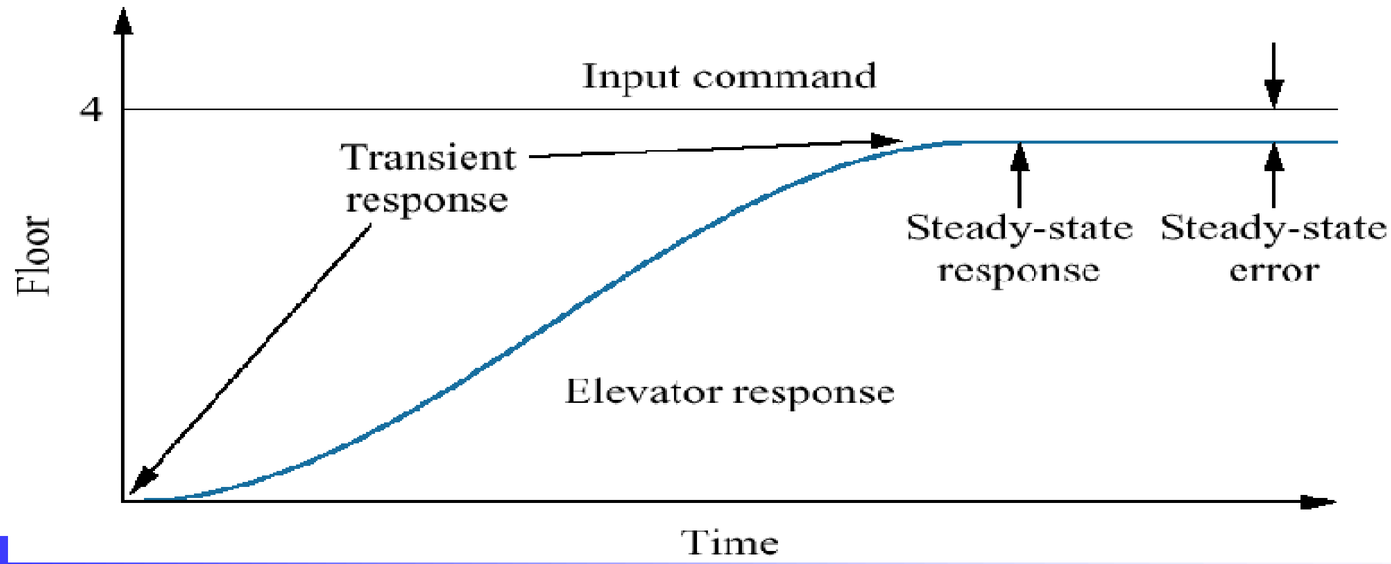
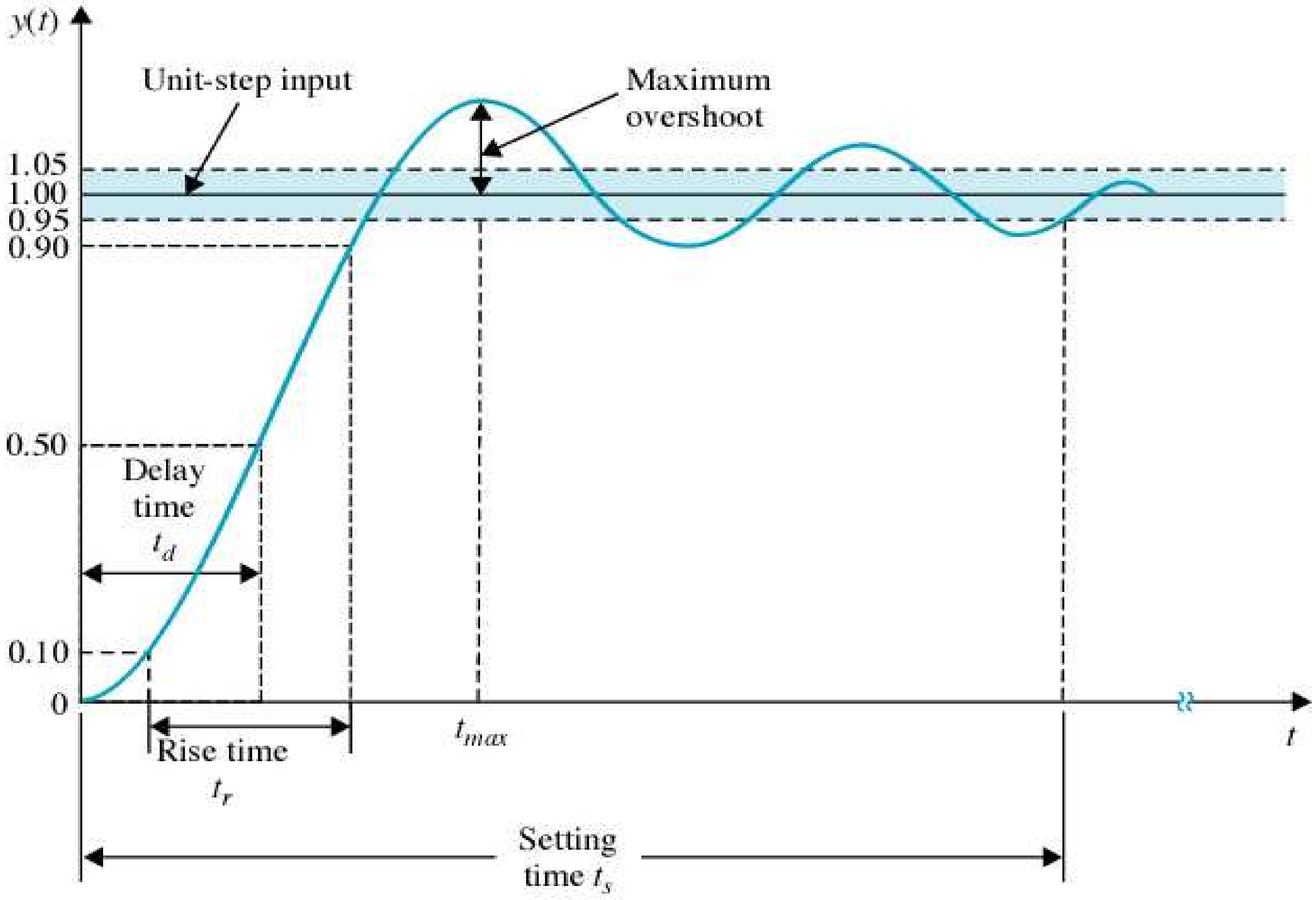
Time-Domain Analysis of Control Systems

Time Response

- Transient response
  - Gradual change of the response from its initial state
- Steady-state response
  - Approximation to the desired response
  - Steady-state error
    - e_{ss} = Desired response – Steady-state response



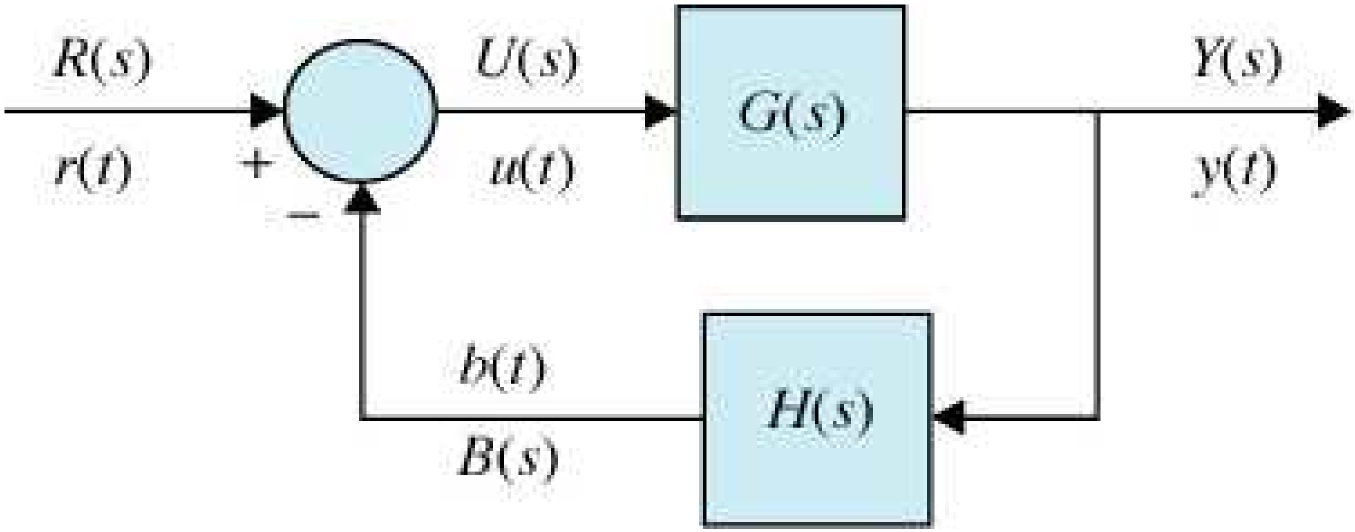
Unit Step Response & Time Domain Specs



Steady-State Error

- Error : $e(t) = r(t) - y(t)$

- Steady State Error : $e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s)$



Type of Control Systems

- Steady State Error :

$$e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{sR(s)}{1 + G(s)H(s)}$$

- System Type : The number of poles of G(s)H(s) at s=0

$$G(s)H(s) = \frac{(s+z_1)(s+z_2)\cdots}{s^n(s+p_1)(s+p_2)\cdots}$$

Step Error Constant

- Steady State Error for Unit Step Function

$$e(\infty) = \lim_{s\to 0}\frac{sR(s)}{1+G(s)H(s)} = \lim_{s\to 0}\frac{R}{1+G(s)H(s)} = \frac{R}{1+\lim_{s\to 0}G(s)H(s)}$$

- Step Error Constant : Kp

$$K_p = \lim_{s\to 0}G(s)H(s)$$

- Type 0 : $e_{ss} = \frac{R}{1+Kp}$
- Type 1 : $e_{ss} = 0$ since $Kp = \infty$

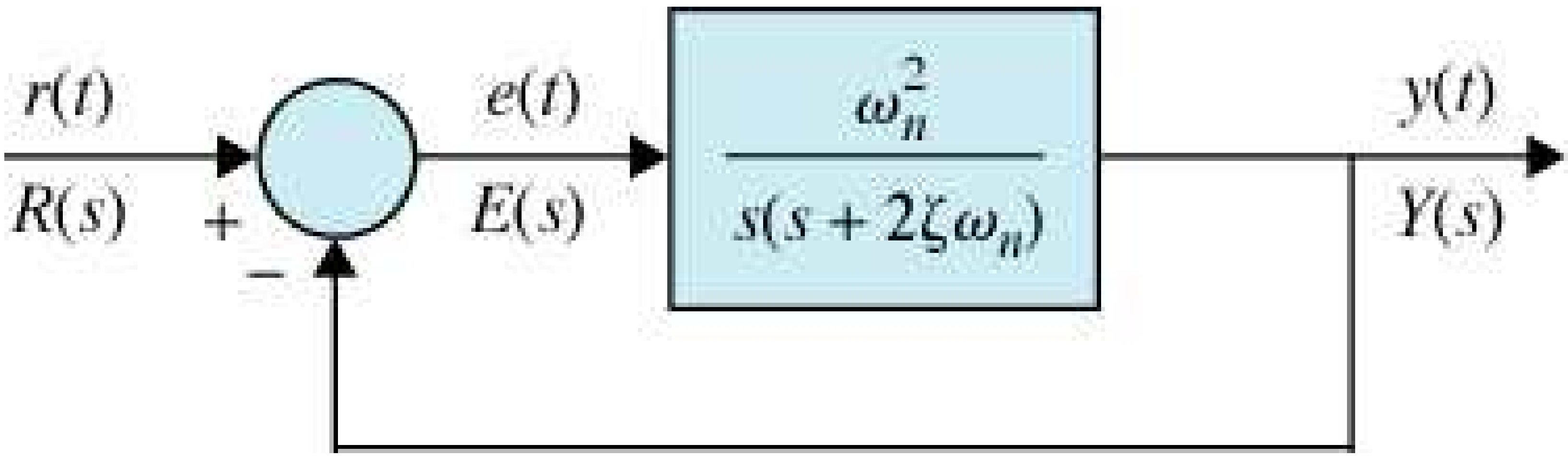
7-6. Second-Order System

- Second-Order System

$$G(s) = \frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

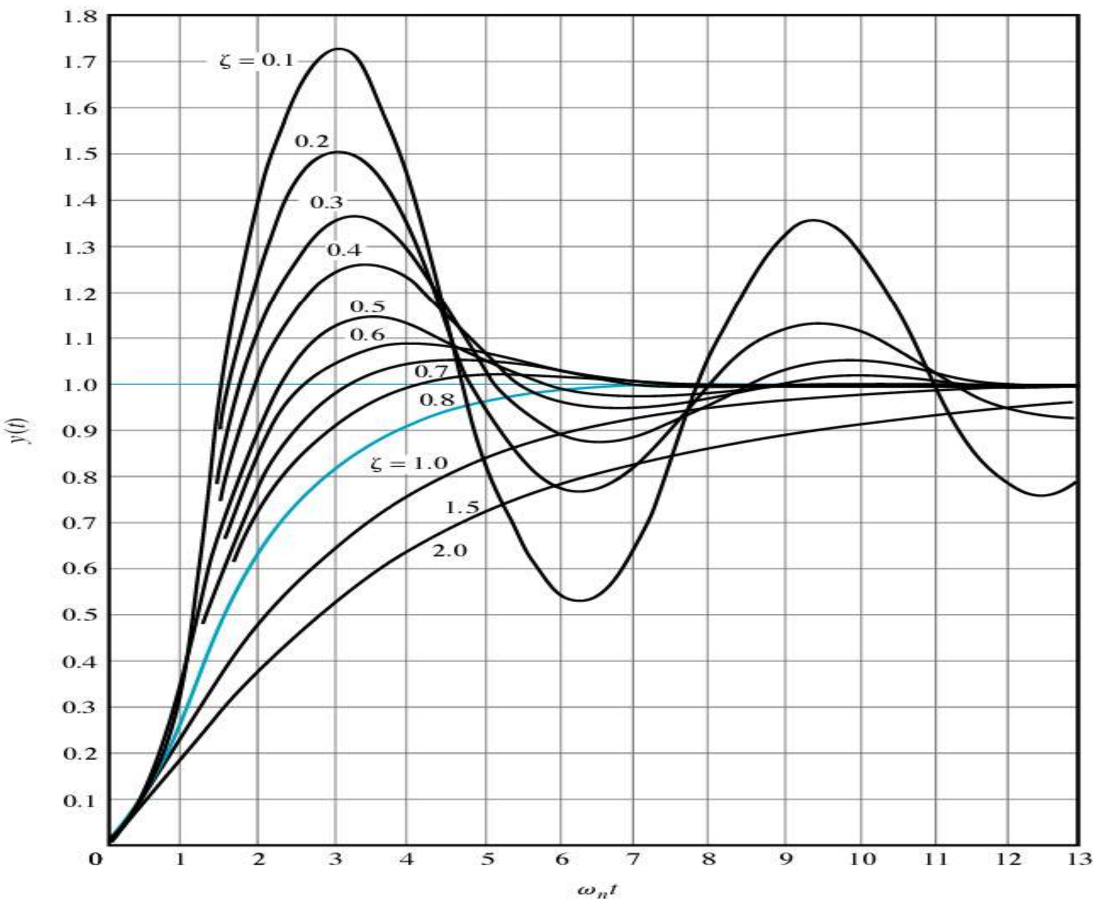
- Example : RLC Circuit

$$LC\frac{d^2 v_c(t)}{dt^2} + RC\frac{dv_c(t)}{dt} + v_c(t) = v_s(t) \qquad \left(LCs^2 + RCs + 1\right)V_c(s) = V(s)$$



Unit Step Response

- Poles : $s = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$
  $= -\sigma \pm j\omega$

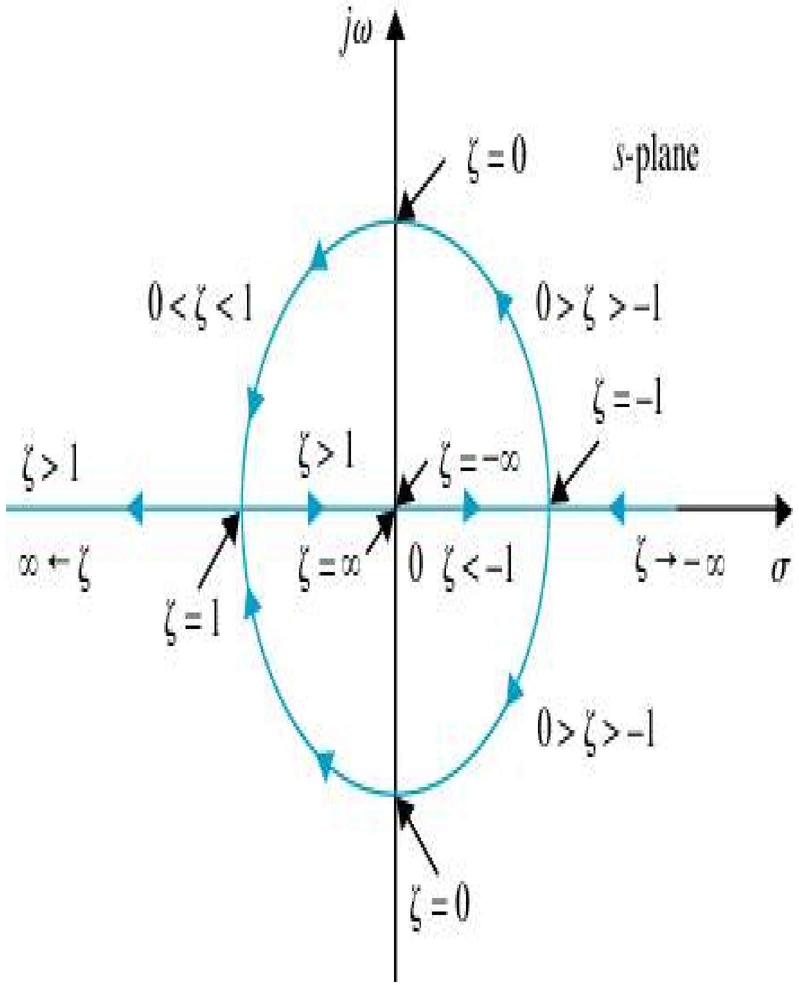


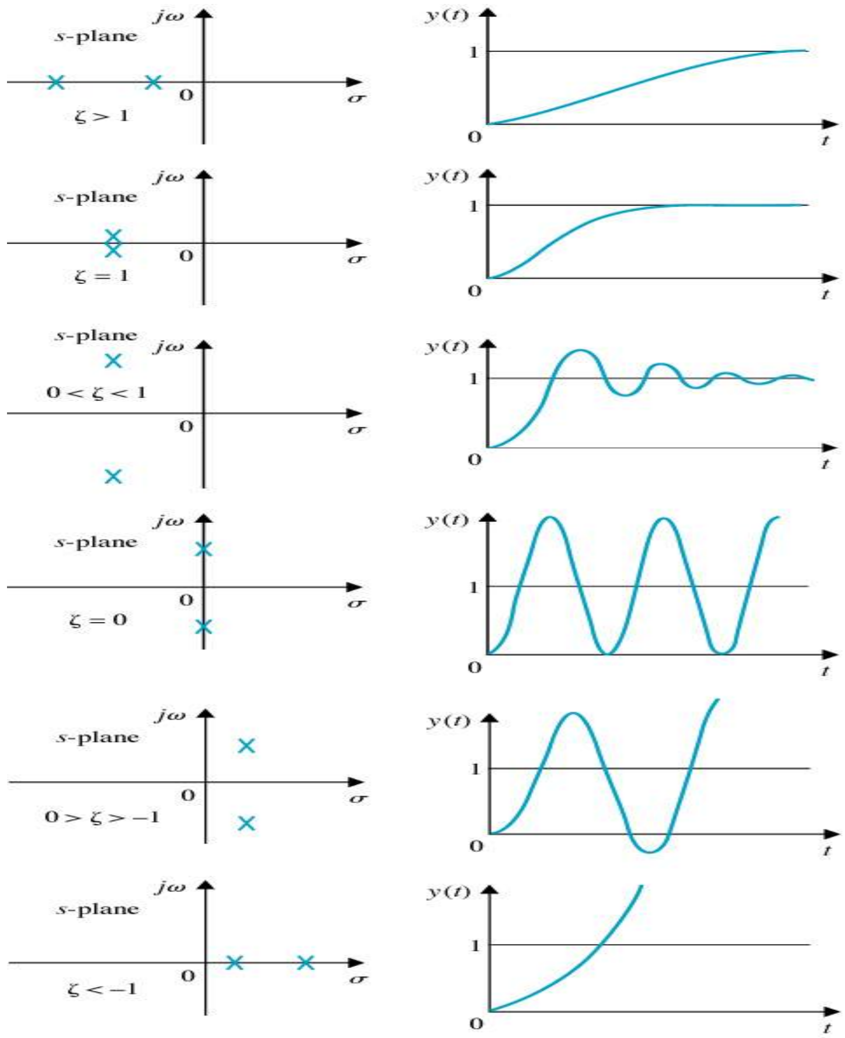
Damping

- Damping Factor : $\xi\omega_n$

- Damping : Damping ratio ξ
  - Overdamping : $\xi > 1$
  - Critical damping : $\xi = 1$
  - Underdamping : $\xi < 1$

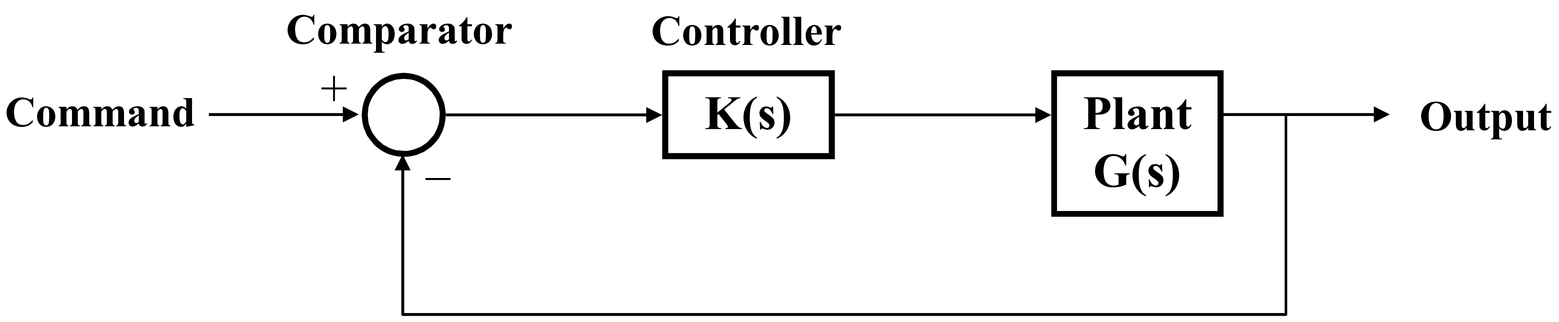
Relation between System Poles & Response





Control System Block Diagram

- Control system block diagram



How to implement

(1) Comparator ?

(2) Controller ?

Control System Example

- TE cooler
  - It converts the electrical energy to thermal energy.
  - Model

$$\frac{dT}{dt} = -k(T(t) - T_i(t) - T_a(t))$$

  - Laplace Trasform

$$sT(s) – T(0) = -k\ T(s) + k\ Ti\ (s) + k\ Ta(s)$$

  - Transfer function of TE Cooler : Ta = 0

$$G(s) = T(s)/Ti(s) = k/(s+k)$$

TEC Control

- Feedback Control : PI (Proportional & Integral) Control

$$T_i(t) = k_p(T(t) - T_s) + k_i \int_0^t (T(t') - T_s)dt'$$

- Transfer Function of PI Control

$$K(s) = \frac{T_i(s)}{E(s)}$$

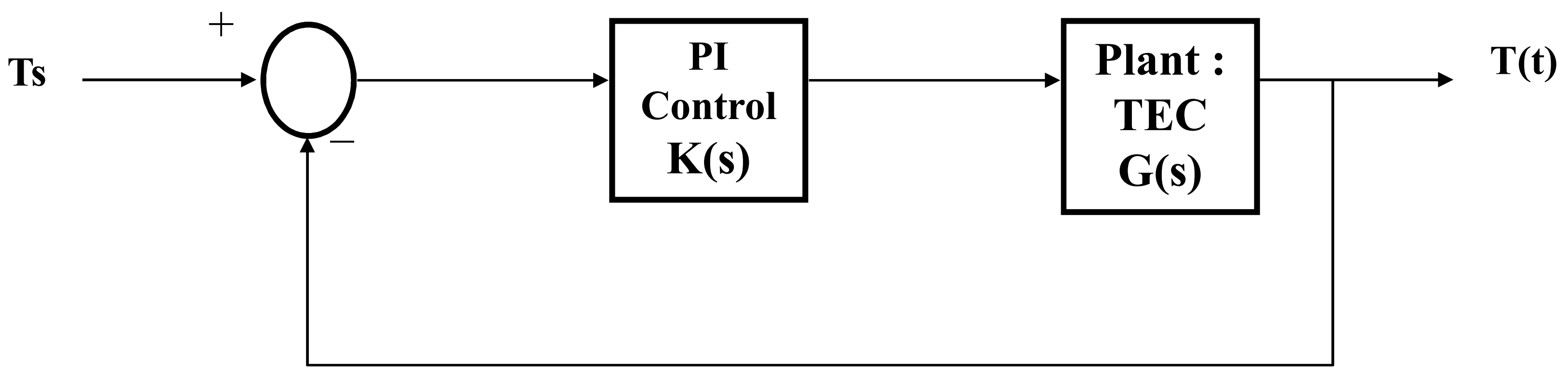
$$= k_p + \frac{k_i}{s}$$

where E(s) is the Laplace transform of the error,

e(t) = T(t) – Ts.

Closed-Loop Transfer Function for TEC Control

- Block Diagram



- Closed-Loop Transfer Function

$$G_{CL}(s) = \frac{T(s)}{T_s(s)}$$

$$= \frac{G(s)K(s)}{1 + G(S)K(s)}$$

$$= \frac{k(k_p s + k_i)}{s^2 + k(1 + k_p)s + kk_i}$$