5 Yield and Fracture

5.1 INTRODUCTION

•The stress-strain($\sigma - \epsilon$) curves of plastics typically deviates from the linearity

$$@\ \epsilon = 1\sim10\% \text{ and } \frac{d\sigma}{d\epsilon} = 0 \text{ (yield) } @\ \epsilon = 10\sim100\%$$

•Brittle plastics (PS, PMMA, SAN...) fracture right after or before yield while ductile plastics (PE, PP, Rubbers...) could reach over 1000% elongation.

•Ductile plastics typically shows yield → necking →strain hardening

•Recovery is driven by entropy elasticity in cross-link.

(Physical entanglements at high MW show similar efect)

•Fracture mechanisms

▷Brittle fracture is initiated by craze formation.

(But accompanied by a localized yielding near the crack tip)

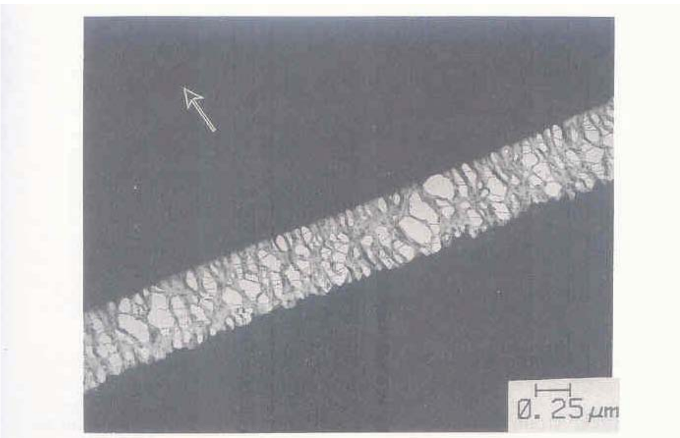
▷Ductile fracture is slip propagated by shear yielding.

•Resistance to yield and crazing determines the mechanism of fracture.

•Mechanism = f (T, strain rate, type of loading, hydrostatic pressure)

Compression, low shear rate, higher T, high p→Ductile fracture

Ex. PS in tension is fractured by brittle but by yielding in compression.



5.1 A craze in polystyrene; the arrow indicates direction of tensile stress (after R. P. Kambour).

Fig. 5.1 A craze in PS

Craze= Crack front where fibrils bridge the two surfaces.

Crack propagates ⊥ Applied stress

5.2 YIELDING (Fig. 5.2)

Standard method to measure modulus and yield= Tensile test

•Brittle polymer (PS) forms craze at very low strain and fracture

•**Ductile polymer** shows **five regions** (at most):

elastic (linear+nonlinear) →

yield (so far homogeneous deformation) →

strain softening (necking initiates**)**→

cold drawing (necking propagates**)**→

strain hardening (necking completed**,** chain fully oriented**)**→fracture

•Nonlinearity may occur in linear region due to the long measure time (Ch 4)

•**Yielding** occurs **by slip** at about **45** degrees to the tensile axis (on plane of maximum shear stress) while **craze** forms on plane **normal** to the tensile stress

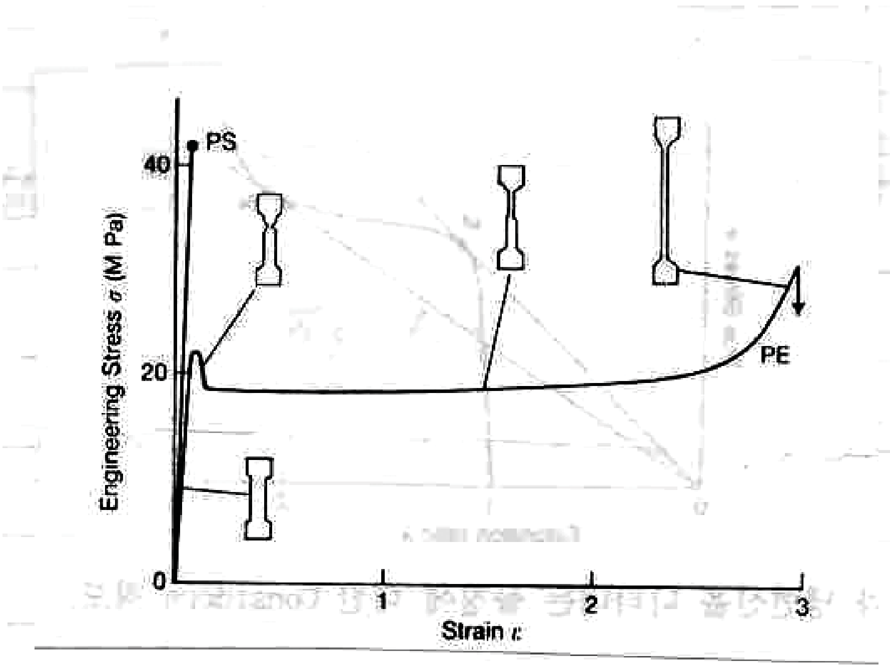


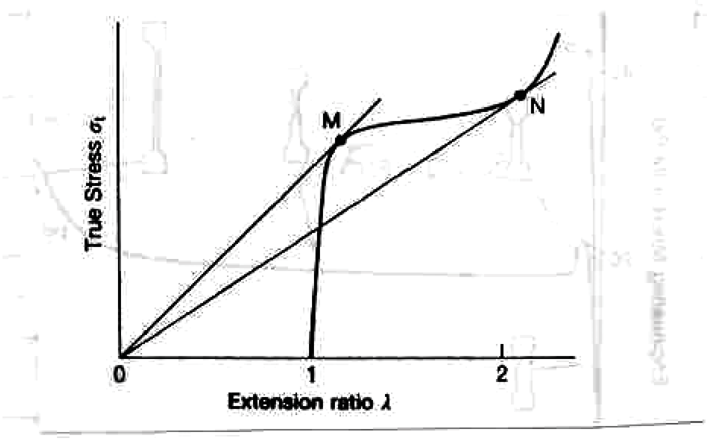
Fig 5.2 S-S ($\sigma - \epsilon$) curves for PS and PE

At yield point, flow takes place instantaneously at the imposed strain rate

$$\Rightarrow \frac{d\sigma}{d\epsilon} = 0$$

5.2.1 Considere's Construction

Useful for interpretation of yield and flow phenomenon



Initial length $L_i \xrightarrow{F} L$

Initial cross-section $A_i \xrightarrow{F} A$

Strain (tensile)

$$\epsilon = \frac{L-L_i}{L_i} = \lambda - 1 \qquad (1) \qquad \lambda = \frac{L}{Li} = \text{ Extension ratio}$$

Assume const volume (ok for ductile polymer)

$$A_i L_i = AL$$

$$A_i = \frac{L}{L_i} A = \lambda A$$

True stress $\qquad$ Nominal stress (=Engineering stress)

$$\sigma_t = \frac{F}{A} \qquad (2) \qquad \sigma = \frac{F}{A_i} \qquad (3)$$

$$\therefore \sigma = \frac{F}{A_i} = \frac{F}{\lambda A} = \frac{\sigma_t}{\lambda} \qquad (4)$$

$$\therefore \frac{d\sigma}{d\lambda} = \frac{1}{\lambda}\frac{d\sigma_t}{d\lambda} - \frac{\sigma_t}{\lambda^2} \qquad (5)$$

←Local slope in $\sigma - \lambda$ plot !

Also from (1)

$$d\epsilon = d\lambda$$

By definition $\frac{d\sigma}{d\epsilon} = 0$ @yield point

So, $\frac{d\sigma}{d\lambda} = 0$ @yield point (6)

Then from (5) $\frac{d\sigma_t}{d\lambda} = \frac{\sigma_t}{\lambda}$ @yield point (7)

local slope $\qquad$ overall slope

If (7) is satisfied yield point exists in engineering S-S curve.

That is, engineering stress-strain curve will show a maximum only if a tangent can be drawn from $\lambda = 0$ to touch the $\sigma_t - \lambda$ curve at a point as M.
A second point like N can also satisfy (7). At this point, neck stabilizes, and begins to extend on either side.

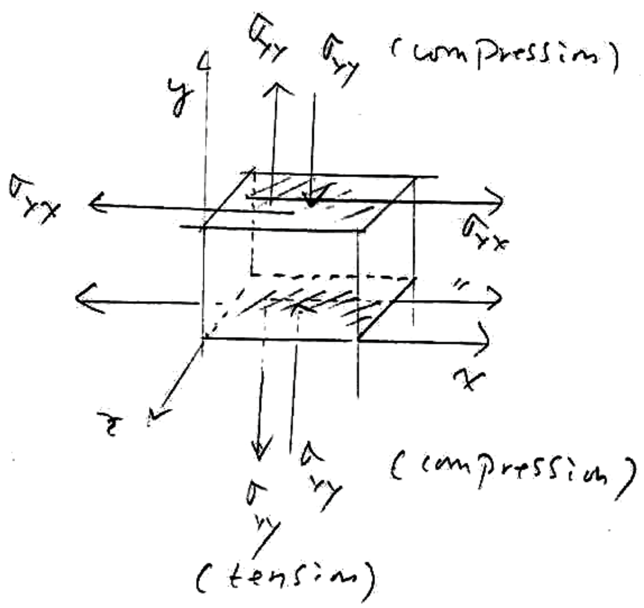
Notes on strain and stress tensor

Stress ····· A second order tensor

- Physical quantity
  - Scalar : Magnitude (1 component)
  - Vector : 〃 + Direction (3 components)
  - Tensor : 〃 + 〃 + Plane orientation (9 components)

σ_{ij}
j: Force is acting along x.
i: Plane is perpendicular to i axis.

- **Stress** $\equiv \frac{\textbf{Force}}{\textbf{Area}}$
- Stress
  - Normal stress(σ_{ii})
    - Tension
    - Compression
  - Shear stress(τ_{ij})



▶Plane = Vector

Magnitude: Area

Direction: Outward unit normal vector

▶Sign convention for stress: + force (or -) on + surface e (or -)→+ stress

+ force (or -) on - surface(or +)→- stress

→ Tension→ +, Compression→ - (Mechanic notation)

<The opposite convention is used for Chem E>

▶Stress tensor = Symmetric

→ $\sigma_{ij} = \sigma_{ji}$

$$\underline{\sigma} = \{\sigma_{ij}\} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix} = \begin{pmatrix} \sigma_{11} & \sigma_{21} & \sigma_{31} \\ \sigma_{12} & \sigma_{22} & \sigma_{32} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{pmatrix}$$

Diagonal elements: Normal stress (often denoted by σ_{ij})

Off-diagonal elements: Shear stress (often denoted by τ_{ij})

▶Elastic materials: $\sigma_{ii} = E\varepsilon_{ii}$ or $\tau_{ij} = G\gamma_{ij}$ (Constitutive eq)

→Strain tensor= Symmetric: $\gamma_{ij}=\gamma_{ji}$

$$\underline{\gamma} = \{\gamma_{ij}\} = \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} \end{pmatrix} = \begin{pmatrix} \gamma_{11} & \gamma_{21} & \gamma_{31} \\ & \vdots & \\ & \vdots & \end{pmatrix}$$

Tensile strain is often denoted by ε_{ii} or ε_i but used interchangeably.

▶ Viscous materials: $\tau_{ij} = \mu\dot{\gamma}_{ij}$ (Constitutive eq)

→Rate of strain tensor= Symmetric ($\tau_{ij} = \mu\dot{\gamma}_{ij}$)

$$\underline{\dot{\gamma}} = \{\dot{\gamma}_{ij}\} = \begin{pmatrix} \dot{\gamma}_{11} & \dot{\gamma}_{12} & \dot{\gamma}_{13} \\ \dot{\gamma}_{21} & \dot{\gamma}_{22} & \dot{\gamma}_{23} \\ \dot{\gamma}_{31} & \dot{\gamma}_{32} & \dot{\gamma}_{33} \end{pmatrix} = \begin{pmatrix} \dot{\gamma}_{11} & \dot{\gamma}_{21} & \dot{\gamma}_{31} \\ & \vdots & \\ & \vdots & \end{pmatrix}$$

Tensile strain rate is often denoted by $\dot{\varepsilon}$

▶Definition of strain

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

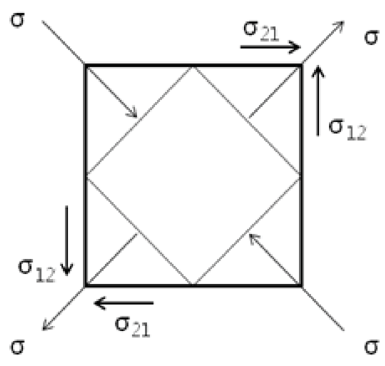
$$\varepsilon_{ij} = \begin{vmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{vmatrix} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{\partial v}{\partial y} & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) & \frac{\partial w}{\partial z} \end{vmatrix}$$

u, v, w: Displacwments in x, y, z directions, respectively

▶Axes along which there are no shear strains are called **principal axes:**
Only pure extension or contraction exists.

▶**Principal strain** axes coincide with the **principal stress** axes for isotropic solids.

▶As the coordinate axes rotate, there exists an angle where the shear stress becomes zero=principal axes



▶Volume change or dilatation (Δ):

$\Delta = (1+\varepsilon_1)(1+\varepsilon_2)(1+\varepsilon_3) - 1 = \varepsilon_1+\varepsilon_2+\varepsilon_3$ for $\varepsilon << 1$

$\Delta = \varepsilon_1+\varepsilon_2+\varepsilon_3$ is the first invarient of strain tensor.

▶With $\Delta = 0 \rightarrow \sum\sigma_{ii}=0$: No volume changing process occurs.

5.2.3 Yield under Multiaxial Stress

So far, uniaxial stress (tensile, compression, shear) was considered.

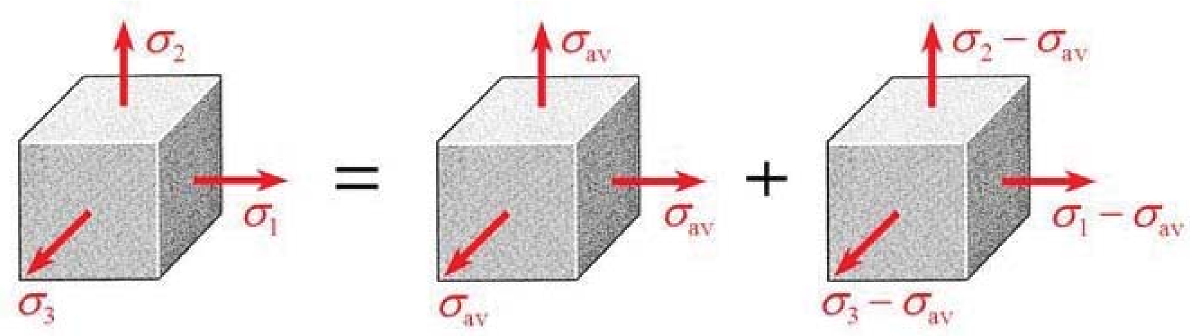
Fracture resistance depends on the ability to develop a yield zone in the region of a crack tip, where it is usually in a state of triaxial tension.

Decomposition of the principal stress

The principal stresses σ_1, σ_2, and σ_3 can be decomposed into the sum of two stresses:

(1) hydrostatic stress due to the stresses σ_{av} acting in each of the principal directions and causing only volume change (static), and

(2) deviatoric stress causing angular distortion without volume change.



Average stress (A stress invarient):

$$\sigma_{av} = \frac{\sigma_1+\sigma_2+\sigma_3}{3}$$

Hydrostatic pressure:

$$p = -\frac{(\sigma_1+\sigma_2+\sigma_3)}{3}$$

Deviatoric stress:

$$\sigma_1 = \sigma_{av} + (\sigma_1 - \sigma_{av})$$

$$\sigma_2 = \sigma_{av} + (\sigma_2 - \sigma_{av})$$

$$\sigma_3 = \sigma_{av} + (\sigma_3 - \sigma_{av})$$

von Mises Yield Criterion

(Below is one of several version)

Yield occurs when

$$(\sigma_{11}-\sigma_{22})^2+(\sigma_{22}-\sigma_{33})^2+(\sigma_{33}-\sigma_{11})^2+6\,({\sigma_{12}}^2+{\sigma_{23}}^2+{\sigma_{31}}^2)\geq 6C^2 \qquad (14)$$

C∝ p (hydrostatic pressure) for polymer,

but Independent of hydrostatic pressure for metal:

$$[(\sigma_{11}-\sigma_{22})^2+(\sigma_{22}-\sigma_{33})^2+(\sigma_{33}-\sigma_{11})^2+6\,({\sigma_{12}}^2+{\sigma_{23}}^2+{\sigma_{31}}^2)]^{1/2}\geq \frac{1}{\sqrt{2}}{\sigma_0}^2 \qquad (14)'$$

for metal where σ_0 the uniaxial yield stress.

Check for uniaxial case; $\sigma_{22}=\sigma_{33}=0$, $\sigma_{ij}=0$ for $i\neq j$

→ LHS=$\sqrt{2}\,\sigma_{11}$

Example 5.3

A sample of linear polyethylene tested at 23°C and 10^{-3} s^{-1} yielded at

30.0 MPa in uniaxial tension, and at 31.5 MPa in uniaxial compression. Assuming that the yield stress is a linear function of hydrostatic pressure, calculate σ_y under superimposed hydrostatic pressure of 500 MPa.

Solution

The difference between tensile and compressive yield stress arises because of the hydrostatic component of the applied stress. For any state of stress, p is an **invariant** given by

$$p = -\tfrac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33}).$$

In tension

$$p = -\tfrac{1}{3}\sigma_{11} = -\tfrac{1}{3}30 = -10 \text{ MPa}.$$

In compression

$$p = -\tfrac{1}{3}\sigma_{11} = -\tfrac{1}{3}(-31.5) = +10.5 \text{ MPa}.$$

Writing $C_0 + C_1 p$ in place of C in eqn 5.14, we have for both cases

$$|\sigma_{11}| = 3^{\frac{1}{2}}(C_0 + C_1 p).$$

Tension

$$30.0 = 3^{\frac{1}{2}}(C_0 - 10C_1).$$

Compression

$$31.5 = 3^{\frac{1}{2}}(C_0 + 10.5C_1).$$

From this we obtain $C_0 = 17.743$ MPa and $C_1 = 0.04225$ (for p in MPa). When tests are conducted in a pressure chamber, the hydrostatic stress term p is the sum of the superimposed pressure and the hydrostatic component of the stress applied by the testing machine. Let σ_y be the nominal yield stress, considering the case of extensional loading. Then

$$p = 500 - \tfrac{1}{3}\sigma_y \text{ MPa}.$$

The condition for yielding then becomes

$$\sigma_y = \sqrt{3}\left(C_0 + 500C_1 - \tfrac{1}{3}\sigma_y C_1\right).$$

Substituting and rearranging, we have

$$\sigma_y = \frac{17.743\sqrt{3} + 0.04225 \times 500\sqrt{3}}{1 + \left(\dfrac{0.04225\sqrt{3}}{3}\right)}$$

$$\sigma_y = 65.7 \text{ MPa}$$