

# LECTURE 4: Production 생산이론

- The Technology of Production
- Production with One Variable Input (Labor)
- Isoquants
- Production with Two Variable Inputs
- Returns to Scale

# The Theory of Production 생산이론

- The Theory of Production 생산이론
  - describes how a firm makes cost-minimizing production decisions and how the firm's resulting cost varies with its output.
- The Production Decisions of a Firm
  - are analogous to the purchasing decisions of consumers, and can likewise be understood in three steps:
    1. Production Technology
    2. Cost Constraints
    3. Input (생산요소) Choices

# Production Decisions of a Firm

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## 1. Production Technology

- Describe how *inputs* can be transformed into *outputs*
  - Inputs: land, labor, capital & raw materials
  - Outputs: cars, desks, books, etc.
- Firms can produce different amounts of outputs using different combinations of inputs

# Production Decisions of a Firm

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## 2. Cost Constraints

- Firms must consider *prices* of labor, capital and other inputs
- Firms want to minimize total production costs partly determined by input prices
- As consumers must consider budget constraints, firms must be concerned about costs of production

# Production Decisions of a Firm

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## 3. Input (생산요소) Choices

- Given input prices and production technology, the firm must choose *how much of each input* to use in producing output
- Given prices of different inputs, the firm may choose different combinations of inputs to minimize costs
  - If labor is cheap, may choose to produce with more labor and less capital

# Production Decisions of a Firm

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- If a firm is a cost minimizer, we can also study
  - ▣ How total costs of production varies with output
  - ▣ How does the firm choose the quantity to maximize its profits
- We can represent the firm's production technology in the form of a **production function**

# The Technology of Production

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- Production Function  $q = f(L, K)$ :
  - Indicates the highest output ( $q$ ) that a firm can produce for every specified combination of inputs.
  - For simplicity, we will consider only labor ( $L$ ) and capital ( $K$ )
  - Shows what is technically feasible when the firm operates efficiently
  - Basically it is an input-output relation  $q = f(L, K)$

# The Technology of Production

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- The production function for two inputs:

$$q = F(K,L)$$

- Contrast production function with utility function,
- Output ( $q$ ) is a function of capital ( $K$ ) and Labor ( $L$ )
- The production function is true for a given technology
  - If technology increases, more output can be produced for a given level of inputs



# The Technology of Production

- Short Run versus Long Run
  - ▣ It takes time for a firm to adjust production from one set of inputs to another
  - ▣ Firms must consider not only what inputs can be varied but over what period of time that can occur
  - ▣ We must distinguish between long run and short run

# The Short Run versus the Long Run

## The Short Run versus the Long Run

**short run:** Period of time in which quantities of one or more production factors cannot be changed.

● **fixed input** 고정생산요소

Production factor that cannot be varied.

● **variable input** 변동생산요소

Production factor that can be varied.in short run

**long run :** Amount of time needed to make all production inputs variable.

# Production: One Variable Input

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- We will begin looking at the short run when only one input can be varied
- We assume capital is fixed and labor is variable
  - ▣ Output can only be increased by increasing labor
  - ▣ Must know how output changes as the amount of labor is changed

# Production: One Variable Input

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<i>Amount of Labor (L)</i>	<i>Amount of Capital (K)</i>	<i>Total Output (q)</i>
0	10	0
1	10	10
2	10	30
3	10	60
4	10	80
5	10	95
6	10	108
7	10	112
8	10	112
9	10	108
10	10	100

# Production: One Variable Input

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## □ Observations:

1. When labor is zero, output is zero as well
2. With additional workers, output ( $q$ ) increases up to 8 units of labor.
3. Beyond this point, output declines
  - Increasing labor can make better use of existing capital initially
  - After a point, more labor is not useful and can be counterproductive

# Production: One Variable Input

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- Firms make decisions based on the benefits with the costs of production
- Sometimes useful to look at benefits and costs on an *incremental basis*
  - ▣ How much more can be produced when at incremental units of an input
- Sometimes useful to make comparison on an *average basis*

# Production: One Variable Input

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- AP  $\frac{Q}{L}$  : Average product of Labor - Output per unit of a particular product
- Measures the productivity of a firm's labor in terms of how much, on average, each worker can produce

$$AP = \frac{\text{Output}}{\text{Labor Input}} = \frac{q}{L}$$

# Production: One Variable Input

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- Marginal Product of Labor – additional output produced when labor increases by one unit
- Change in output divided by the change in labor

$$MP_L = \frac{\Delta Output}{\Delta Labor Input} = \frac{\Delta q}{\Delta L}$$



# Average and Marginal Products

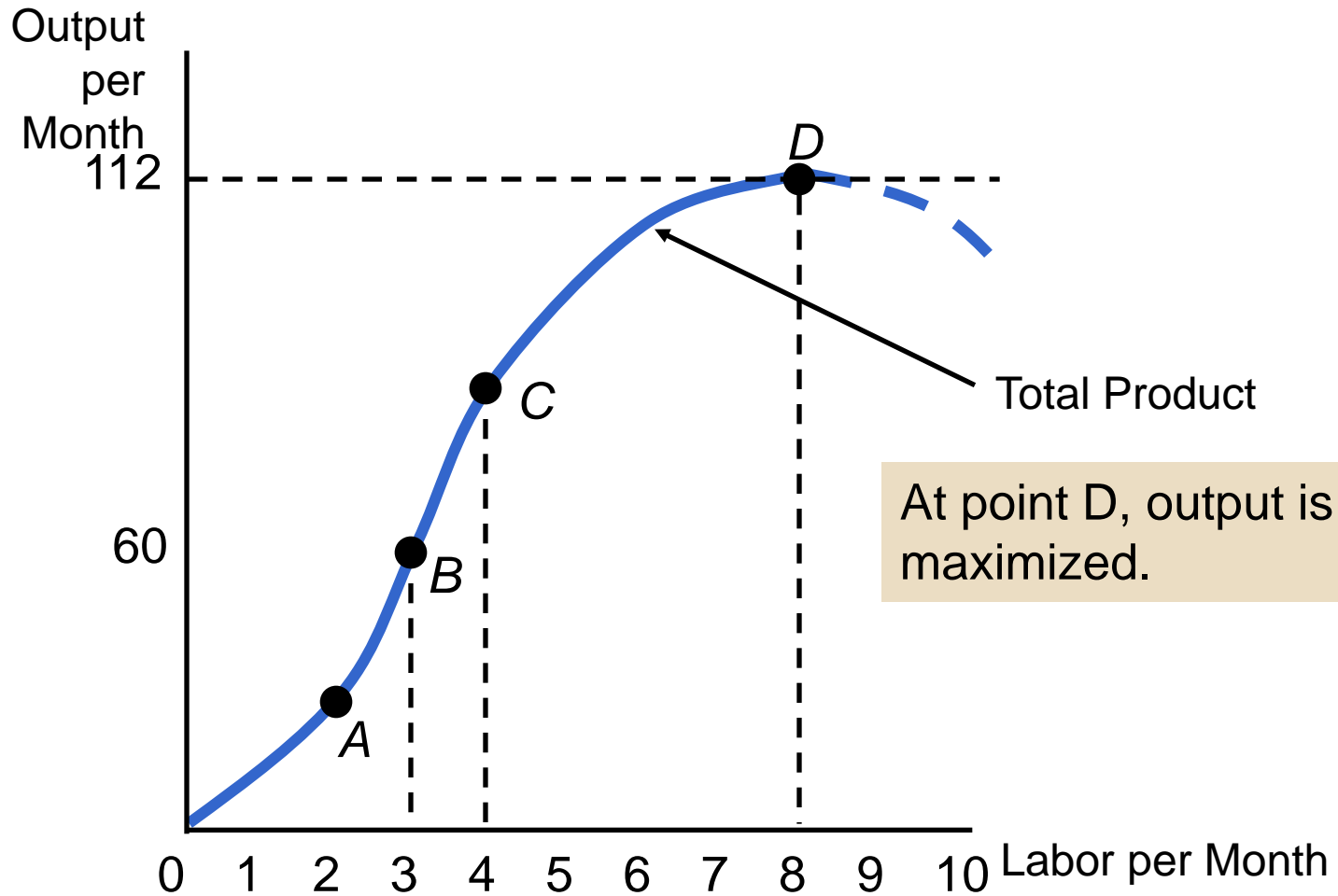
- **AP: average product** 평균생산
  - Output per unit of a particular input.
- **MP: marginal product** 한계생산
  - Additional output produced as an input is increased by one unit.

Average product of labor = Output/labor input =  $q/L$

Marginal product of labor = Change in output/change in labor input  
=  $\Delta q/\Delta L$

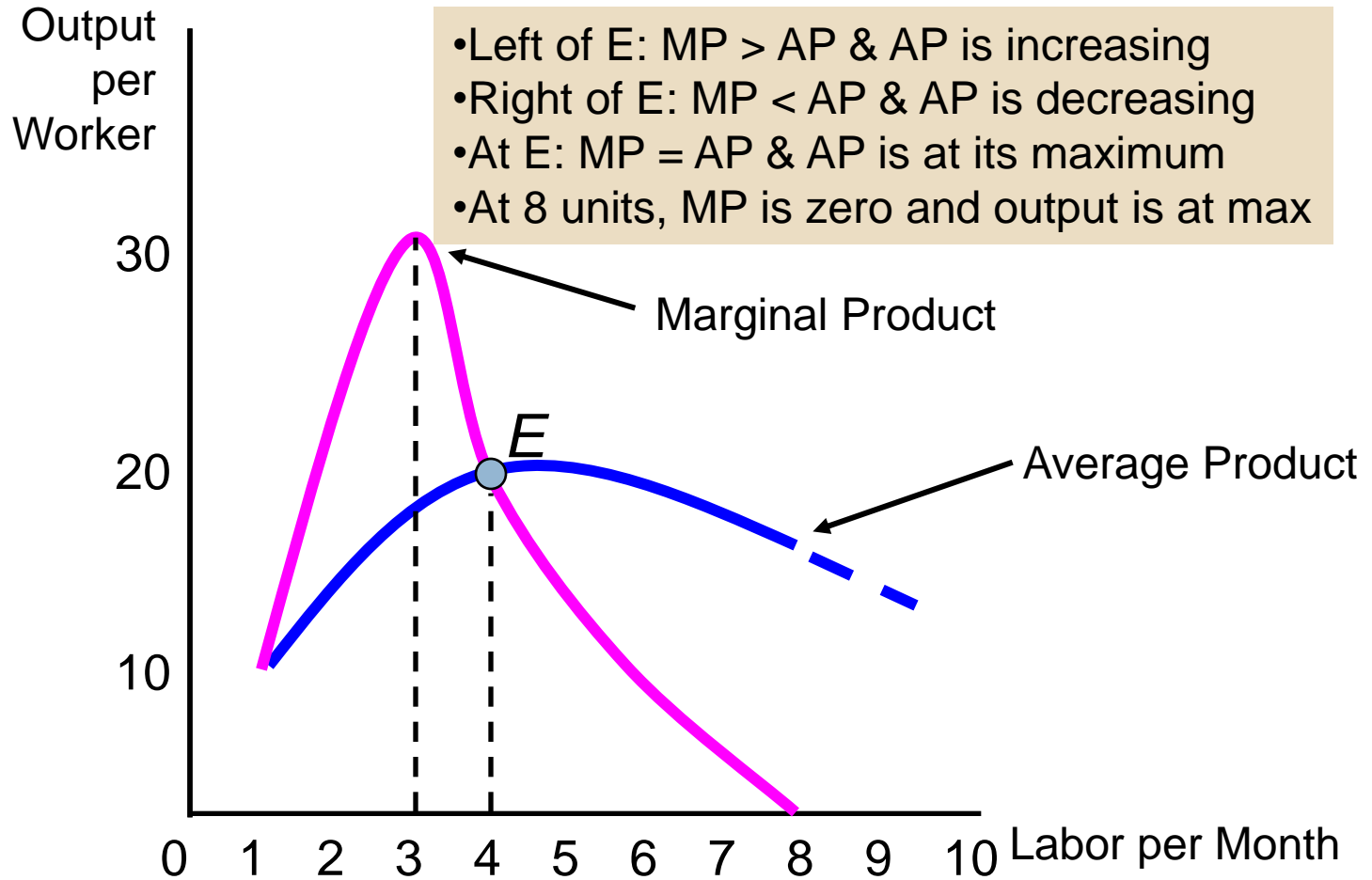
# Production: One Variable Input

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# Production: One Variable Input

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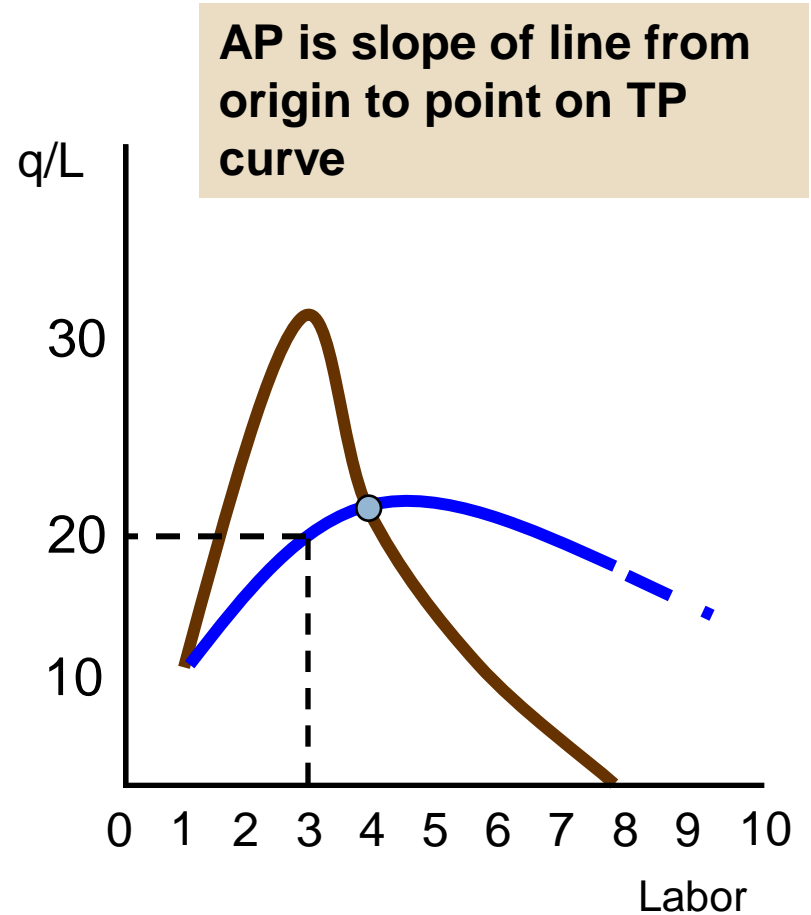
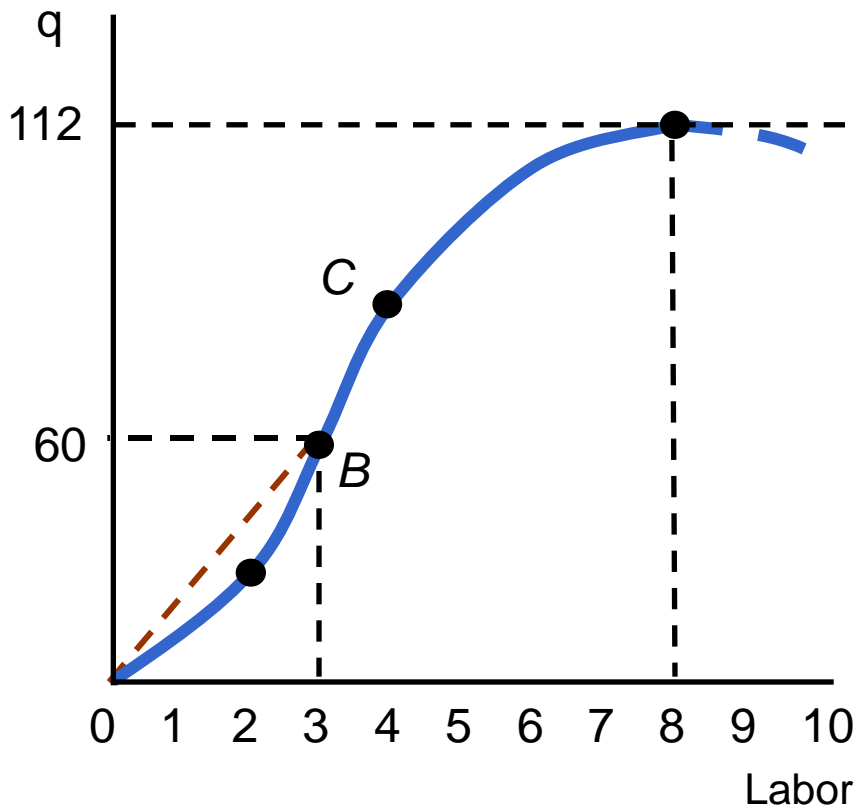
# Marginal & Average Product

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- When marginal product is greater than the average product, the average product is increasing
- When marginal product is less than the average product, the average product is decreasing
- When marginal product is zero, total product (output) is at its maximum
- Marginal product crosses average product at its maximum

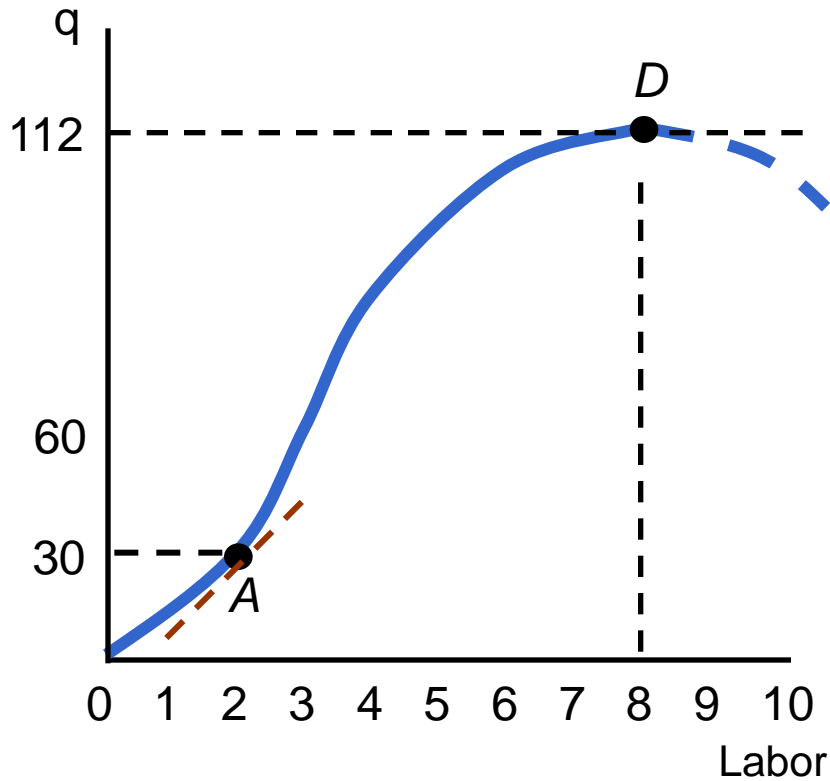
# Product Curves

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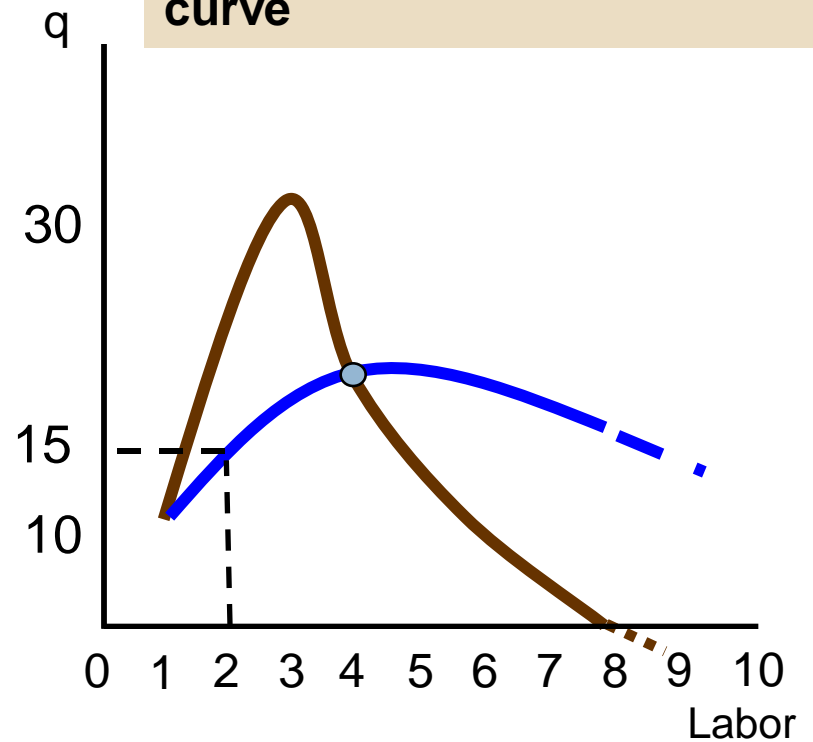


# Product Curves

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MP is slope of line tangent to corresponding point on TP curve



# Production: One Variable Input

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- From the previous example, we can see that as we increase labor the additional output produced declines
- Law of Diminishing Marginal Returns  
한계수확체감의 법칙: As the use of an input increases with other inputs fixed, the resulting additions to output will eventually decrease.

# Law of Diminishing Marginal Returns

## 한계수확(익체감)의 법칙

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- When the labor input is small and capital is fixed, output increases considerably since workers can begin to specialize and MP of labor increases
- When the labor input is large, some workers become less efficient and MP of labor decreases



# Law of Diminishing Marginal Returns

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- Usually used for short run when one variable input is fixed
- Can be used for long-run decisions to evaluate the trade-offs of different plant configurations
- Assumes the quality of the variable input is constant

# Law of Diminishing Marginal Returns

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- Easily confused with negative returns – decreases in output.
- Explains a *declining* marginal product, **not** necessarily a negative one
  - ▣ *Additional* output can be declining while *total* output is increasing

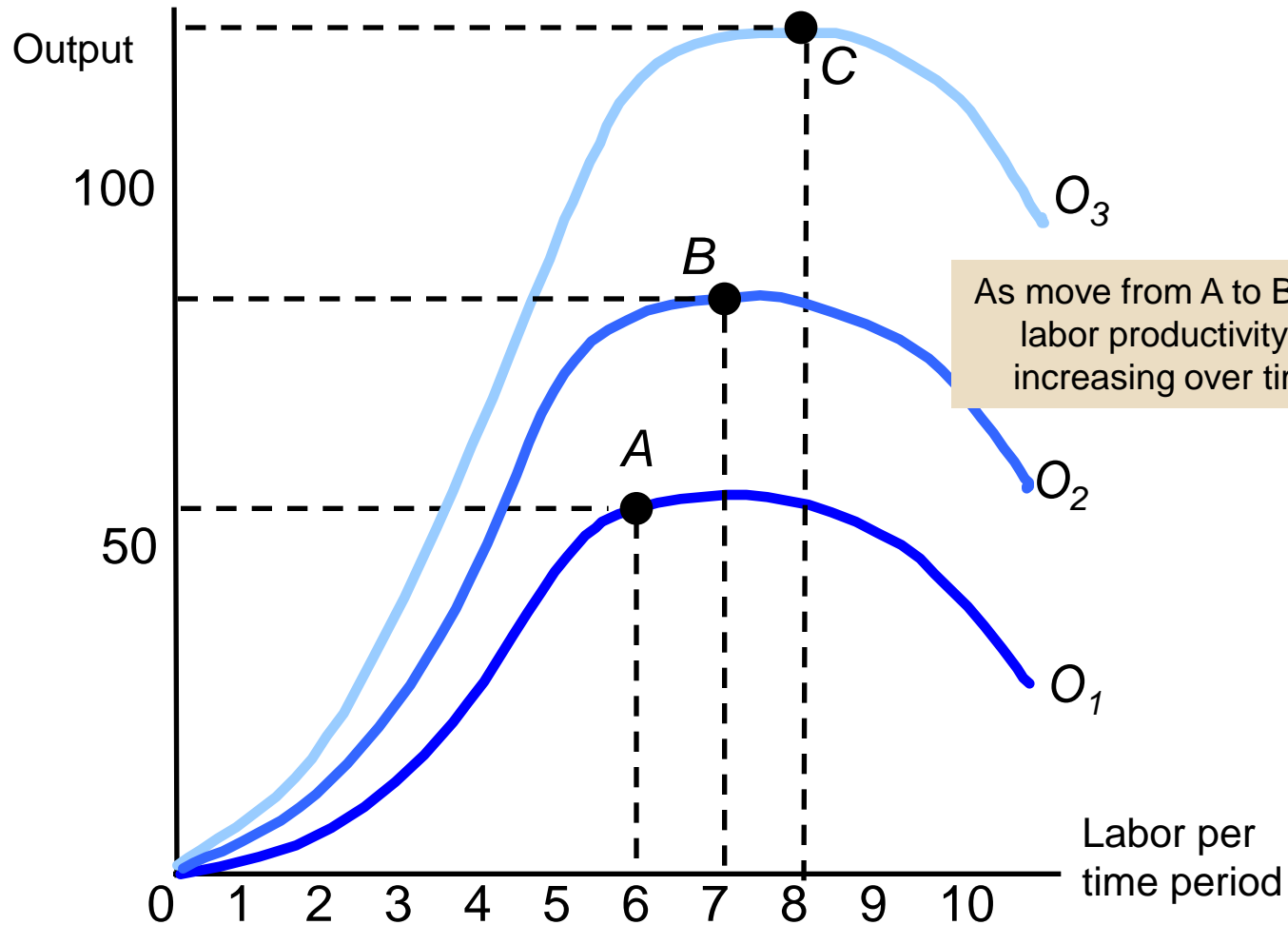
# Law of Diminishing Marginal Returns

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- Assumes a constant technology
  - ▣ Changes in technology will cause shifts in the total product curve
  - ▣ More output can be produced with same inputs
  - ▣ Labor productivity can increase if there are improvements in technology, even though any given production process exhibits diminishing returns to labor.

# The Effect of Technological Improvement

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# Malthus and the Food Crisis

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- Malthus predicted mass hunger and starvation as diminishing returns limited agricultural output and the population continued to grow.
- Why did Malthus' prediction fail?
  - ▣ Did not take into account changes in technology
  - ▣ Although he was right about diminishing marginal returns to labor

# Labor Productivity

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- Macroeconomics are particularly concerned with **labor productivity**
  - The average product of labor for an entire industry or the economy as a whole
  - Links macro and microeconomics
  - Can provide useful comparisons across time and across industries

$$\text{Average Productivity} = \frac{q}{L}$$

# Labor Productivity

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- Link between labor productivity and standard of living
  - ▣ Consumption can increase only if productivity increases.
  - ▣ Growth of Productivity
    1. Growth in stock of capital – total amount of capital available for production
    2. Technological change – development of new technologies that allow factors of production to be used more efficiently

# Production: Two Variable Inputs

- Firm can produce output by combining different amounts of labor and capital
- In the long-run, capital and labor are both variable.
- We can look at the output we can achieve with different combinations of capital and labor



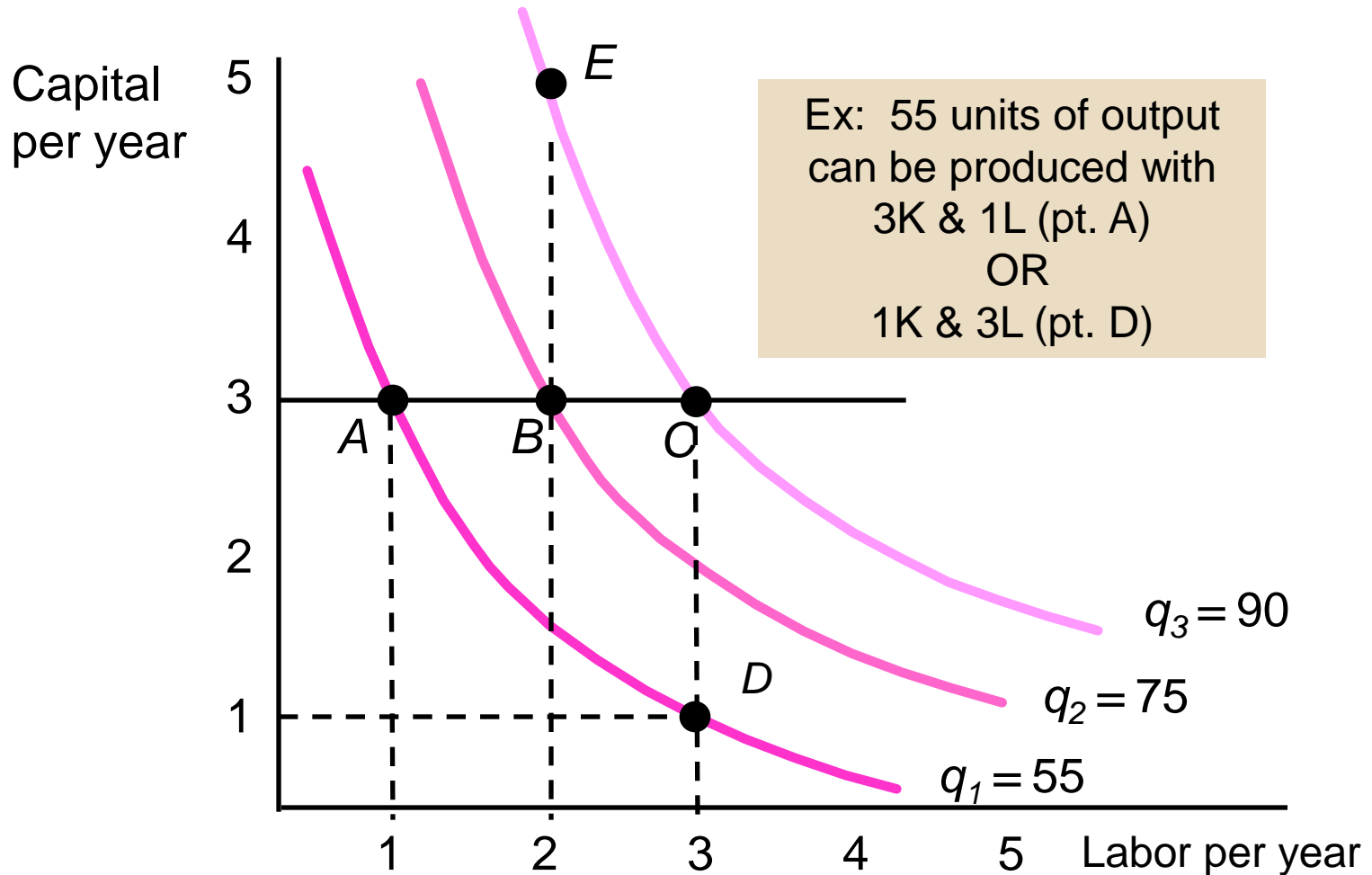
# Production: Two Variable Inputs

<i>Capital Input</i>	<i>Labor Input</i>				
	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>
1	20	40	55	65	75
2	40	60	75	85	90
3	55	75	90	100	105
4	65	85	100	110	115
5	75	90	105	115	120

# Production: Two Variable Inputs

- The information can be represented graphically using **isoquants** 등생산(량) 곡선, 또는 등량곡선
  - Curves showing all possible combinations of inputs that yield the same output
- Curves are smooth to allow for use of fractional inputs
  - Curve 1 shows all possible combinations of labor and capital that will produce 55 units of output

# Isoquant Map



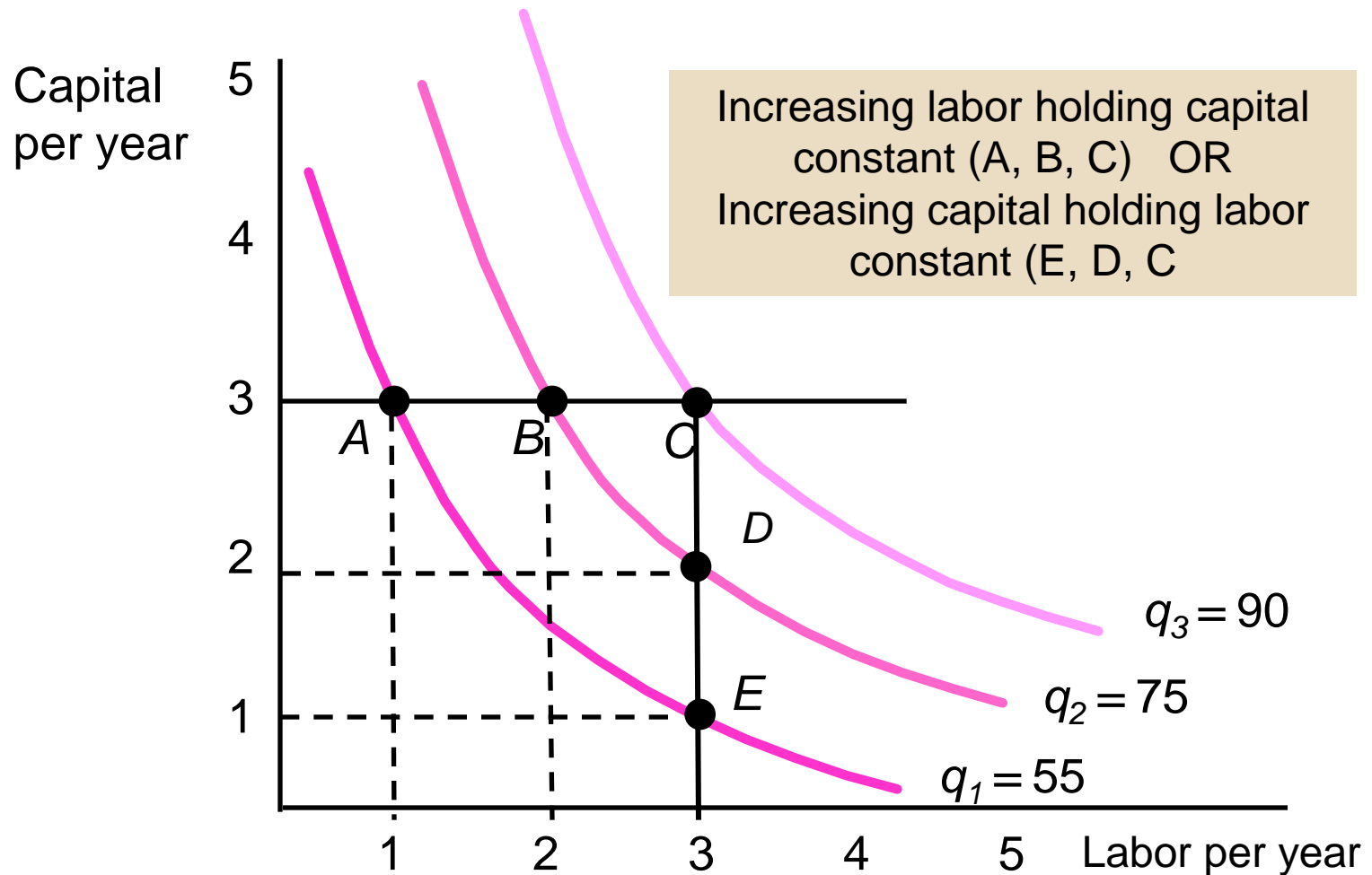
# Production: Two Variable Inputs

- Diminishing Returns to Labor with Isoquants
- Holding capital at 3 and increasing labor from 0 to 1 to 2 to 3.
  - ▣ Output increases at a decreasing rate (0, 55, 20, 15) illustrating diminishing marginal returns from labor in the short-run and long-run.

# Production: Two Variable Inputs

- Diminishing Returns to Capital with Isoquants
- Holding labor constant at 3 increasing capital from 0 to 1 to 2 to 3.
  - Output increases at a decreasing rate (0, 55, 20, 15) due to diminishing returns from capital in short-run and long-run.

# Diminishing Returns



# Production: Two Variable Inputs

- Substituting Among Inputs
  - ▣ Companies must decide what combination of inputs to use to produce a certain quantity of output
  - ▣ There is a trade-off between inputs allowing them to use more of one input and less of another for the same level of output.

# Production: Two Variable Inputs

- Substituting Among Inputs
  - ▣ Slope of the isoquant shows how one input can be substituted for the other and keep the level of output the same.
  - ▣ Positive slope is the **marginal rate of technical substitution (MRTS)**
    - Amount by which the quantity of one input can be reduced when one extra unit of another input is used, so that output remains constant.



# Production: Two Variable Inputs

- MRTS 한계기술대체율 the marginal rate of technical substitution equals:

$$MRTS = \frac{\text{Change in Capital input}}{\text{Change in Labor input}}$$

$$MRTS = -\frac{\Delta K}{\Delta L} \text{ (for a fixed level of } q\text{)}$$

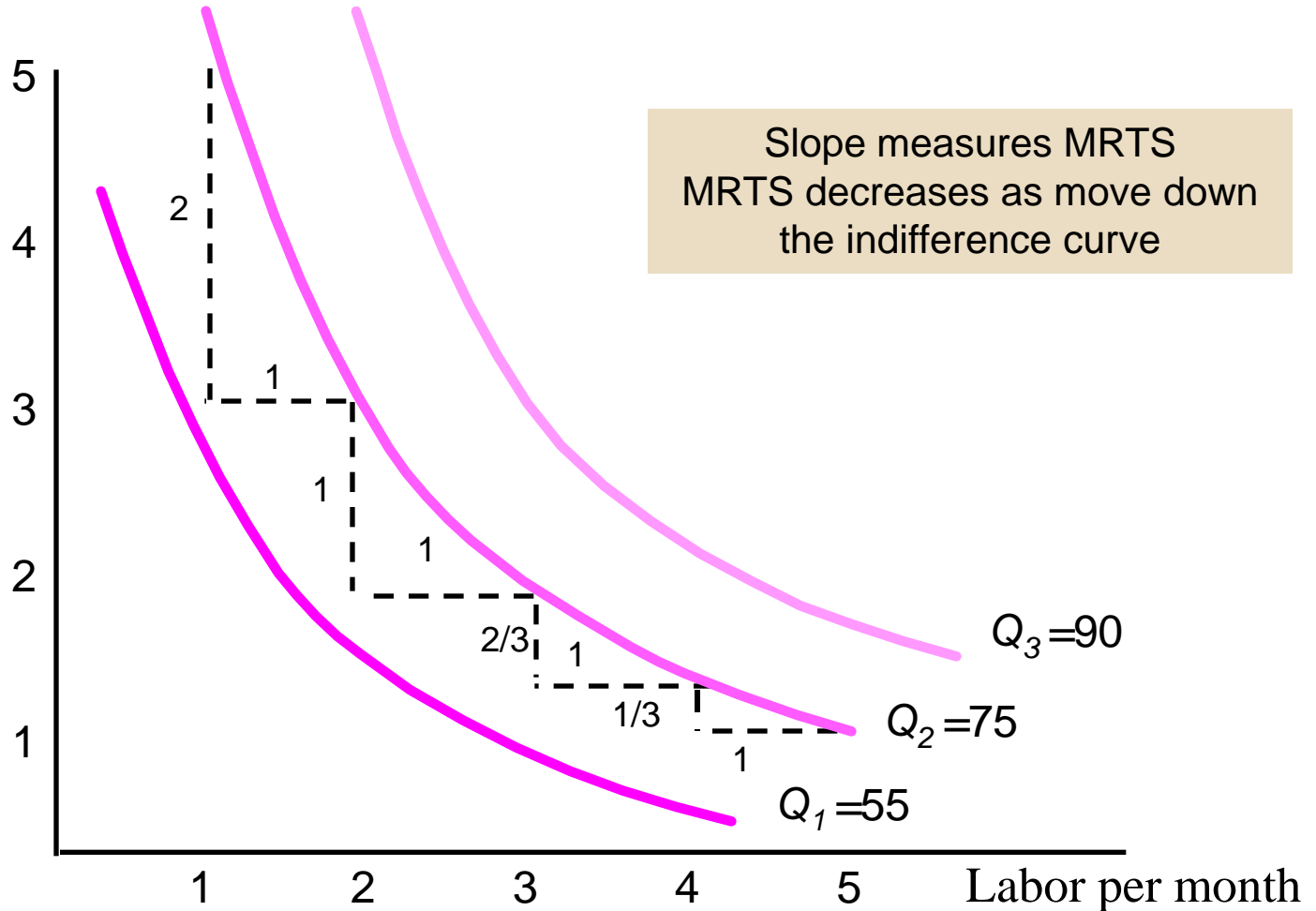
# Production: Two Variable Inputs

- As increase labor to replace capital
  - Labor becomes relatively less productive
  - Capital becomes relatively more productive
  - Need less capital to keep output constant
  - Isoquant becomes flatter

# Marginal Rate of Technical Substitution

한계기술대체율

Capital  
per year



# MRTS and Isoquants

- We assume there is diminishing MRTS
  - ▣ Increasing labor in one unit increments from 1 to 5 results in a decreasing MRTS from 1 to  $1/2$ .
  - ▣ Productivity of any one input is limited
- Diminishing MRTS occurs because of diminishing returns and implies isoquants are convex.
- There is a relationship between MRTS and marginal products of inputs

# MRTS and Marginal Products

- If we increase labor and decrease capital to keep output constant, we can see how much the increase in output is due to the increased labor
  - Amount of labor increased times the marginal productivity of labor

$$= (MP_L)(\Delta L)$$

# MRTS and Marginal Products

- Similarly, the decrease in output from the decrease in capital can be calculated
  - ▣ Decrease in output from reduction of capital times the marginal produce of capital

$$= (MP_K)(\Delta K)$$

# MRTS and Marginal Products

- If we are holding output constant, the net effect of increasing labor and decreasing capital must be zero
- Using changes in output from capital and labor we can see

$$(MP_L)(\Delta L) + (MP_K)(\Delta K) = 0$$

# MRTS and Marginal Products

- Rearranging equation, we can see the relationship between MRTS and MPs

$$(MP_L)(\Delta L) + (MP_K)(\Delta K) = 0$$

$$(MP_L)(\Delta L) = - (MP_K)(\Delta K)$$

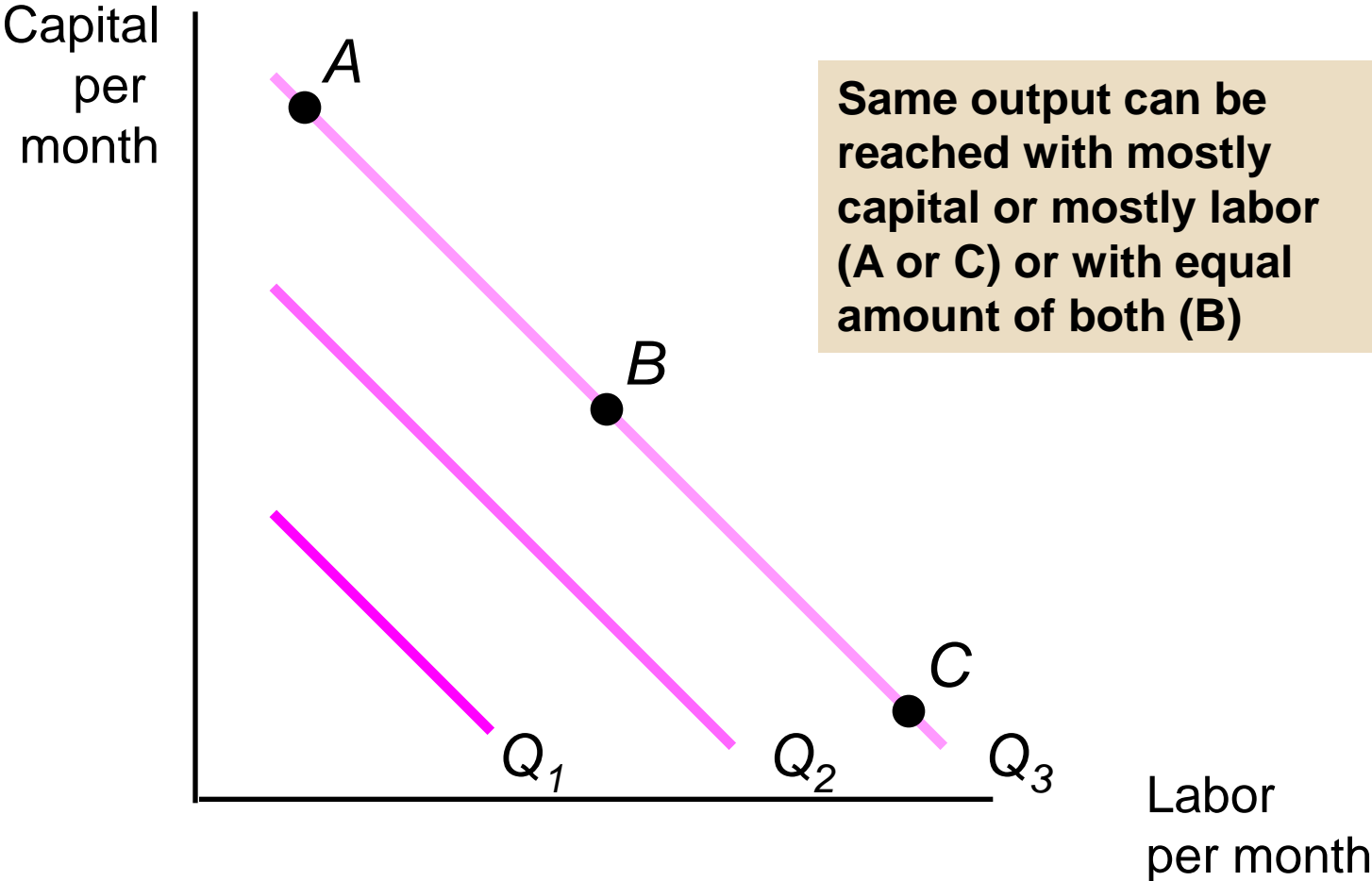
$$\frac{(MP_L)}{(MP_K)} = -\frac{\Delta L}{\Delta K} = MRTS$$



# Isoquants: Special Cases

- Two extreme cases show the possible range of input substitution in production
  1. Perfect substitutes
    - MRTS is constant at all points on isoquant
    - Same output can be produced with a lot of capital or a lot of labor or a balanced mix

# Perfect Substitutes



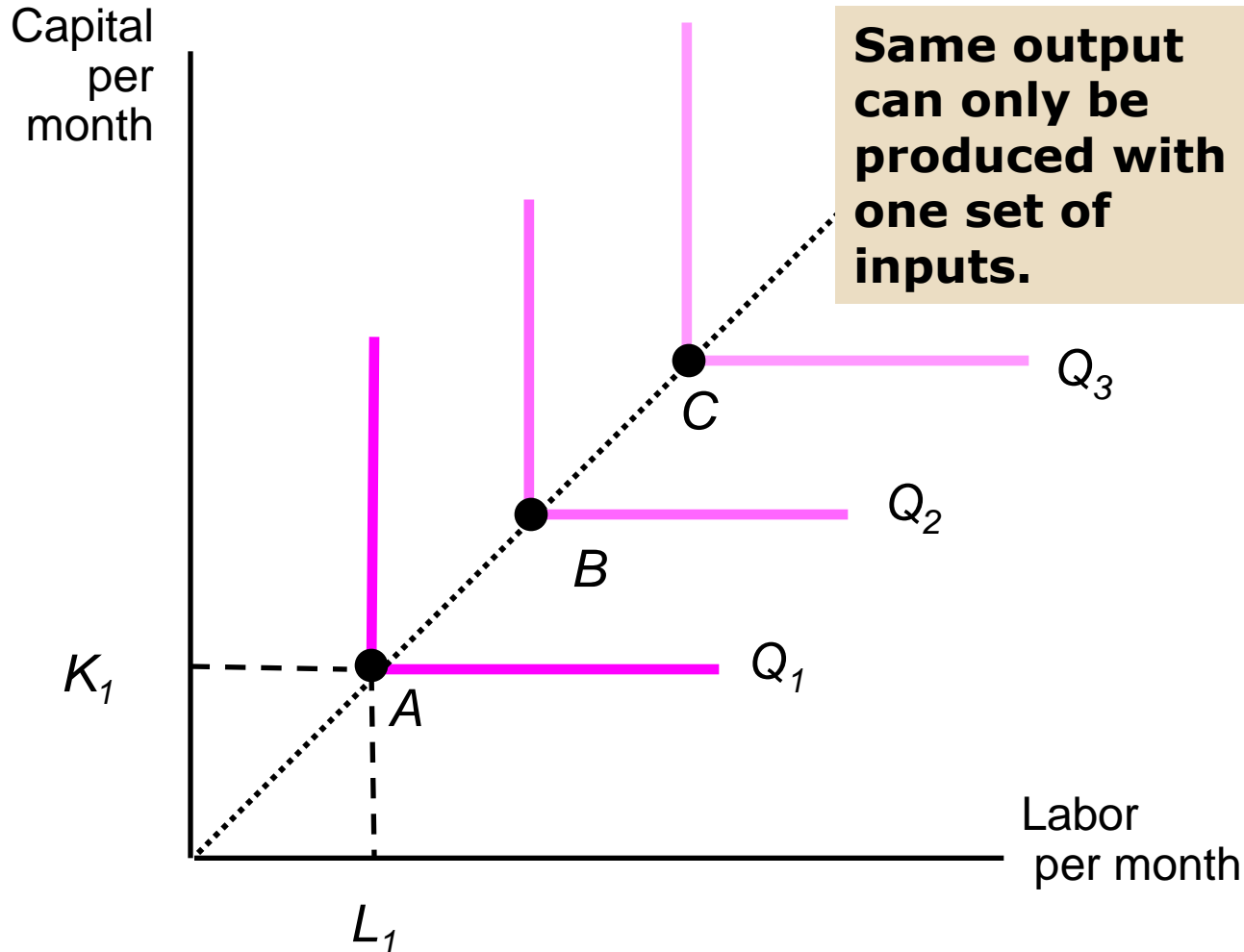
# Isoquants: Special Cases

- Extreme cases (cont.)

- 2. Perfect Complements

- Fixed proportions production function
- There is no substitution available between inputs
- The output can be made with only a specific proportion of capital and labor
- Cannot increase output unless increase both capital and labor in that specific proportion

# Fixed-Proportions Production Function



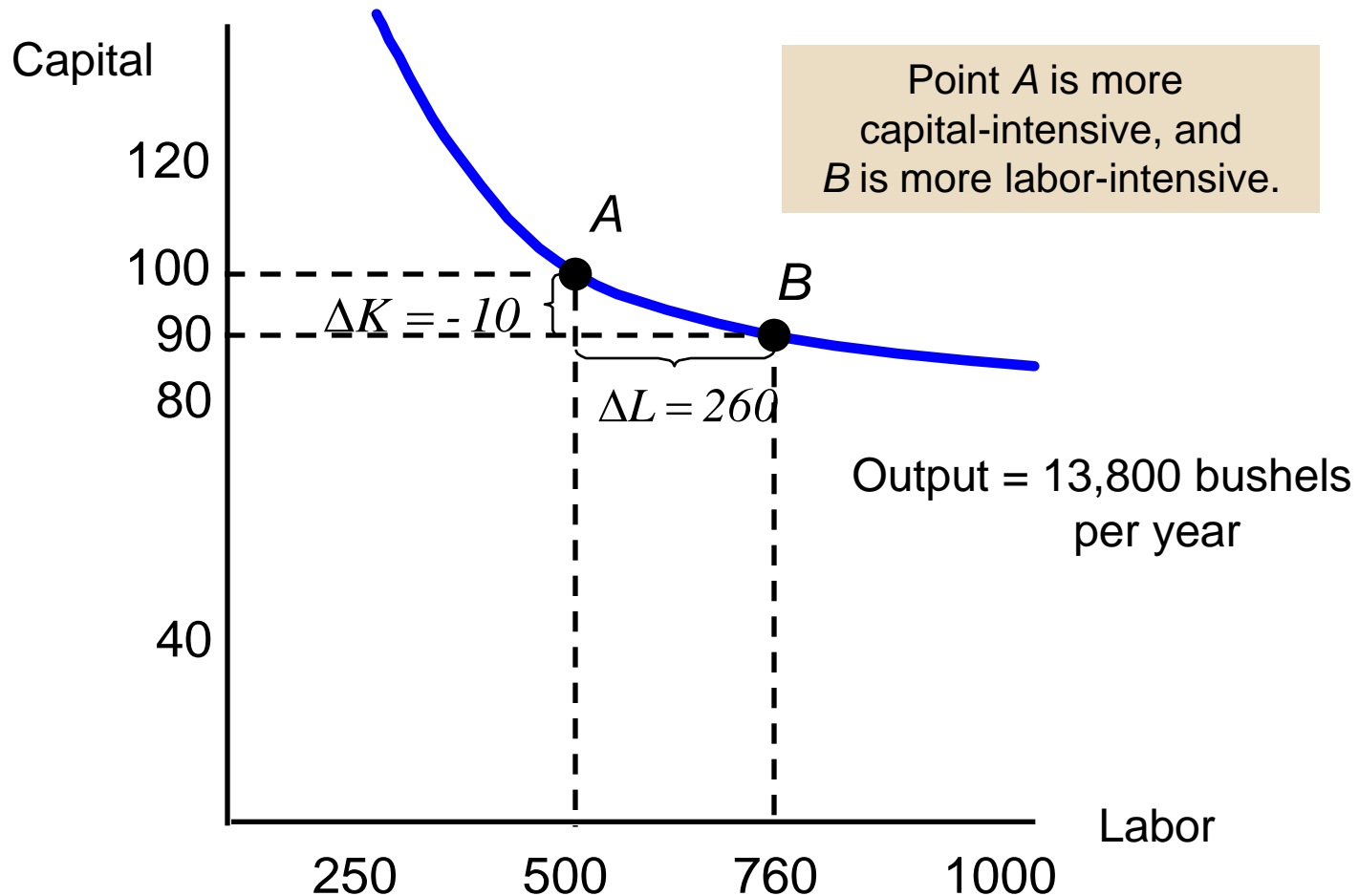
# A Production Function for Wheat

- Farmers can produce crops with different combinations of capital and labor.
  - ▣ Crops in US are typically grown with capital-intensive technology
  - ▣ Crops in developing countries grown with labor intensive productions
- Can show the different options of crop production with isoquants

# A Production Function for Wheat

- Manger of a farm can use the isoquant to decide what combination of labor and capital will maximize profits from crop production
  - ▣ A: 500 hours of Labor, 100 units of capital
  - ▣ B: decreases unit of capital to 90, but must increase hours of labor by 260 to 760 hours.
  - ▣ This experiment shows the farmer the shape of the isoquant

# Isoquant Describing the Production of Wheat



# A Production Function for Wheat

- Increase L to 760 and decrease K to 90 the  $MRTS = 0.04 < 1$

$$MRTS = - \frac{\Delta K}{\Delta L} = -(10 / 260) = 0.04$$

- When wage is equal to cost of running a machine, more capital should be used
- Unless labor is much less expensive than capital, production should be capital intensive



# Returns to Scale

## 규모수익

- In addition to discussing the tradeoff between inputs to keep production the same
- How does a firm decide, in the long run, the best way to increase output
  - ▣ Can change the scale of production by increasing all inputs in proportion
  - ▣ If double inputs, output will most likely increase but by how much?

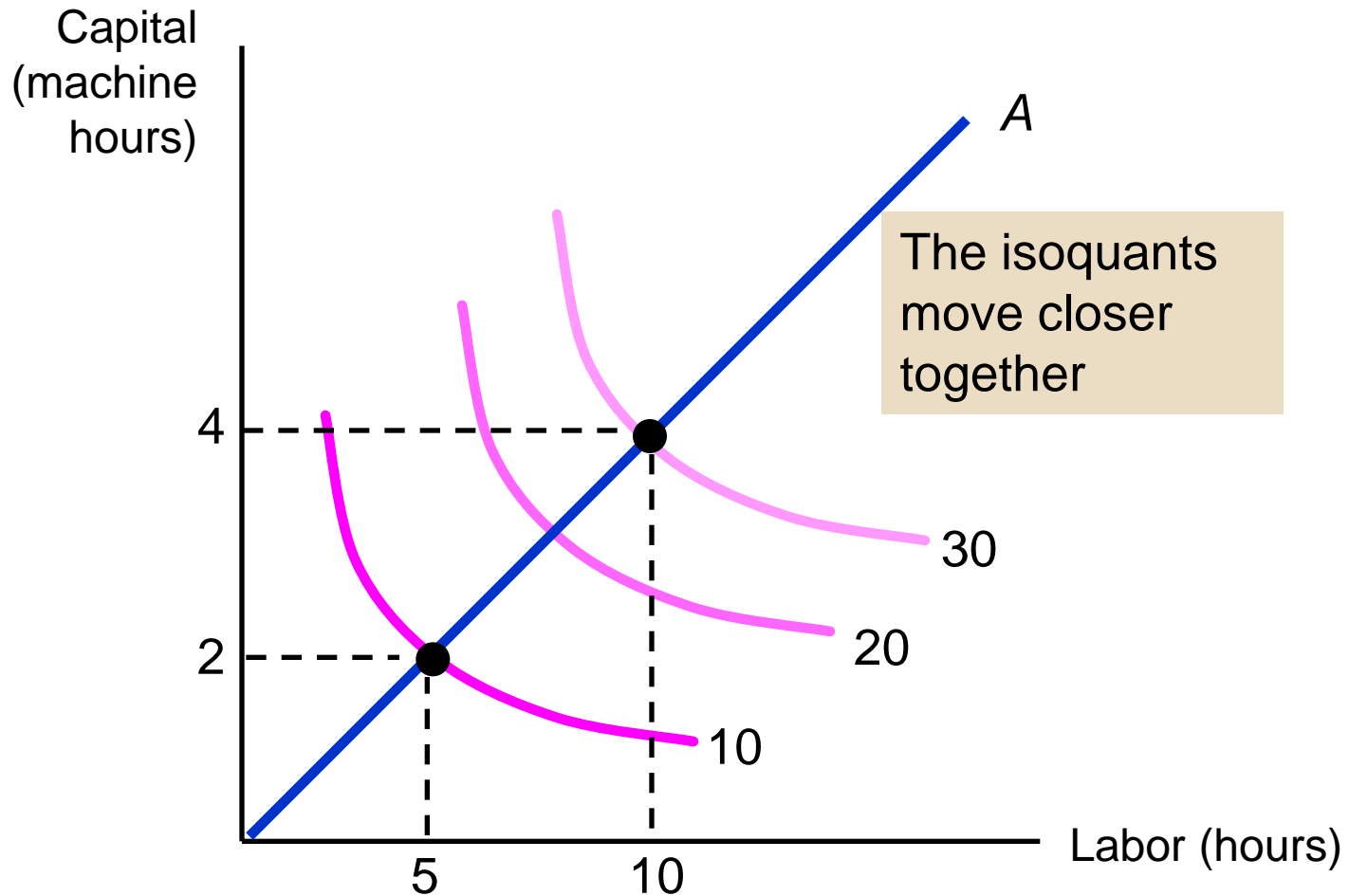
# Returns to Scale

- Rate at which output increases as inputs are increased proportionately
  - Increasing returns to scale 체증규모수확
  - Constant returns to scale 일정 (불변) 규모수확
  - Decreasing returns to scale 체감규모수확

# Returns to Scale

- **Increasing returns to scale:** output more than doubles when all inputs are doubled
  - Larger output associated with lower cost (cars)
  - One firm is more efficient than many (utilities)
  - The isoquants get closer together

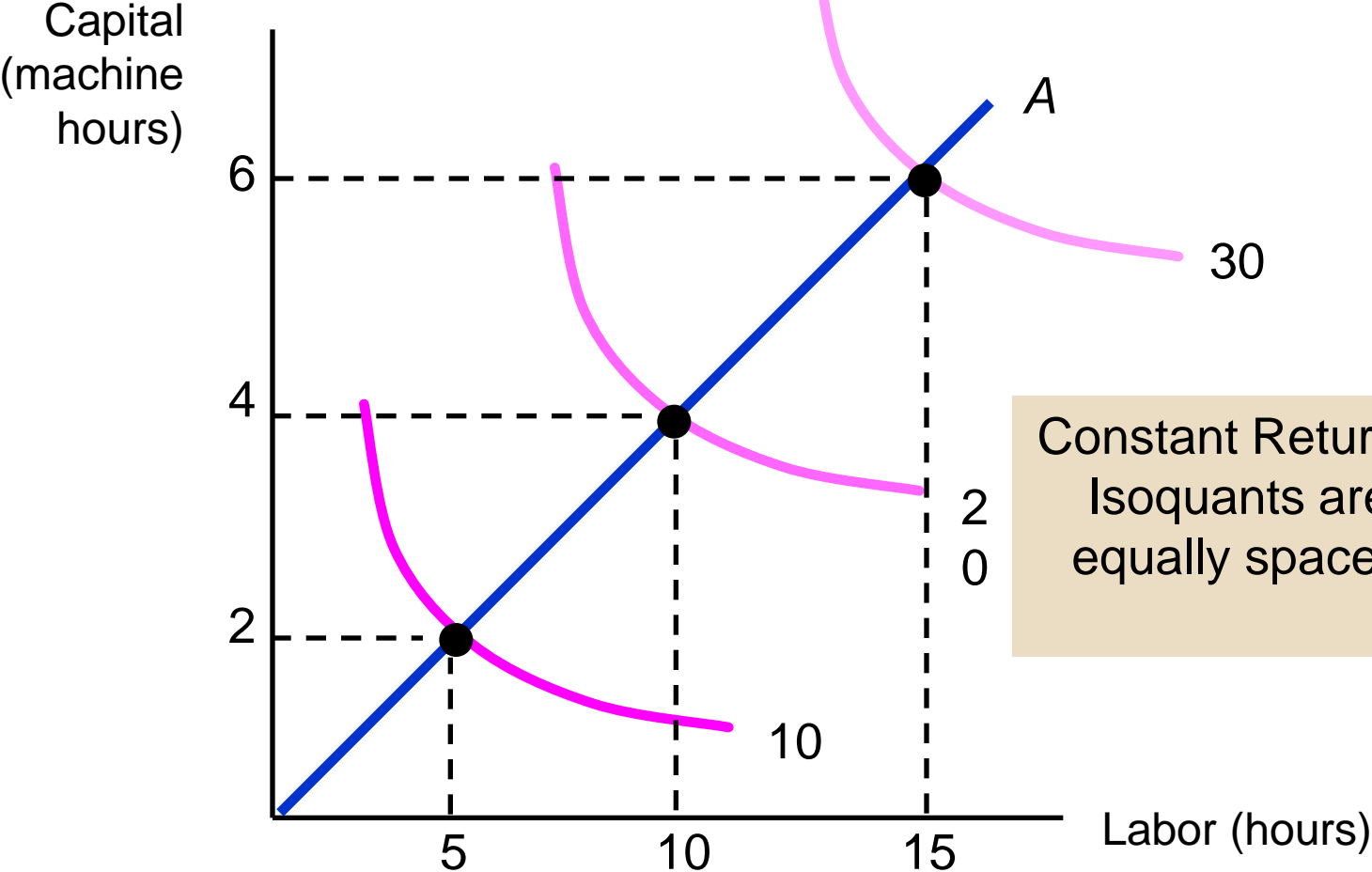
# Increasing Returns to Scale 체증규모수확



# Constant Returns to Scale 일정 (불변) 규모수확

- **Constant returns to scale:** output doubles when all inputs are doubled
  - Size does not affect productivity
  - May have a large number of producers
  - Isoquants are equidistant apart

# Returns to Scale



# Decreasing Returns to Scale 체감 규모수확

- **Decreasing returns to scale:** output less than doubles when all inputs are doubled
  - Decreasing efficiency with large size
  - Reduction of entrepreneurial abilities
  - Isoquants become farther apart

# Returns to Scale

