Lecture Note for Solid Mechanics

- Pressure Vessels and Axial Loading Applications -

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- **Text book : Mechanics of Materials, 6th ed.,**

  W.F. Riley, L.D. Sturges, and D.H. Morris, 2007.

- **Prerequisite : Knowledge of Statics, Basic Physics, Mathematics, etc.**

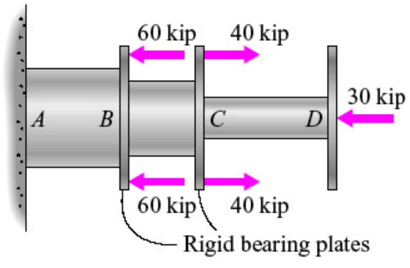
Pressure Vessels and Axial Loading Applications

- Theory of elasticity
  - Determining internal forces and deformations at all points subjected to external forces
  - To obtain general solutions for complex loading and geometry
- Mechanics of materials
  - Real structural elements are analyzed as idealized models subjected to simplified loading and restraints
  - To obtain approximate solutions
  - Practical to most design problems
- Deformations of axial loaded members
  - Uniform member

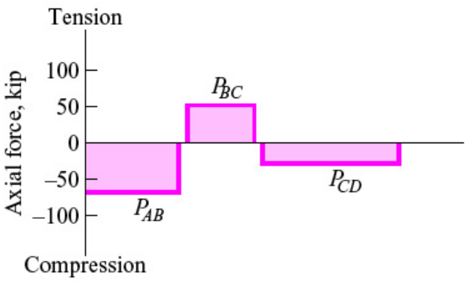
$$\delta = \varepsilon L = \frac{\sigma L}{E} \quad \textit{and} \quad \delta = \frac{PL}{EA}$$

  - Multiple Loads/sizes

$$\delta = \sum_{i=1}^{n} \delta_i = \sum_{i=1}^{n} \frac{P_i L_i}{E_i A_i}$$

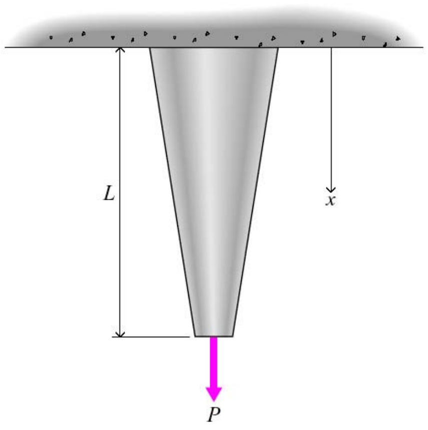


(a)



(b)

Pressure Vessels and Axial Loading Applications



- Nonuniform deformation (Varying axial force or geometry)

  - Axial strain : $\varepsilon = \dfrac{d\delta}{dL} = \dfrac{d\delta}{dx}$

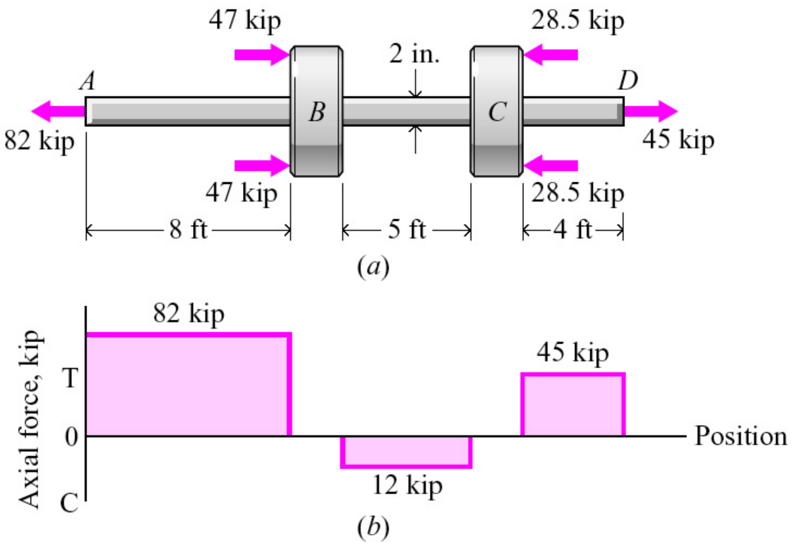
  - Incremental deformation : $d\delta = \varepsilon \, dx$

  - Hooke's law : $\varepsilon = \dfrac{\sigma}{E} = \dfrac{P_x}{EA_x}$

$$d\delta = \frac{P_x}{EA_x} dx \quad \Rightarrow \quad \delta = \int_0^L d\delta = \int_0^L \frac{P_x}{EA_x} dx$$

Pressure Vessels and Axial Loading Applications

(Example) Rigid yokes B and C are fastened to 2-in square steel (E=30,000 ksi) bar AD. Determine (a) maximum normal stress in the bar (b) change in length of the segment AB (c) change in length of segment BC (d) change in length of complete bar.



(Sol)

(a) $$\sigma_{\max} = \frac{P_{\max}}{A} = \frac{82}{4} = 20.5\, ksi\ (T)$$

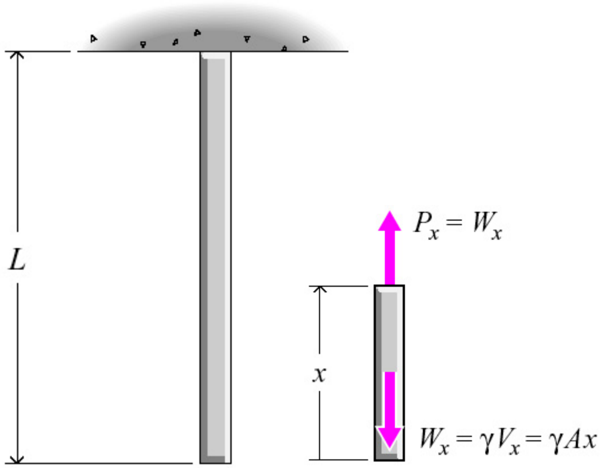
(b) $$\delta_{AB} = \frac{P_{AB} L_{AB}}{E_{AB} A_{AB}} = \frac{+82(8)(12)}{30{,}000(4)} = +0.0656\ in$$

(c) $$\delta_{BC} = \frac{P_{BC} L_{BC}}{E_{BC} A_{BC}} = \frac{-12(5)(12)}{30{,}000(4)} = -0.006\ in$$

(d) $$\delta_{CD} = \frac{P_{CD} L_{CD}}{E_{CD} A_{CD}} = \frac{+45(4)(12)}{30{,}000(4)} = +0.018\ in$$

$$\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD}$$
$$= +0.0656 - 0.006 + 0.018 = +0.0776\ in$$

(Example) Determine (a) elongation of the bar due to its own weight W in terms of W, L, A and E (b) elongation of the bar if the bar is also subjected to an axial tensile force P at its lower end



(a) (b)

(Sol)

(a) Axial force is a function of x, the distance from the free end of the bar.
The weight of the segment is

$$W_x = \gamma V_x = \gamma Ax$$

$$\delta = \int_0^L \frac{P_x}{EA_x} dx = \frac{1}{EA}\int_0^L \gamma Axdx = \frac{\gamma}{E}\int_0^L xdx$$

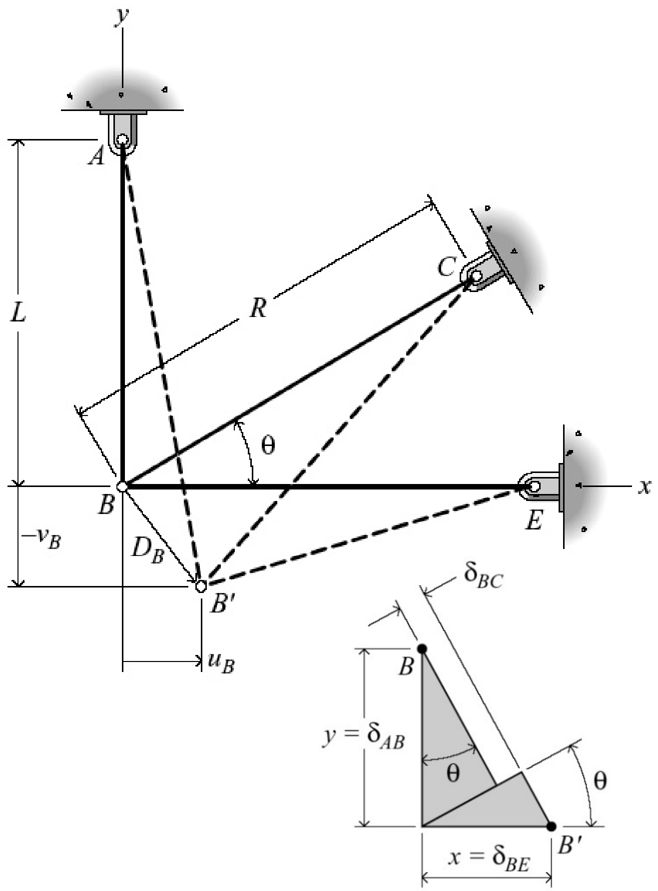
$$\delta = \frac{\gamma}{E}\int_0^L xdx = \left.\frac{\gamma x^2}{2E}\right]_0^L = \frac{\gamma L^2}{2E}$$

$$\delta = \frac{\gamma L^2}{2E} = \frac{W}{AL}\left[\frac{L^2}{2E}\right] = \frac{WL}{2AE} \qquad \Leftarrow W = \gamma AL$$

(b) Elongation would be found using the method of superposition.

$$\delta = \frac{WL}{2EA} + \frac{PL}{EA} = \frac{L}{EA}\left(\frac{W}{2} + P\right) \qquad \Leftarrow \delta = \frac{PL}{EA}$$

- Deformations in a system of axially loaded bars
  - Determining axial deformations and strains in pin-connected deformable bars
  - Axial deformations of the bars in the system through a study of geometry of the deformed system



Axial deformation in the bar AB

$$\delta_{AB} = L_f - L_i = \sqrt{(L + v_B)^2 + u_B^2} - L$$

$$\delta_{AB}^2 + 2L\delta_{AB} + L^2 = L^2 + 2Lv_B + v_B^2 + u_B^2 \quad \Rightarrow \delta_{AB} \cong v_B$$

In a similar manner

$$-\delta_{BE} = L_f - L_i = \sqrt{(L - u_B)^2 + v_B^2} - L$$

$$\delta_{BE}^2 - 2L\delta_{BE} + L^2 = L^2 - 2Lu_B + u_B^2 + v_B^2 \quad \Rightarrow \delta_{BE} \cong u_B$$

Axial deformation in the bar BC

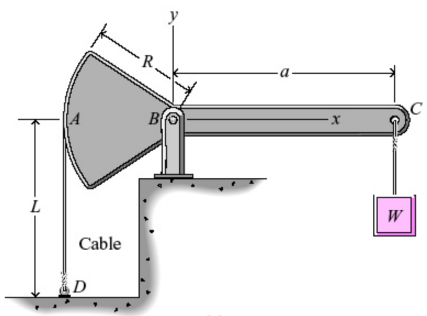
$$\delta_{BC} = \sqrt{(R\cos\theta - u_B)^2 + (R\sin\theta + v_B)^2} - R$$

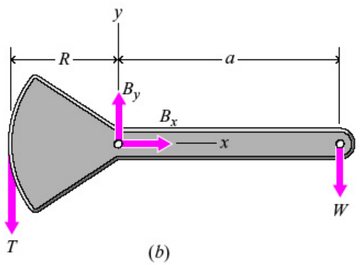
$$\delta_{BC}^2 + 2R\delta_{BC} + R^2 = R^2\cos^2\theta - 2Ru_B\cos\theta + u_B^2 + R^2\sin^2\theta + 2Rv_B\sin\theta + v_B^2$$

$$\delta_{BC} \cong v_B\sin\theta - u_B\cos\theta \cong \delta_{AB}\sin\theta - \delta_{BE}\cos\theta$$

Pressure Vessels and Axial Loading Applications

- Statically indeterminate axially loaded members
  - Statically determinate system
    - reactions at supports and the forces in the individual members can be found by solving equilibrium equations
  - Statically indeterminate system
    - equilibrium equations are not sufficient for the determinations of reactions at supports and the forces in the individual members
    - additional equations involving the geometry of deformations are needed



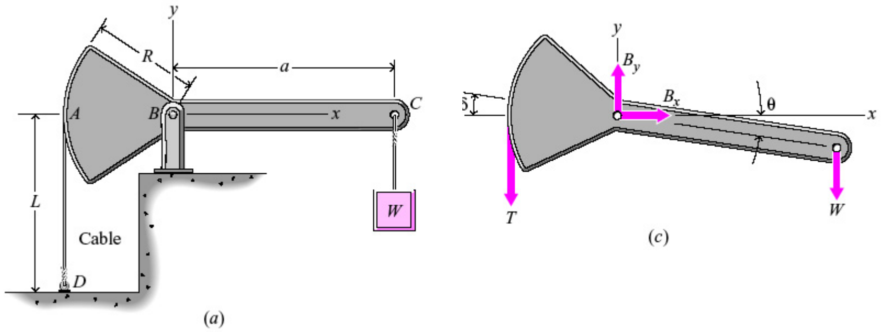


(1) Assume that cable and lever are rigid

$$\sum M_B = TR - Wa = 0$$

$$T = \frac{Wa}{R}$$

Pressure Vessels and Axial Loading Applications



(2) Assume that cable is deformable

$$\sum M_B = TR - Wa\cos\theta = 0$$

$$T = \frac{Wa}{R}\cos\theta$$

$$\delta = \frac{TL}{EA}$$

$$\frac{\delta EA}{L} = \frac{Wa}{R}\cos\theta$$

$$\delta = R\theta$$

$$R^2 EA\theta = WaL\cos\theta$$

(Example) The cable is rigid.
If W=100 lb, a=30 in, R=15 in, therefore T=200 lb

(Example) The cable is a 3/32 in diameter steel (E=29,000 psi) wire.
If W=100 lb, a=30 in, R=15 in, L=45 in,
Then θ=0.002997 rad=0.1717 deg, therefore T=199.999 lb

$$\%D = \frac{200 - 199.999}{199.999}(100) = 0.0005\%$$

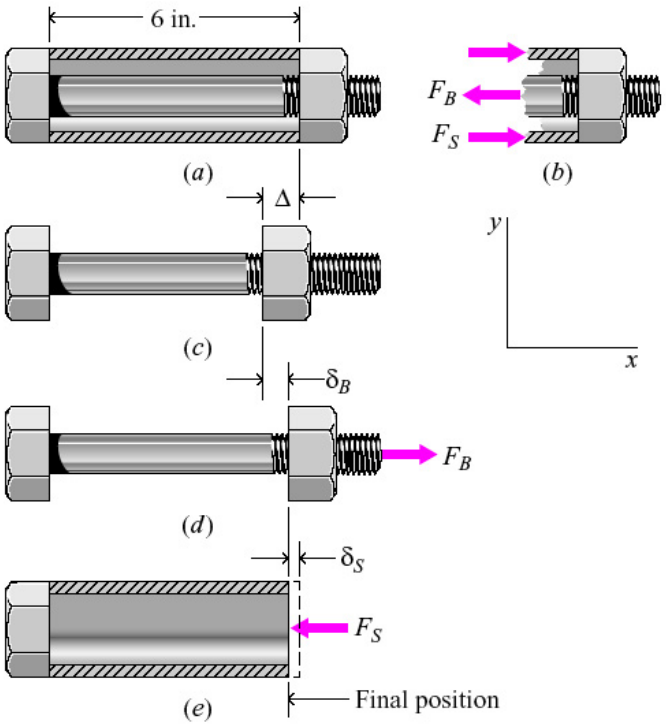
(Example) The cable is a 3/32 in diameter aluminum (E=10,600 psi) wire
If W=100 lb, a=30 in, R=15 in, L=45 in,
Then θ=0.00820 rad=0.4698 deg, therefore T=199.993 lb.

$$\%D = \frac{200 - 199.993}{199.993}(100) = 0.0035\%$$

Tension in the wire changes very little when the stiffness of the wire is changed by a factor of almost 3 and both values are essentially the same as that obtained using rigid wire assumption.

(Example) A ½-in diameter alloy-steel bolt (E=30,000 ksi) passes through a cold rolled brass sleeve (E=15,000 ksi). The cross sectional area of the sleeve is 0.375 in^2. Determine the normal stresses produced in the bolt and sleeve by tightening the nut ¼ turn (0.020 in).

(Sol)



The FBD contains two unknowns : statically indeterminate

As the nut is turned, Δ=0.020 in, if the sleeve is not present.

If sleeve is present, the movement is resisted.

⇨ Tensile stress in the bolt and compressive stress in the sleeve

$$\sum F_x = 0: \quad F_S - F_B = 0$$

$$0.375\sigma_S = \frac{\pi}{4}\left(\frac{1}{2}\right)^2 \sigma_B$$

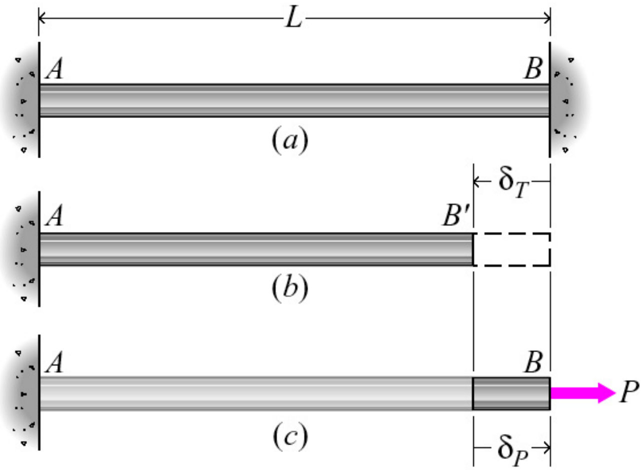
$$\sigma_S = 0.5236\sigma_B$$

$$\delta_s + \delta_B = \Delta \quad \Rightarrow \quad \frac{\sigma_B L_B}{E_B} + \frac{\sigma_S L_S}{E_S} = \Delta$$

$$\frac{\sigma_B(6)}{30{,}000} + \frac{\sigma_S(6)}{15{,}000} = 0.020 \quad \Rightarrow \quad \sigma_B + 2\sigma_S = 100$$

$$\sigma_B = 48.8\,ksi\ (T) \quad \sigma_S = 25.6\,ksi\ (C)$$

- Thermal effects
  - When temperature change takes place while a member is restrained, thermal stresses are induced in the member



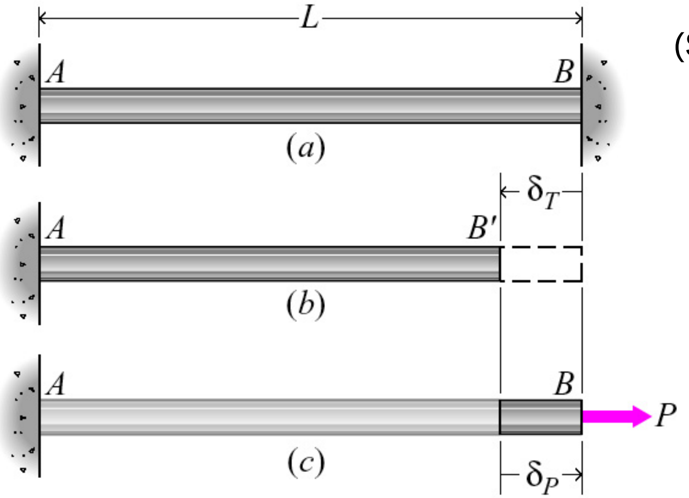
$$\delta_{total} = \delta_T + \delta_\sigma = \varepsilon_T L + \varepsilon_\sigma L$$

$$\mathbf{0} = \alpha \,\Delta T\, L + \frac{\sigma}{E} L$$

  - If the temperature of the bar increases (ΔT positive), the induced stress must be negative and the wall must push on the ends of the rod.
  - If the temperature of the bar decreases (ΔT negative), the induced stress must be positive and the wall must pull on the ends of the rod.

Pressure Vessels and Axial Loading Applications

(Example) The 10-m section of steel rail (E=200GPa, α=11.9x10^{-6}/^{o}C) has a cross section of 7500mm^2. Both ends of the rail are tight against adjacent rigid rail. For increase in temperature of 50^{o}C, determine (a) the normal stress in the rail (b) the internal force on the cross section of the rail.



(Sol) Since temperature increases, the deformation shown in figures are reversed.

(a) $\delta_{total} = \delta_T + \delta_\sigma = \varepsilon_T L + \varepsilon_\sigma L$

$$0 = \alpha\,\Delta T\,L + \frac{\sigma}{E} L$$

$$\sigma = -\alpha\,\Delta T\,E = -(11.9 \times 10^{-6})(50)(200 \times 10^{9})$$

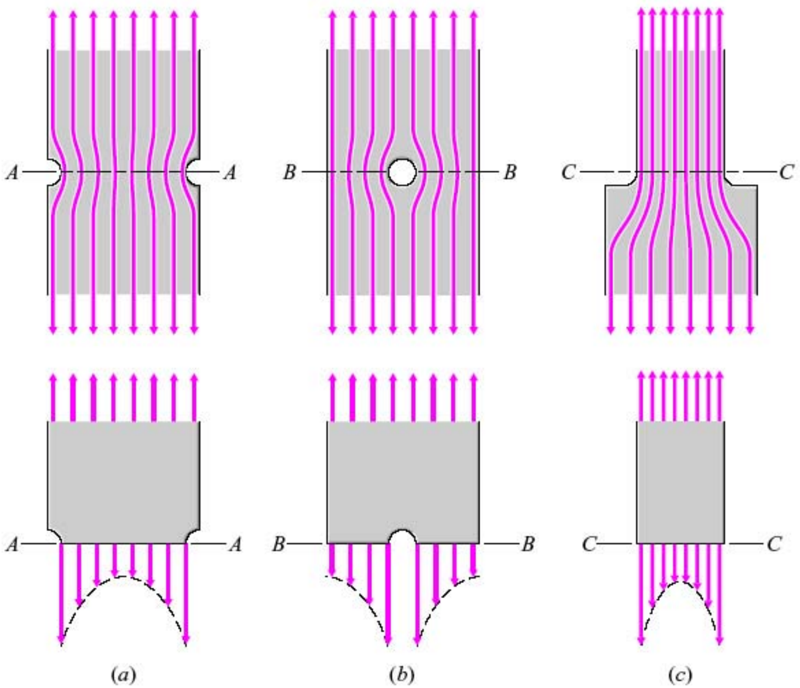
$$= -119.0(10^{6})\,N/m^{2} = 119.0\,MPa\;(C)$$

(b) $F = \sigma A = 119.0(10^{6})(7500)(10^{-6})$

$$= 892.5(10^{3})\,N \cong 893\,kN\;(C)$$

Pressure Vessels and Axial Loading Applications

- Stress concentrations
  - If there exists a discontinuity in the structural or machine element, the stress at the discontinuity may be considerably greater than the nominal stress on the section
  - Stress concentration factor (K) : Ratio of the maximum stress to the nominal stress



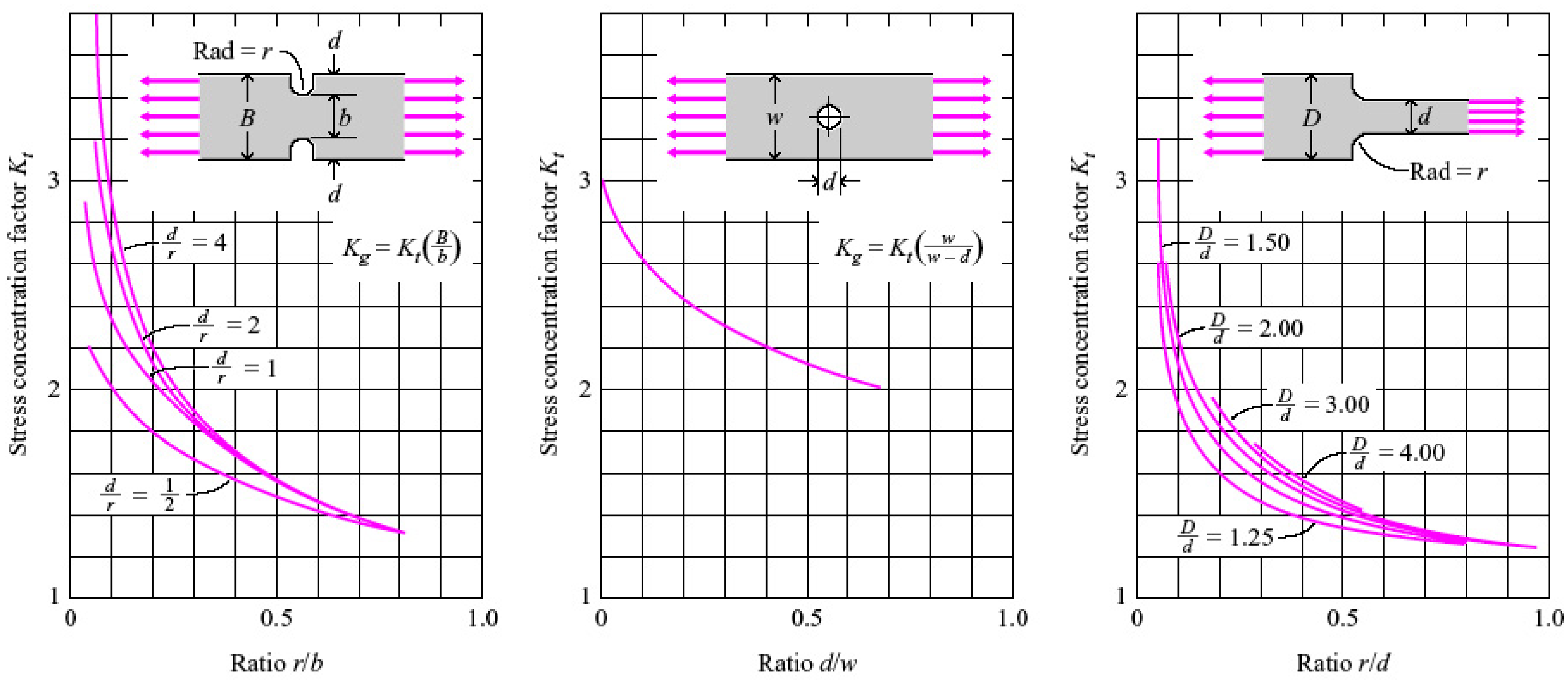
- Maximum normal stress

$$\sigma = K \frac{P}{A}$$

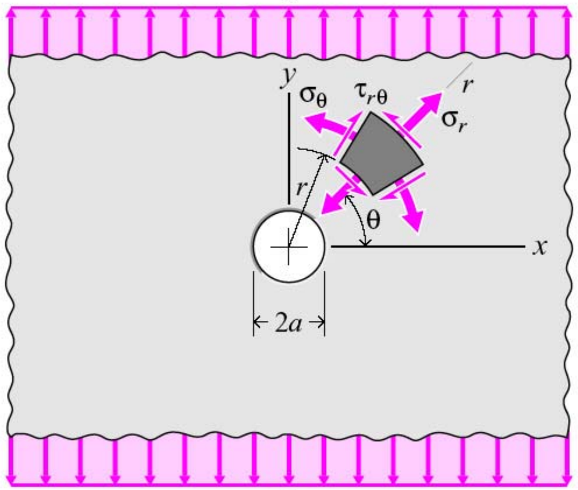
A : gross area or net area depending on the value used for K

$$\sigma = K_g \frac{P}{A_g} \quad \text{or} \quad \sigma = K_t \frac{P}{A_t}$$

Pressure Vessels and Axial Loading Applications



(a) (b) (c)



Stress distributions based on theory of elasticity

$$\sigma_r = \frac{\sigma}{2}\left(1 - \frac{a^2}{r^2}\right) - \frac{\sigma}{2}\left(1 - \frac{4a^2}{r^2} + \frac{3a^4}{r^4}\right)\cos 2\theta$$

$$\sigma_\theta = \frac{\sigma}{2}\left(1 + \frac{a^2}{r^2}\right) + \frac{\sigma}{2}\left(1 + \frac{3a^4}{r^4}\right)\cos 2\theta$$

$$\tau_{r\theta} = \frac{\sigma}{2}\left(1 + \frac{2a^2}{r^2} - \frac{3a^4}{r^4}\right)\sin 2\theta$$

On the boundary of the hole at r=a

$$\sigma_r = 0$$

$$\sigma_\theta = \sigma(1 + 2\cos 2\theta)$$

$$\tau_{r\theta} = 0$$

At θ=0, $\sigma_\theta = 3\sigma$

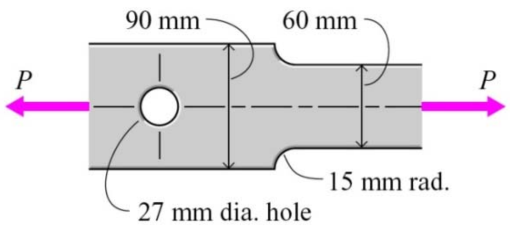
σ : uniform tensile stress in the plane in the regions far from the hole

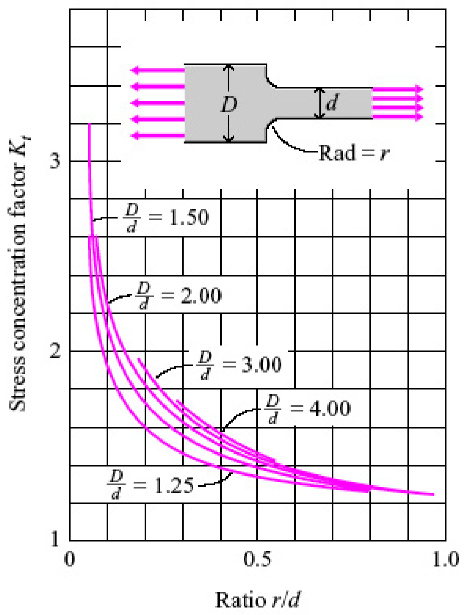
The localized nature of a stress concentration can be evaluated by the consideration of tangential stress along the x-axis

$$\sigma_\theta = \frac{\sigma}{2}\left(2 + \frac{a^2}{r^2} + \frac{3a^4}{r^4}\right) \Rightarrow \sigma_\theta = 1.074\sigma \quad at\ r = 3a$$

Pressure Vessels and Axial Loading Applications

(Example) Machined part is 20-mm thick. Determine the maximum safe load P if a safety factor of 2.5 with respect to failure by yielding is specified.





(Sol)

The yield strength is 360 MPa.
The allowable stress based on safety factor of 2.5 is 360/2.5=144MPa

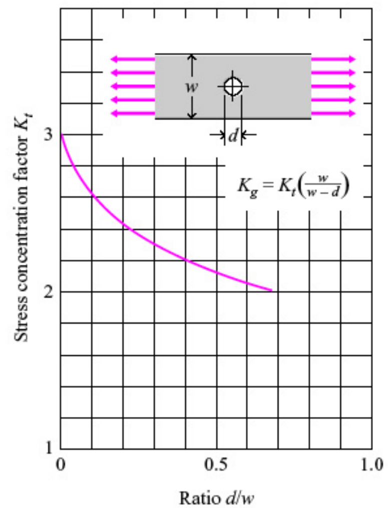
At the fillet,

$$D/d = 90/60 = 1.5 \qquad r/d = 15/60 = 0.25 \quad \Rightarrow \quad K_t = 1.62$$

$$P = \sigma_a A_t / K_t = 144(10^6)(60)(20)(10^{-6})/1.62$$
$$= 106.7(10^3)\,N = 106.7\,kN$$

At the hole,

$$d/w = 27/90 = 0.3 \quad \Rightarrow \quad K_t = 2.30$$



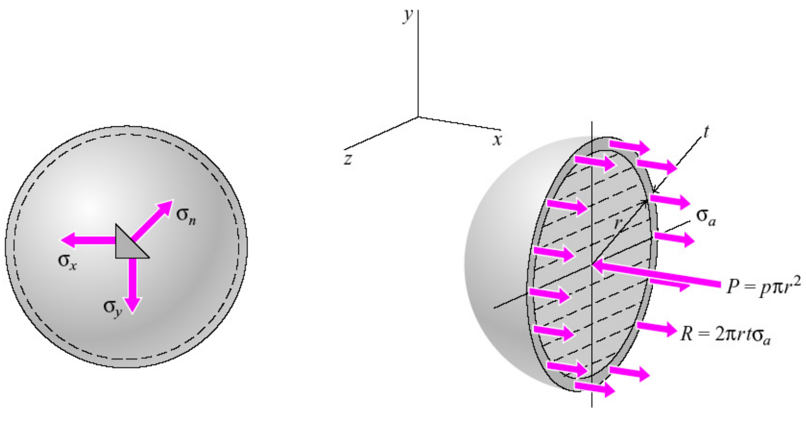
$$P = \sigma_a A_t / K_t = 144(10^6)(90-27)(20)(10^{-6})/2.30$$
$$= 78.9(10^3)\,N = 78.9\,kN$$

Therefore, $P_{\max} = 78.9\,kN$

Pressure Vessels and Axial Loading Applications

- Thin-walled pressure vessels
  - Thin-walled when the ratio of the wall thickness to the radius of the vessel is so small that the distribution of the normal stress on a plane perpendicular to the surface of the vessel is uniform throughout the thickness of the vessel
  - If the ratio of the wall thickness to the inner radius of the vessel is less than 0.1, the maximum normal stress is less than 5% greater than the average

(1) Spherical pressure vessels



(a) (b)

Based on symmetry of loading and geometry

$$\sigma_x = \sigma_y = \sigma_n$$

Axial (or meridional) stress in a sphere

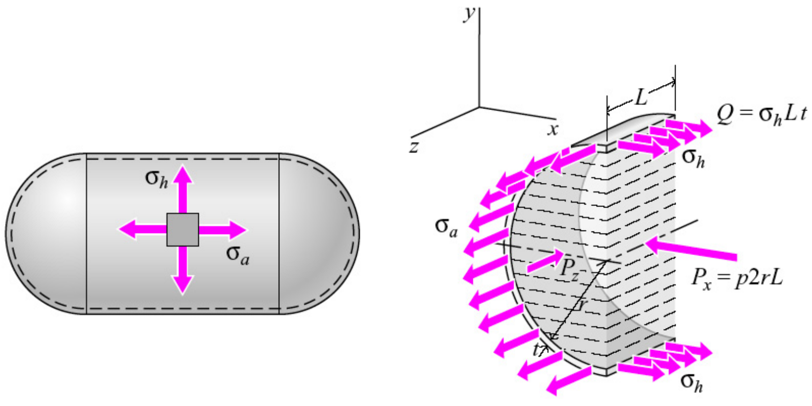
$$R - P = 0$$

$$2\pi r t \sigma_a = p \pi r^2$$

$$\sigma_a = \frac{pr}{2t}$$

Pressure Vessels and Axial Loading Applications

(2) Cylindrical pressure vessels



(a) (b)

From a summation of forces in the x-direction

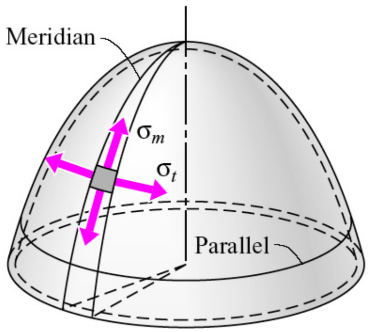
$$2Q - P_x = 0$$

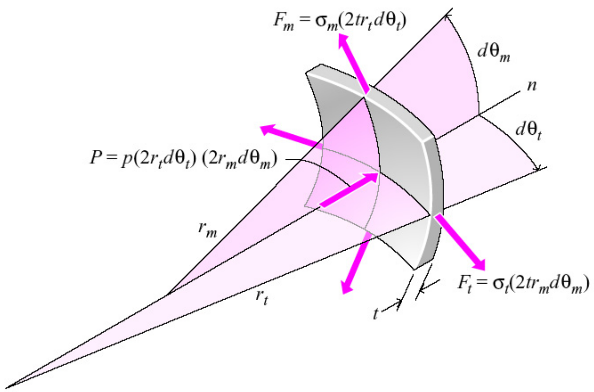
$$2\sigma_h L t = p2rL$$

$$\sigma_h = \frac{pr}{t}$$

$$\sigma_a = \frac{pr}{2t}$$

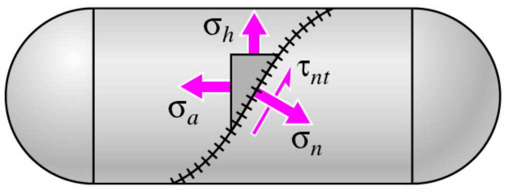
(3) Thin shells of revolution



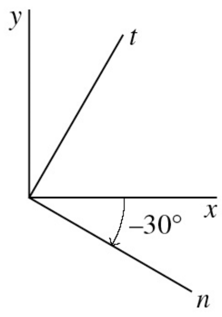


$$\frac{\sigma_m}{r_m} + \frac{\sigma_t}{r_t} = \frac{p}{t}$$

(Example) Cylindrical pressure vessel with inside diameter of 1.5m is constructed by wrapping a 15mm thick steel plate. The butt welded seams form an angle of 30deg with a transverse plane through the cylinder. Determine the normal stress and shear stress when the internal pressure in the vessel is 1500kPa.



(a)



(b)

$$\sigma_h = \frac{pr}{t} = \frac{(1500)(10^3)(0.75)}{0.015} = 75.0(10^6)\, N/m^2 = 75.0\, MPa$$

$$\sigma_a = \frac{pr}{2t} = \frac{(1500)(10^3)(0.75)}{2(0.015)} = 37.5(10^6)\, N/m^2 = 37.5\, MPa$$

Therefore,

$$\sigma_x = \sigma_a = +37.5\, MPa \qquad \sigma_y = \sigma_h = +75.0\, MPa \qquad \tau_{xy} = 0$$

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta \cos\theta$$
$$= (37.5)\cos^2(-30) + (75.0)\sin^2(-30) + 0$$
$$= 46.875\, MPa \cong 46.9\, MPa\ (T)$$

$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$
$$= -(37.5 - 75.0)\sin(-30)\cos(-30) + 0$$
$$= -16.238\, MPa \cong 16.24\, MPa\ (C)$$

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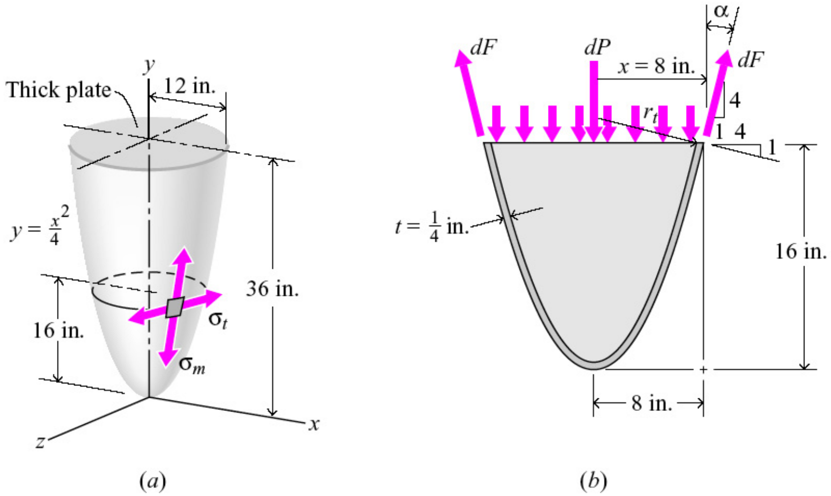

- Determining σ_m

$$-\int_{A_p} dP + \int_{A_\sigma} dF \cos\alpha = 0$$

$$-p\pi x^2 + \sigma_m 2\pi x t \cos\alpha = 0$$

$$-250\pi(8^2) + \sigma_m 2\pi(8)(1/4)(4/\sqrt{17}) = 0$$

$$\sigma_m = 4123\ psi \cong 4120\ psi\ (T)$$

- Determining σ_t

$$r_m = \frac{\left[1+(dy/dx)^2\right]^{1.5}}{d^2y/dx^2} = \frac{(1+4^2)^{1.5}}{1/2} = 140.19\ in$$

$$r_m = 8\left(\sqrt{17}/4\right) = 8.246\ in$$

$$\frac{p}{t} = \frac{\sigma_m}{r_m} + \frac{\sigma_t}{r_t} \quad \Rightarrow \quad \frac{250}{1/4} = \frac{4123}{140.19} + \frac{\sigma_t}{8.246}$$

$$\sigma_t = 8003\ psi \cong 8000\ psi$$

(Homework)

(5.4), (5.19), (5.49), (5.59), (5.79), (5.100)