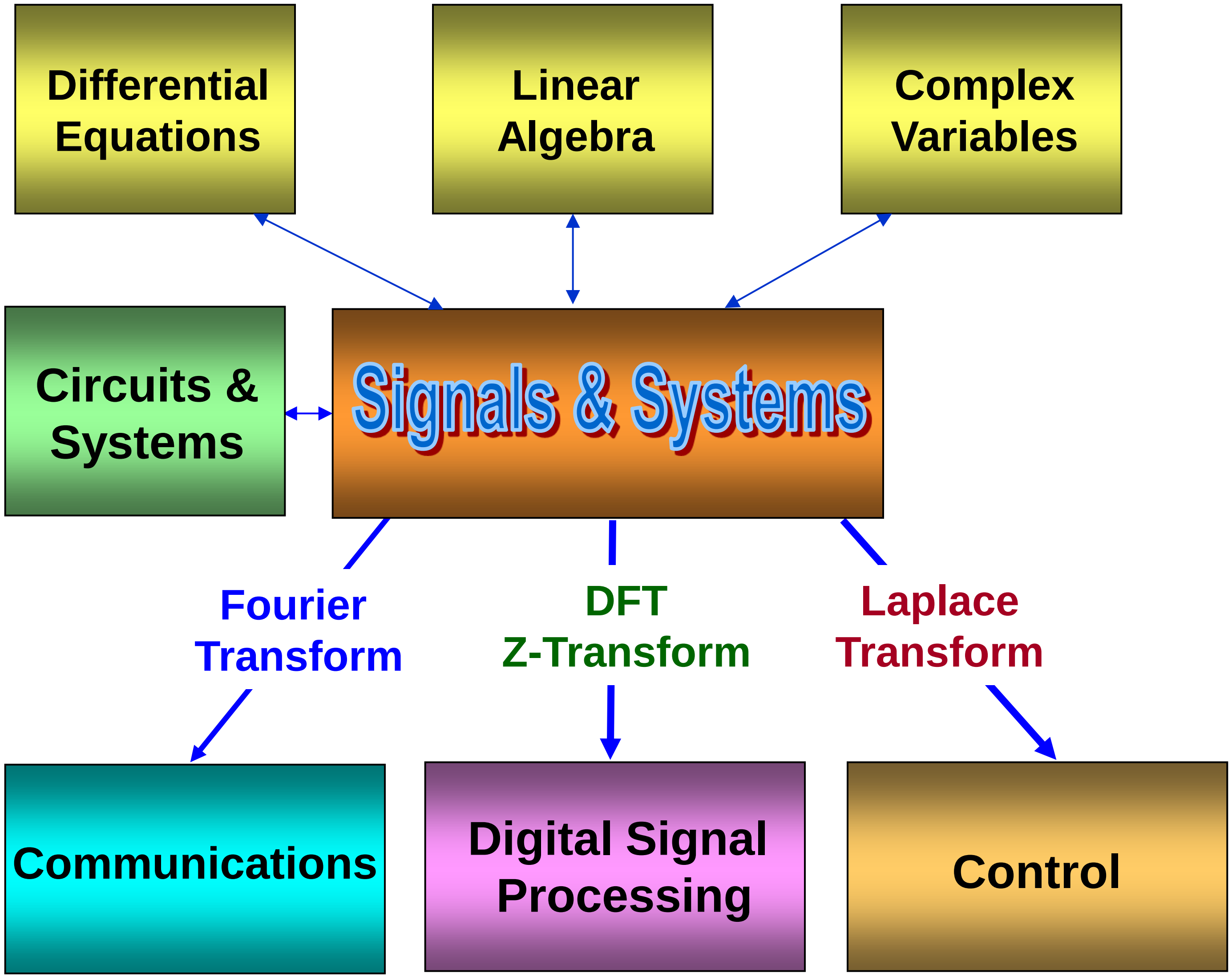
- Text : A. V. Oppenheim, A. S. Willsky, and S. H. Nawab, *Signals & Systems*, 2nd Ed., Prentice-Hall, NJ, 1997

- Course Evaluation
  - MATLAB-Programming Reports: 6%,
  - Problem-Solving Reports: 4%
  - Quiz: 30% (15% each)
  - Midterm Exam: 40%
  - Final exam 40%


Differential Equations
Linear Algebra
Complex Variables
Circuits & Systems
Signals & Systems
Fourier Transform
DFT
Z-Transform
Laplace Transform
Communications
Digital Signal Processing
Control

1. Signals and Systems

- Signals
  - Detectable physical quantities or variables by means of which messages or information can be transmitted.
  - Represented as functions of one or more independent variables

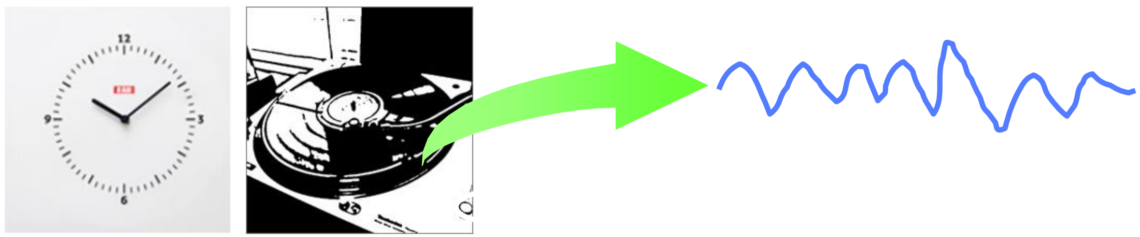




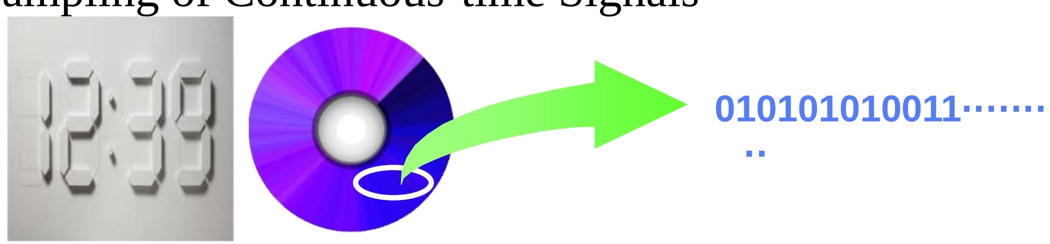



1. Signals and Systems

- Continuous-time signal: independent variable $t \Rightarrow x(t)$
  - Voltage and Current in RC circuit
  - Audio Signal, Audio Tape, Video Tape
  - Analog TV Signal



- Discrete-time signal: independent variable $n \Rightarrow x[n]$
  - Stock market record
  - Digital Audio Tape (DAT)
  - Input to DAC
  - Sampling of Continuous-time Signals



• Continuous-time signal processing

- RC Circuit

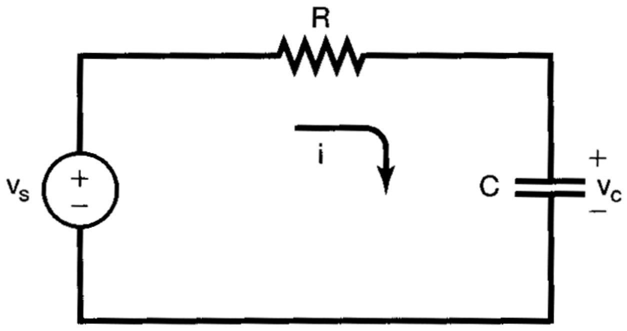


Figure 1.1 A simple *RC* circuit with source voltage v_s and capacitor voltage v_c.

$$V_S(t) = A\cos(\omega_1 t + \theta) \Rightarrow V_C(t)?$$

- Solution in Time-domain?

- Solution in Frequency domain?

$$\frac{V_C(j\omega)}{Vs(j\omega)} = H(j\omega) = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1+\omega^2 (RC)^2}}$$

- Cut-off frequency : $\omega = \dfrac{1}{RC}$

• Continuous-time and discrete-time signal

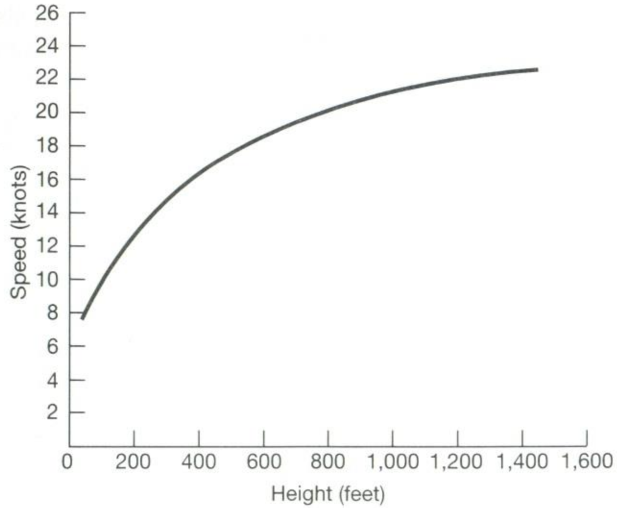


Figure 1.5 Typical annual vertical wind profile. (Adapted from Crawford and Hudson, National Severe Storms Laboratory Report, ESSA ERLTM-NSSL 48, August 1970.)

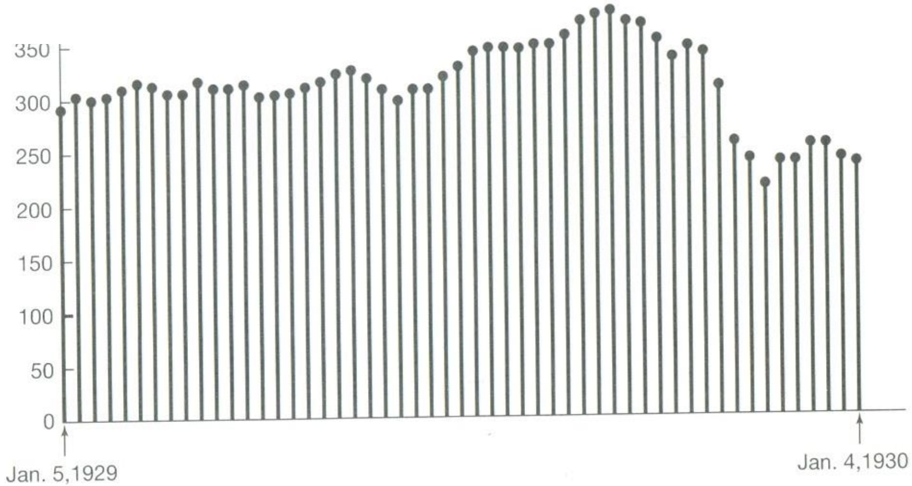


Figure 1.6 An example of a discrete-time signal: The weekly Dow-Jones stock market index from January 5, 1929, to January 4, 1930.

• Continuous-time and discrete-time signal

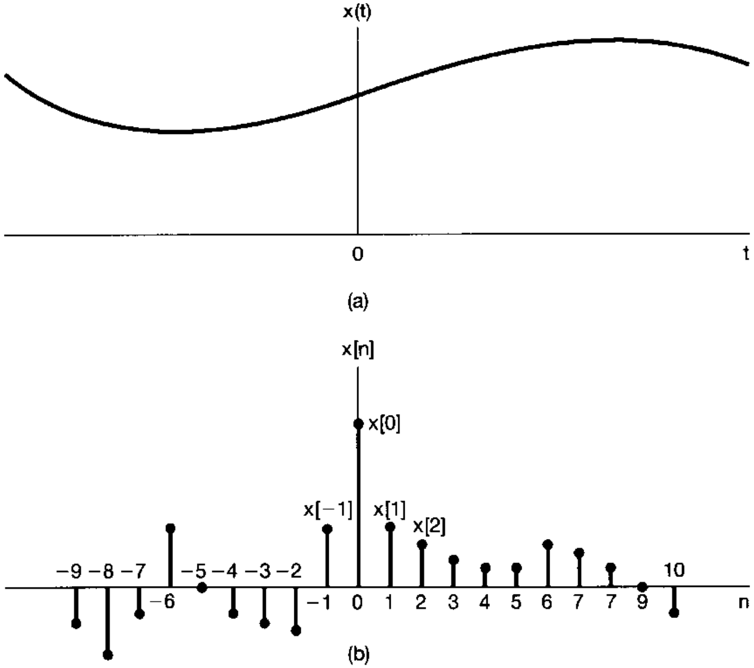


Figure 1.7 Graphical representations of (a) continuous-time and (b) discrete-time signals.

1.1.2 Signal energy and power

- Power and Energy in Electric Circuits

$$p(t) = v(t)i(t) = \frac{1}{R}v^2(t)$$

$$E = \int_{t_1}^{t_2} p(t)dt = \frac{1}{R}\int_{t_1}^{t_2} v^2(t)dt$$

- Average power *P*

$$P = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} p(t)dt = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} \frac{1}{R}v^2(t)dt$$

- Continuous-time signal: energy $\int_{t_1}^{t_2} |x(t)|^2\, dt$

- Discrete-time signal : energy $\sum_{n=n_1}^{n_2} |x[n]|^2$

- Energy over an infinite time interval

$$E_\infty \stackrel{\Delta}{=} \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2\, dt = \int_{-\infty}^{\infty} |x(t)|^2\, dt$$

$$E_\infty \stackrel{\Delta}{=} \lim_{N\to\infty} \sum_{n=-N}^{N} |x[n]|^2 = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

- Average Power

$$P_{\infty} \stackrel{\Delta}{=} \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} | x(t) |^2 \, dt$$

$$P_{\infty} \stackrel{\Delta}{=} \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} | x[n] |^2$$

- Classification of Signals

i) $E_{\infty} < \infty \Rightarrow P_{\infty} = 0$

ii) $P_{\infty} < \infty, P_{\infty} > 0 \Rightarrow E_{\infty} = \infty$

iii) $E_{\infty} = \infty, P_{\infty} = \infty$ (ex : $x(t) = t$)

1.2 Transformation of the Independent Variable

1.2.1 Example of transformations of the independent variables

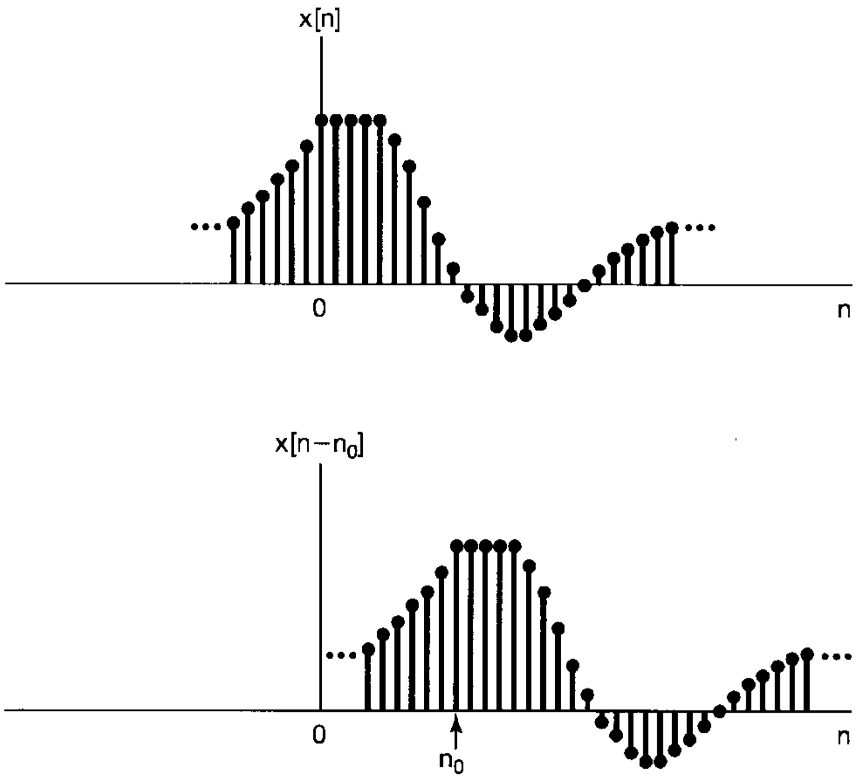


Figure 1.8 Discrete-time signals related by a time shift. In this figure $n_0 > 0$, so that $x[n - n_0]$ is a delayed verson of $x[n]$ (i.e., each point in $x[n]$ occurs later in $x[n - n_0]$).

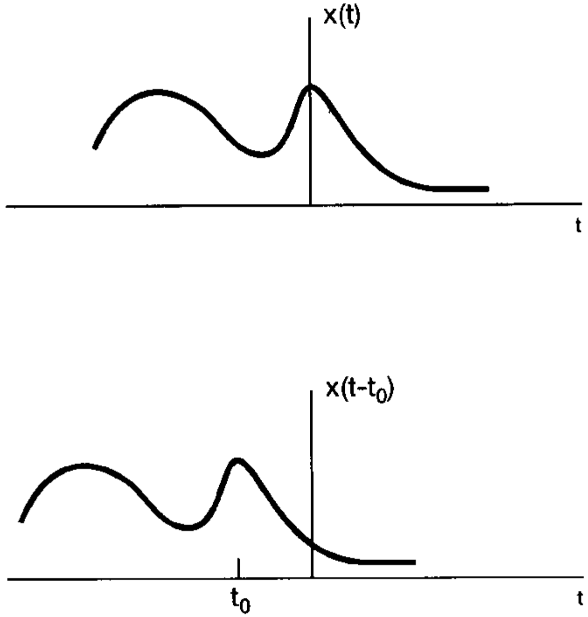


Figure 1.9 Continuous-time signals related by a time shift. In this figure $t_0 < 0$, so that $x(t - t_0)$ is an advanced version of $x(t)$ (i.e., each point in $x(t)$ occurs at an earlier time in $x(t - t_0)$).

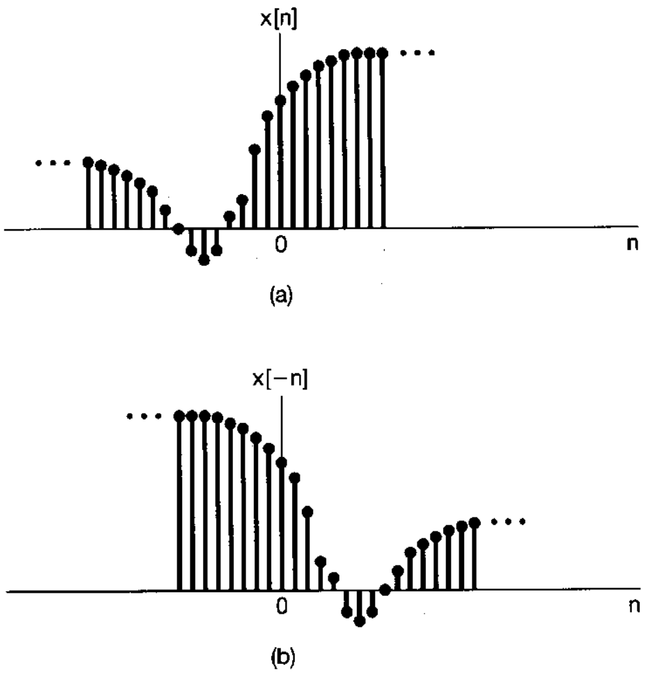


Figure 1.10 (a) A discrete-time signal $x[n]$; (b) its reflection $x[-n]$ about $n = 0$.

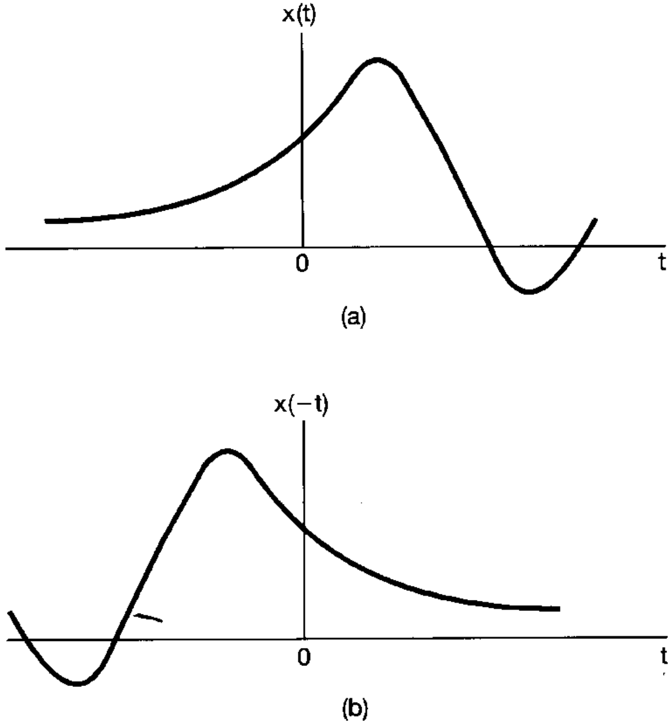


Figure 1.11 (a) A continuous-time signal $x(t)$; (b) its reflection $x(-t)$ about $t = 0$.

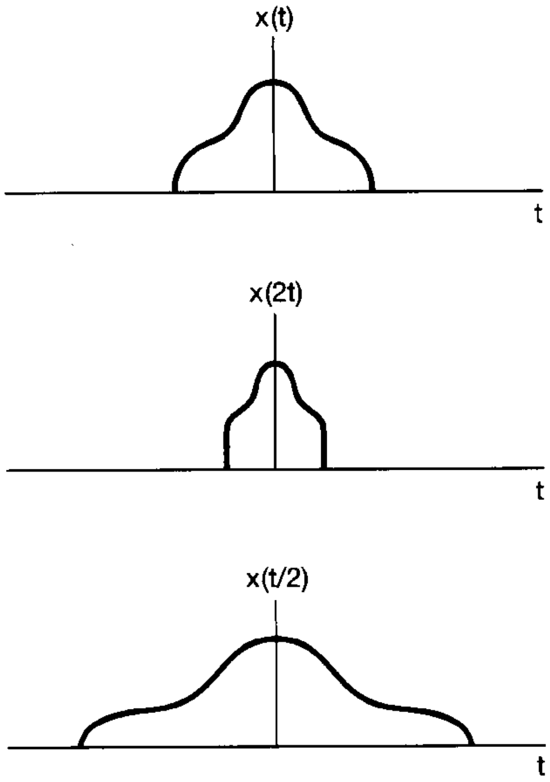


Figure 1.12 Continuous-time signals related by time scaling.

Example 1.1

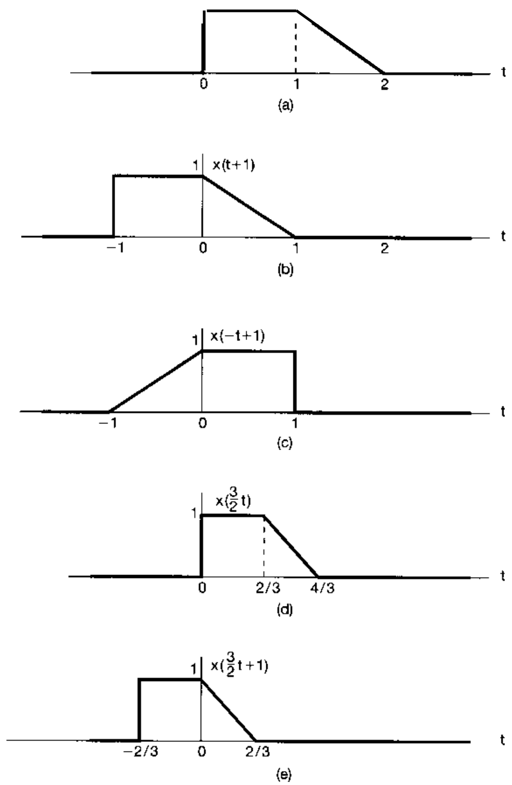


Figure 1.13 (a) The continuous-time signal $x(t)$ used in Examples 1.1–1.3 to illustrate transformations of the independent variable; (b) the time-shifted signal $x(t+1)$; (c) the signal $x(-t+1)$ obtained by a time shift and a time reversal; (d) the time-scaled signal $x(\frac{3}{2}t)$; and (e) the signal $x(\frac{3}{2}t+1)$ obtained by time-shifting and scaling.

$$x(t)$$

$$x(t+1)$$

$$x(-t+1)$$

$$x\left(\frac{3}{2}t\right)$$

$$x\left(\frac{3}{2}t+1\right)$$

1.2.2 Periodic signals

$$x(t) = x(t+T)$$
$$x[n] = x[n+N]$$

$$T : \text{period}, \; x(t) = x(t \pm T) = x(t \pm 2T) = \cdots$$
$$N : \text{period}, \; x[n] = x[n \pm N] = x[n \pm 2N] = \cdots$$

- Fundamental Period : The smallest positive value among the periods

$$T_0, N_0$$

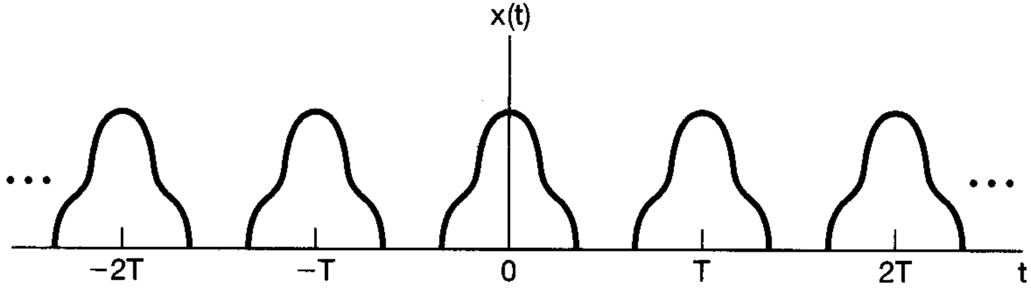


Figure 1.14 A continuous-time periodic signal.

$$x[n] = x[n+N]$$

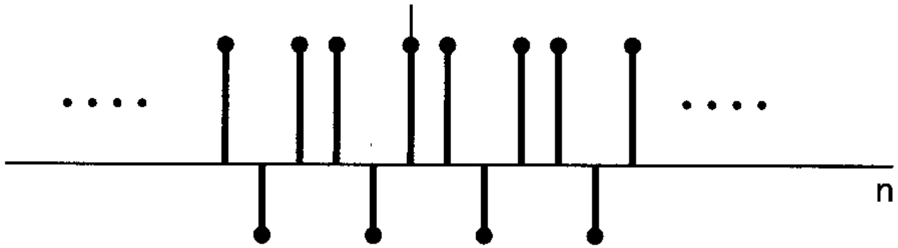


Figure 1.15 A discrete-time periodic signal with fundamental period $N_0 = 3$.

Example 1.4 Check of periodicity

$$x(t) = \begin{cases} \cos(t) & \text{if } t < 0 \\ \sin(t) & \text{if } t \geq 0 \end{cases}$$

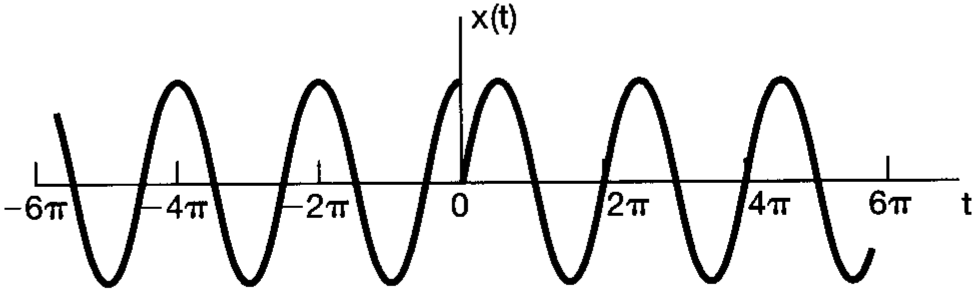


Figure 1.16 The signal $x(t)$ considered in Example 1.4.

1.2.3 Even and odd signals

- Even signal : $x(-t) = x(t)$ $\quad x[-n] = x[n]$
- Odd signal : $x(-t) = -x(t)$ $\quad x[-n] = -x[n]$

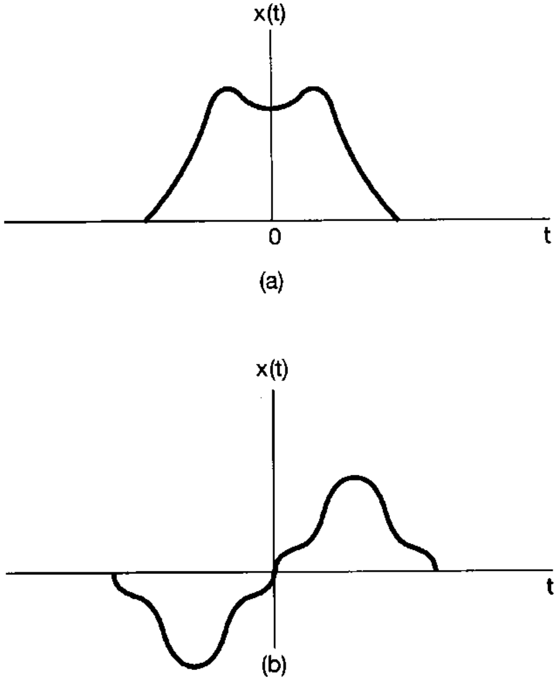


Figure 1.17 (a) An even continuous-time signal; (b) an odd continuous-time signal.

• Even-odd decomposition of a signal

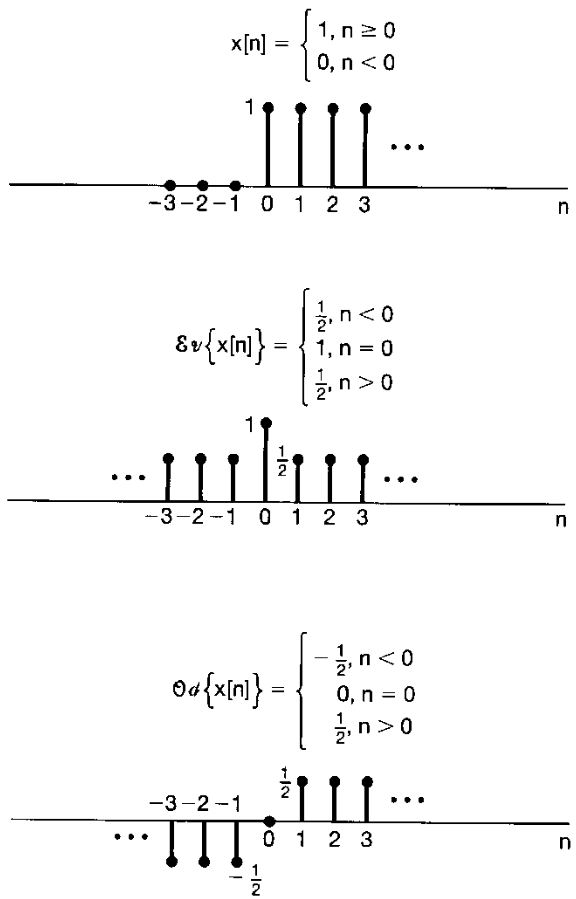


Figure 1.18 Example of the even-odd decomposition of a discrete-time signal.

$$Ev\{x(t)\} = \frac{1}{2}[x(t) + x(-t)]$$

$$Od\{x(t)\} = \frac{1}{2}[x(t) - x(-t)]$$

1.3 Exponential and sinusoidal signal

1.3.1 Continuous-Time Complex Exponential and Sinusoidal Signal

$$x(t) = Ce^{at}$$

- *Real Exponential Signals*

$C, a : \text{Real}$

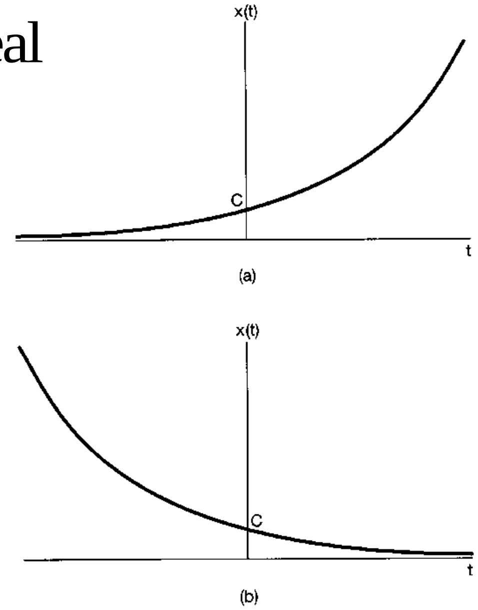


Figure 1.19 Continuous-time real exponential $x(t) = Ce^{at}$: (a) $a > 0$; (b) $a < 0$.

- *Periodic complex exponential and sinusoidal signals*

a : purely imaginary

$$x(t) = e^{j\omega_0 t}$$

$$e^{j\omega_0 (t+T)} = e^{j\omega_0 t} e^{j\omega_0 T}$$

$$e^{j\omega_0 T} = 1 \text{ if } T_0 = \frac{2\pi}{|\omega_0|}, \quad \text{the smallest positive value of } T$$

$$e^{j\omega_0 t} = e^{j\omega_0 (t+T)}$$

- Euler's Relation: $e^{j\omega_0 t} = \cos\omega_0 t + j\sin\omega_0 t$

$$A\cos(\omega_0 t + \phi) = A\,\mathrm{Re}\left\{e^{j(\omega_0 t + \phi)}\right\}$$

$$A\sin(\omega_0 t + \phi) = A\,\mathrm{Im}\left\{e^{j(\omega_0 t + \phi)}\right\}$$

$$A\cos(\omega_0 t + \phi) = \frac{A}{2} e^{j\phi} e^{j\omega_0 t} + \frac{A}{2} e^{-j\phi} e^{-j\omega_0 t}$$

- Periodic Signals: infinite total energy but finite average power

$$E_{period} = \int_0^{T_0} \left| e^{j\omega_0 t} \right|^2 dt = \int_0^{T_0} 1 \cdot dt = T_0$$

$$P_{period} = \frac{1}{T_0} E_{period} = 1$$

$$P_\infty \equiv \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left| e^{j\omega_0 t} \right|^2 dt = 1$$