제 3장 정상 열전도

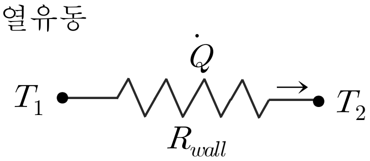
3-1 평면 벽에서의 정상 열전도

$$\dot{Q}_{cond,wall} = kA\frac{T_1 - T_2}{L}$$

열저항 개념

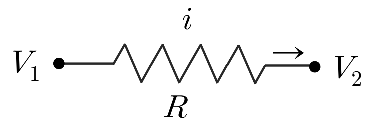
$$\dot{Q}_{cond,wall} = \frac{T_1 - T_2}{R_{wall}} \qquad R_{wall} = \frac{L}{kA}$$ (R_{wall}: 열저항 또는 전도저항)

i) 열유동



$$\dot{Q} = \frac{T_1 - T_2}{R_{wall}}$$

ii) 전류



$$i = \frac{V_1 - V_2}{R}$$

$$i \leftrightarrow \dot{Q}$$
$$V \leftrightarrow T$$
$$R \leftrightarrow R_{wall}$$

$$\dot{Q} = hA(T_s - T_\infty) = \frac{T_s - T_\infty}{R_{conv}} \qquad R_{conv} = \frac{1}{hA}$$

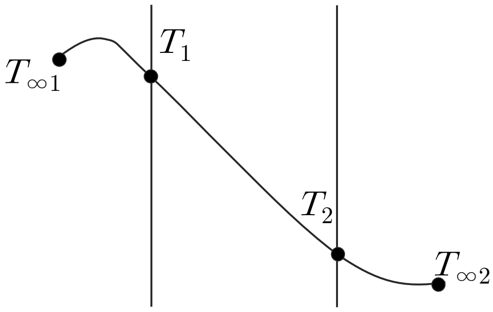
$$\dot{Q}_{rad} = \epsilon\sigma A(T_s^4 - T_{surr}^4) = h_{rad}A(T_s - T_{surr})$$

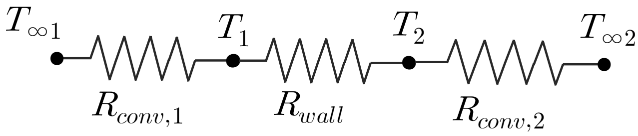
$$= \frac{T_s - T_{surr}}{R_{rad}} \qquad R_{rad} = \frac{1}{h_{rad}A}$$

$$h_{rad} = \frac{\dot{Q}_{rad}}{A(T_s - T_{surr})} = \epsilon\sigma(T_s^2 + T_{surr}^2)(T_s + T_{sur})$$

열저항 회로

$$\dot{Q} = h_1 A(T_{\infty 1} - T_1) = kA\frac{T_1 - T_2}{L} = h_2 A(T_2 - T_{\infty 2})$$





$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{conv,1} + R_{wall} + R_{conv,2}} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

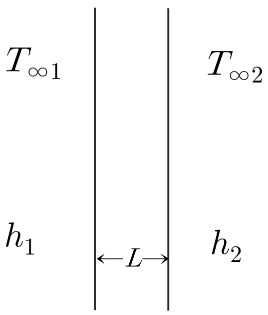
$$= \frac{T_{\infty 1} - T_1}{1/h_1 A} = \frac{T_1 - T_2}{L/kA} = \frac{T_2 - T_{\infty 2}}{1/h_2 A}$$

열관류율

$\dot{Q} = UA\Delta T \qquad U$: 열관류율

$$UA = \frac{1}{R_{total}}$$

(Ex 3-2) 한 장의 유리 창문을 통한 열손실



$$R_i = R_{conv,1} = \frac{1}{h_1 A}$$

$$R_{glass} = \frac{L}{kA}$$

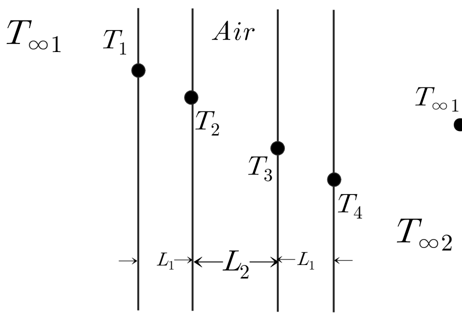
$$R_o = R_{conv,2} = \frac{1}{h_2 A}$$

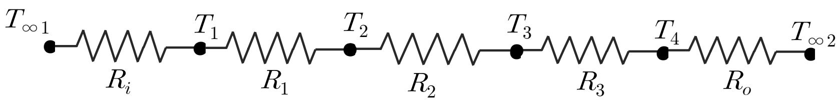
$$R_{total} = R_i + R_{glass} + R_o$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = 266(W)$$

$$= \frac{T_{\infty 1} - T_1}{R_{conv,1}} \quad \therefore T_1 = T_{\infty 1} - \dot{Q} R_{conv,1}$$

(Ex 3-3) 2중 유리창을 통한 열손실





$$R_i = R_{conv,1} = \frac{1}{h_1 A}$$

$$R_i = R_3 = R_{glass} = \frac{L_1}{k_1 A}$$

$$R_2 = R_{air} = \frac{L_2}{k_2 A}$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A}$$

$$R_{total} = R_i + R_1 + R_2 + R_3 + R_o$$

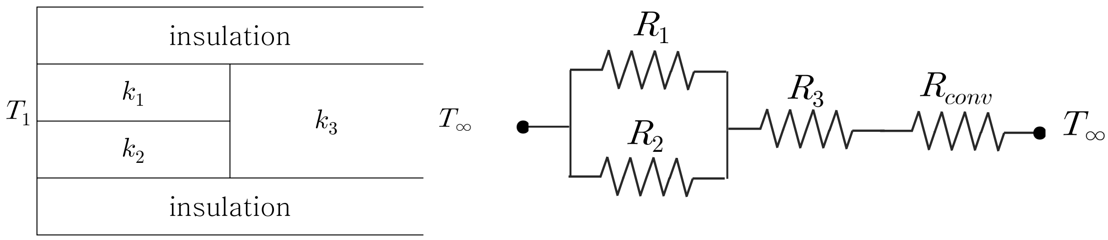
$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{T_{\infty 1} - T_1}{R_{conv,1}}$$

3-2 열 접촉 저항

$$R_c = \frac{1}{h_c} = \frac{\Delta T_{interface}}{\dot{Q}/A} \qquad (h_c : \text{열 접촉 컨덕턴스})$$

$$R_{interface} = \frac{1}{h_c A_c} \qquad (A_c : \text{접촉면})$$

3-3 열저항 회로의 일반화



3-4 원형관과 구에서의 열전도

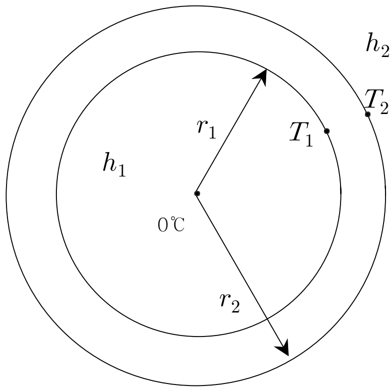
$$\dot{Q}_{cond,\,cyl} = -kA\frac{dT}{dr} \qquad A = 2\pi rL$$

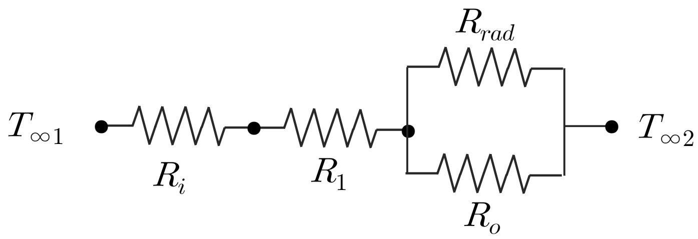
$$\dot{Q}_{cond,cyl} = 2\pi Lk\frac{T_1 - T_2}{\ln(r_2/r_1)} = \frac{T_1 - T_2}{R_{cyl}}$$

$$R_{cyl} = \frac{\ln(r_2/r_1)}{2\pi Lk}$$

$$R_{sph} = \frac{r_2 - r_1}{4\pi r_1 r_2 k}$$

(Ex 3-7) 구형탱크로의 열전달





$A_1 = \pi D_1^2 \qquad A_2 = \pi D_2^2$

$h_{rad} = \epsilon\sigma(T_2^2 + T_{\infty 2}^2)(T_2 + T_{\infty 2})$

$$R_i = R_{conv,1} = \frac{1}{hA_1}$$

$$R_1 = R_{sph} = \frac{r_2 - r_1}{4\pi k r_1 r_2}$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A_2}$$

$$R_{rad} = \frac{1}{h_{rad} A_2}$$

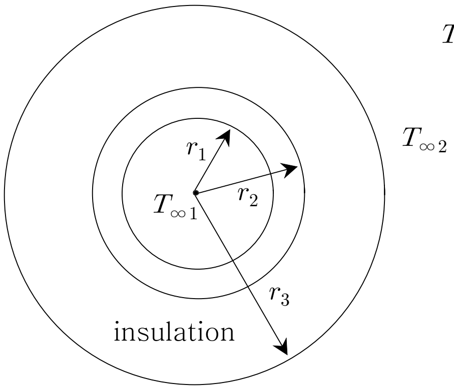
$$\frac{1}{R_{eq}} = \frac{1}{R_o} + \frac{1}{R_{rad}}$$

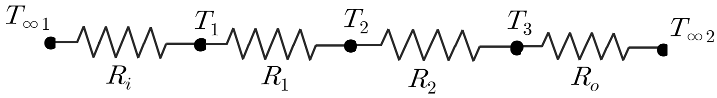
$R_{total} = R_i + R_1 + R_{rad}$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{T_{\infty 2} - T_2}{R_{eq}}$$

$$Q = \dot{Q}\Delta t \qquad m_{ice} = \frac{Q}{h_{iq}}$$

(Ex 3-8) 단열된 증기관을 통한 열손실





$A_1 = 2\pi r_1 L$
$A_3 = 2\pi r_3 L$

$$R_i = R_{conv,1} = \frac{1}{hA_1}$$

$$R_1 = R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi k_1 L}$$

$$R_2 = R_{insulation} = \frac{\ln(r_3/r_2)}{2\pi k_2 L}$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A_3}$$

$$R_{total} = R_i + R_1 + R_2 + R_o$$

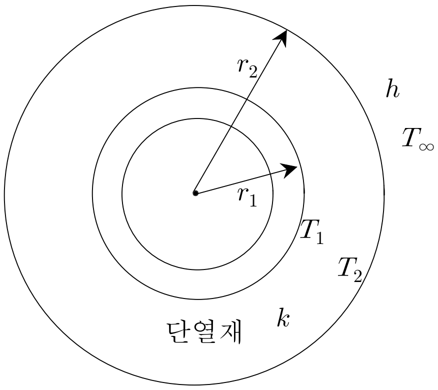
$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

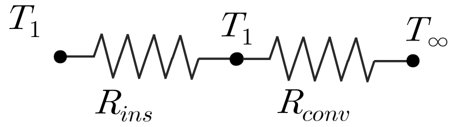
$$\Delta T_{pipe} = \dot{Q} R_{pipe}$$

$$\Delta T_{insulation} = \dot{Q} R_{insulation}$$

3-5 임계 단열 반지름

원통이나 구형 셀에 단열재를 추가할 때 열전달률이 최대가 되는 반지름





$$\dot{Q} = \frac{T_1 - T_\infty}{R_{ins} + R_{conv}} = \frac{T_1 - T_2}{\dfrac{\ln(r_2/r_1)}{2\pi Lk} + \dfrac{1}{h(2\pi)}}$$

$$\frac{d\dot{Q}}{dr} = 0 \quad \Rightarrow \quad r_{cr,cylinder} = \frac{k}{h}$$

$$r_{cr,\, sphere} = \frac{2k}{h}$$

3-6 단열

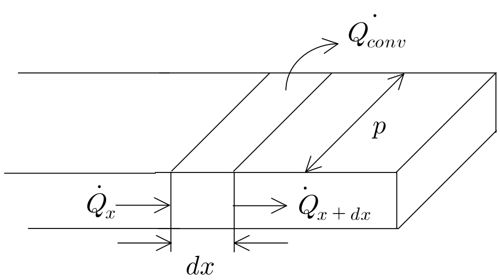
단열재는 일차적으로 열유동에 저항을 주기위하여 이용되는 물질

단열재의 선택시 고려사항은 목적, 환경, 취급과 설치의 용이 비용등

단열재의 유효도는

$$R = \frac{L}{R} \quad \text{(평판단열)}$$

3-7 휜표면으로 부터의 열전달



$$\dot{Q}_{cond,x} = \dot{Q}_{cond,x+\Delta x} + \underline{\underline{\dot{Q}_{conv}}}$$

$$h(p\Delta x)(T - T_{\infty})$$

$$\therefore \quad \frac{\dot{Q}_{cond,x+\Delta x} - \dot{Q}_{cond,x+\Delta x}}{\Delta x} + hp(T - T_{\infty}) = 0$$

$$\frac{d\dot{Q}_{cond}}{dx} + hp(T - T_{\infty}) = 0$$

$\dot{Q}_{cond} = -kA_c \frac{dT}{dx}$ 이므로

$$\frac{d}{dx}(kA_c \frac{dT}{dx}) - hp(T - T_{\infty}) = 0$$

$a^2 = \frac{hp}{kA_c} \quad \theta = T - T_{\infty}$ 라면

$$\frac{d^2\theta}{dx^2} - a^2\theta = 0$$

$$\theta = C_1 e^{ax} + C_2 e^{-ax}$$

경계조건

i) $\theta(0) = \theta_b = T_b - T_{\infty}$

ii) a) 무한히 긴 휜 ($T_{fin\ tip} = T_{\infty}$) 경우

$L \rightarrow \infty \qquad \theta(L) = 0$

b) 단열된 휜 끝 ($\dot{Q}_{fin\ tip} = 0$)

$$\frac{d\theta}{dx}|_{x=L} = 0$$

c) 휜 끝에서의 대류

$$-kA\frac{dT}{dx})_{x=L} = hA(T_L - T_{\infty})$$

i) 무한히 긴 휜

$$\frac{\theta}{\theta_b} = e^{-ax}$$

$$\dot{Q} = -kA_c \frac{dT}{dx}|_{x=0} = \sqrt{hpkA}\,\theta_b$$

ii) 단열된 휜 끝

$$\frac{\theta}{\theta_b} = \frac{\cosh a(L-x)}{\cosh aL}$$

$$\dot{Q} = -kA_c \frac{dT}{dx}|_{x=0} = \sqrt{hpkA_c}\,\theta_b \tanh aL$$

iii) 휜 끝에서의 대류

$$\frac{\theta}{\theta_b} = \frac{\cosh a(L-x) + \frac{h}{ak} sinh\, a(L-x)}{\cosh aL + \frac{h}{ak} sinh\, aL}$$

그러나 이식보다 수정길이 L_c 를 도입하여 ii)의 단열된 휜 끝공식을 사용한다.

$$L_c = L + \frac{A_c}{p}$$

사각단면 $A_c = wt$, $p = 2w$

원통형 $A_c = \frac{\pi}{4} D^2$, $p = \pi D$

휜 효율

$$\eta_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{fin\max}} = \frac{\text{휜으로부터 실제 열전달}}{\text{휜 전체가 바닥의 온도와 같을 때 열전달}}$$

i) 무한히 긴 휜 열효율

$$\eta = \frac{\sqrt{hpkA_c}(T_b - T_\infty)}{hA_{fin}(T_b - T_\infty)} \qquad A_{fin} = pL$$

ii) 단열된 휜 끝의 열효율

$$\eta = \frac{\sqrt{hpkA_c}(T_b - T_\infty)\tanh aL}{hA_{fin}(T_b - T_\infty)}$$

휜 유효도

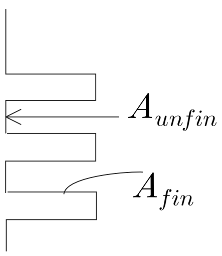
$$\epsilon_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\,fin}} = \frac{\dot{Q}_{fin}}{hA_b(T_b - T_\infty)}$$

$$= \frac{\eta_{fin} hA_{fin}(T_b - T_\infty)}{hA_b(T_b - T_\infty)} = \frac{A_{fin}}{A_b}\eta_{fin}$$

총괄 유효도

$$\dot{Q}_{total,fin} = \dot{Q}_{unfin} + \dot{Q}_{fin}$$

$$= hA_{unfin}(T_b - T_\infty) + \eta_{fin} hA_{fin}(T_b - T_\infty)$$



$$\epsilon_{fin,overall} = \frac{\dot{Q}_{total,fin}}{Q_{total,nofin}}$$

3-8, 일반 형태에서의 열전달

$\dot{Q} = Sk(T_1 - T_2)$ S: 전도형상계수 , k: 표면사이 매체 열전도