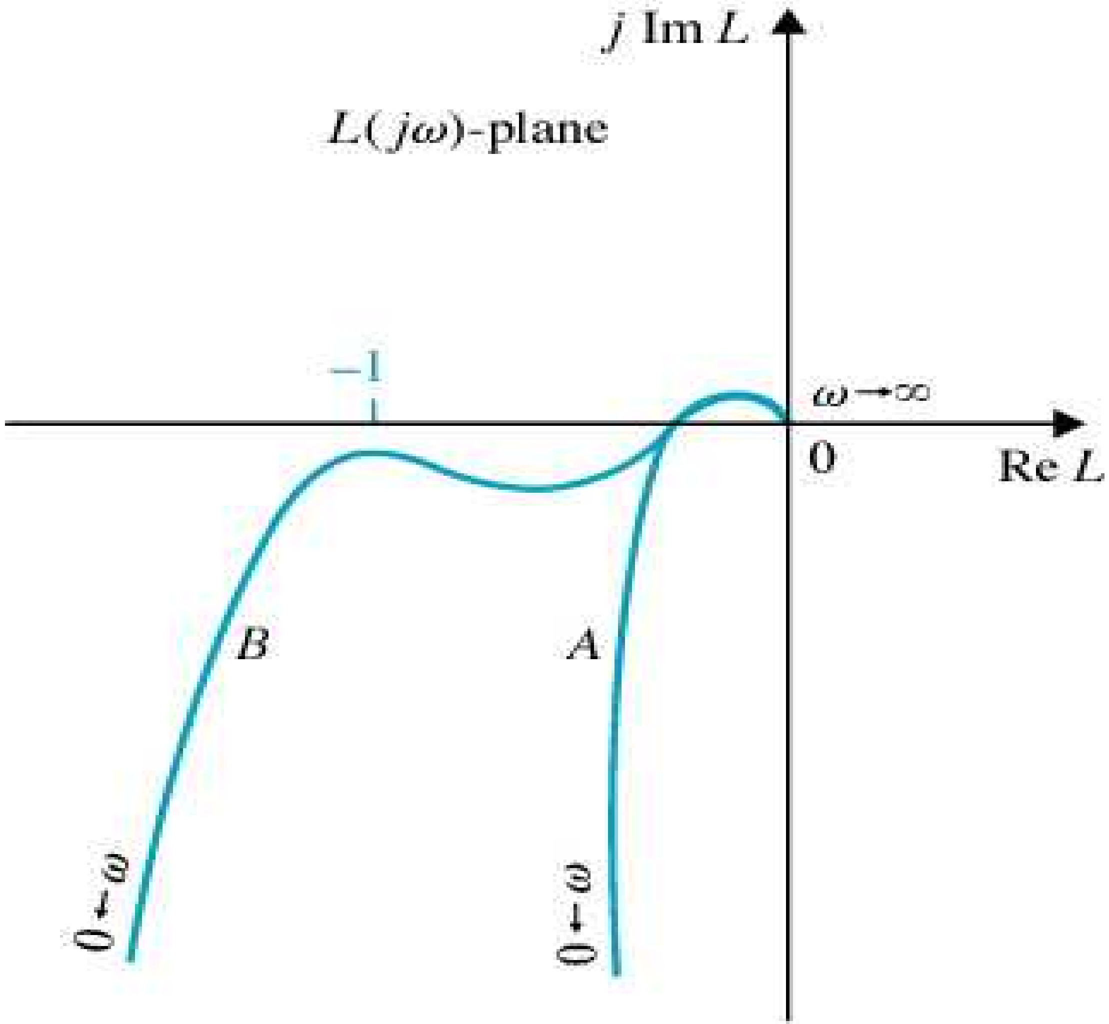
(a) Stable and well-damped system
(b) Stable but oscillatory system
(c) Marginally unstable system
(d) Unstable system

Gain Margin & Phase Margin

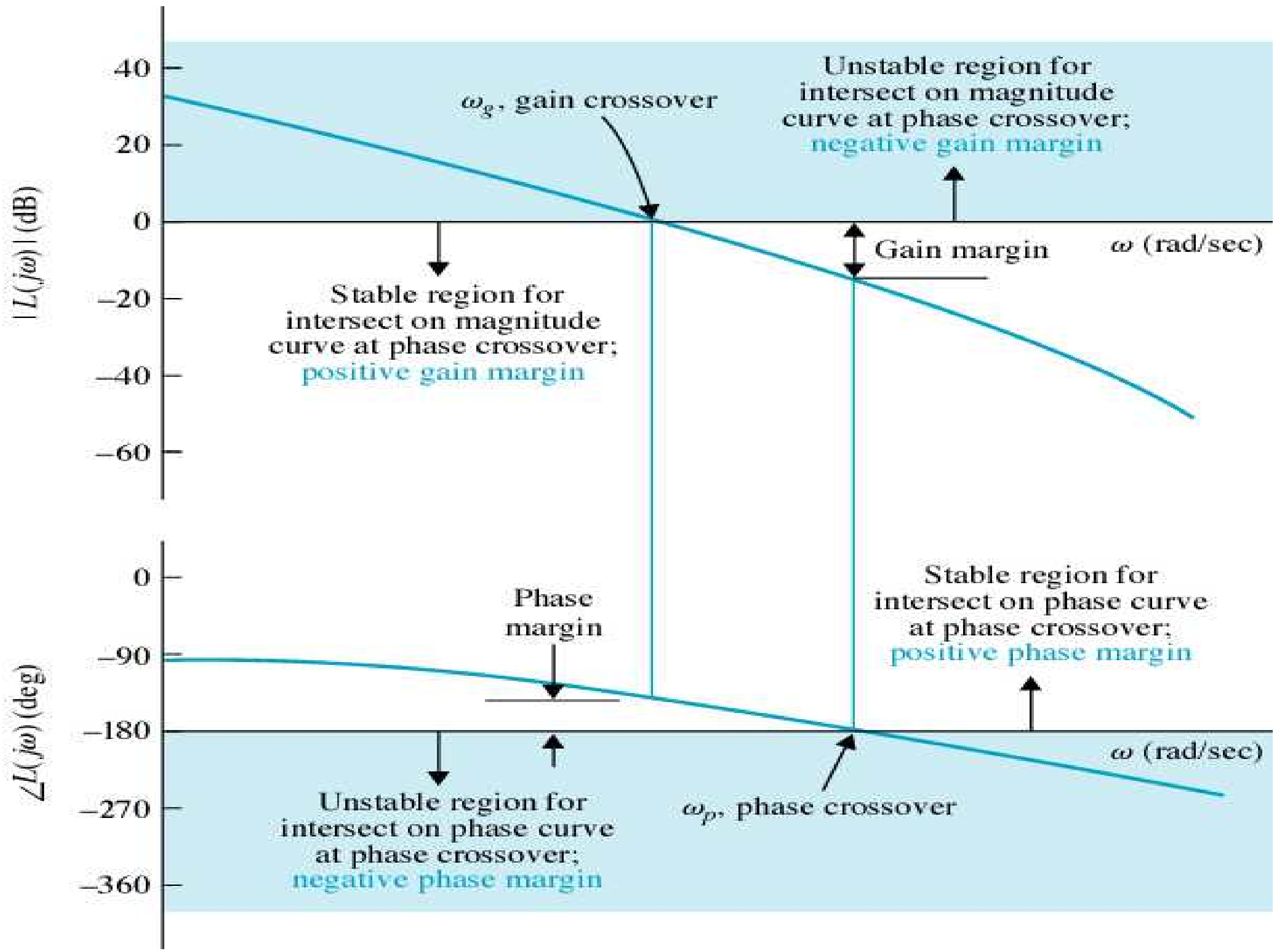




Relative Stability



Gain Margin & Phase Margin in Bode Plot



Bode plot of $L(s) = \dfrac{2500}{s(s+5)(s+50)}$.