Comment

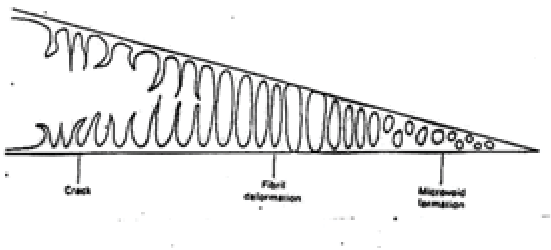
This reveals a large pressure sensitivity of σ_y, quite unlike the case of metal, in which the yield stress is essentially insensitive to the hydrostatic pressure. Plastics used for submarine parts are hardly yielded.

5.3 Crazing

- Time dependant failure mode which occurs for most glassy thermoplastics (PMMA,PS…), but sometimes with semi-crystalline polymers such as PP.
- Wedge type regions where highly oriented fibrils bridge the two surfaces
- Crack : Regions where fibrils are torn, and the bulk surfaces are separated

  Fibril fracture → crack initiation

  Craze = crack front



▸Craze nucleates at points of high stress concentration, viz., at crack tip, surface, and inclusion.

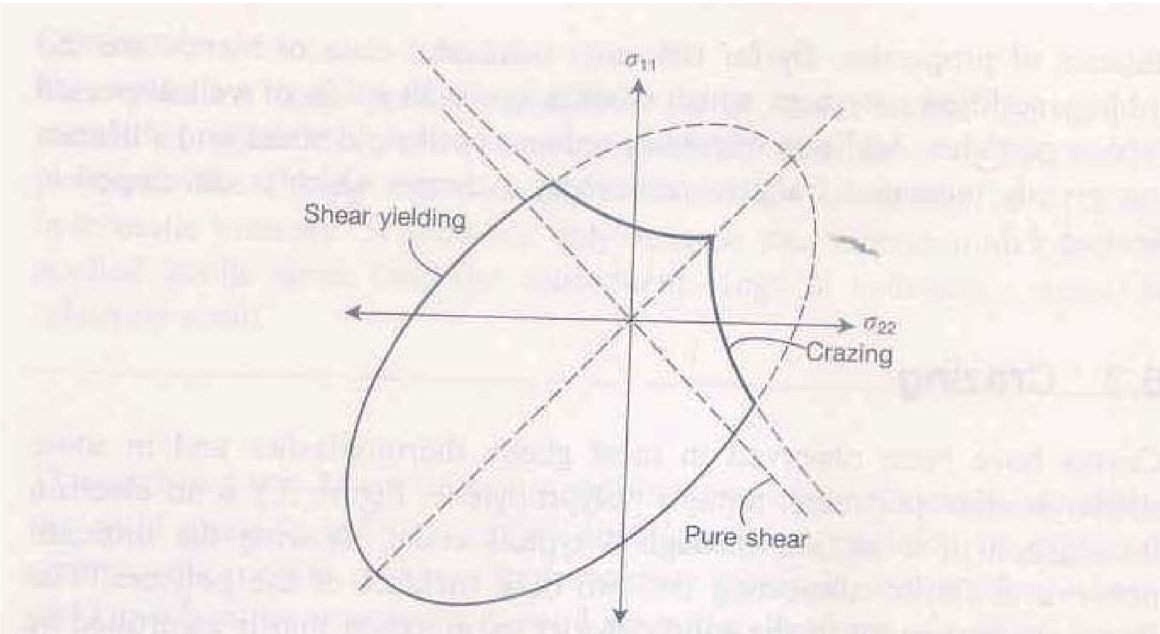
▸Internal as well as external stress causes nucleation.

▸Internal stress arises from: differential contraction during cooling in the mold for injection molding, and relaxation of oriented molecules i.e., entropic recovery force eliminates the orientation by heating or by absorption of liquid

▸ Craze initiation, growth rates depend on the applied stress and T.

▸Kinetics of crazing and yielding determines brittle or ductile fracture.

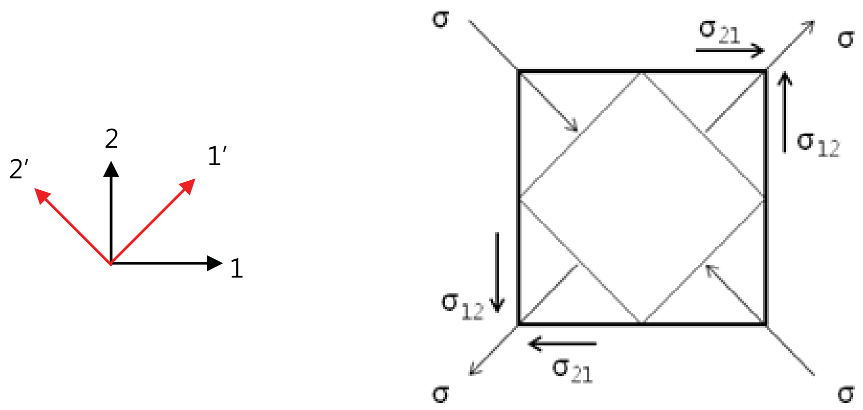
▸Major difference between the deformation mechanisms in Fig 7:
Failure envelop of PMMA under biaxial loading in 1, 2 directions.



5.7 Failure envelope (heavy continuous line) for PMMA under biaxial stress ($\sigma_{33}=0$) at room temperature, showing intersection of crazing and shear yielding envelopes (after S. Sternstein and L. Ongchin).

▹**Pure shear** line (σ_{11}=-σ_{22} line) = Boundary between hydrostatic compression and hydrostatic tension :

State of stress for pure shear:



$$\{\sigma_{ij}\} = \begin{pmatrix} 0 & \sigma & 0 \\ \sigma & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{In rotated axes:} \quad \{\sigma_{ij}\}' = \begin{pmatrix} \sigma & 0 & 0 \\ 0 & -\sigma & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

In both coordinates, $\boldsymbol{\sum\sigma_{ii} = 0}$ →No stress driving volume change occurs.

All deformations are driven by shear.

Zero mean stress: No hydrostatic pressure

Note p is independent of coordinate axis.

▷Below this line: **Compression**

Pressure tends to reduce the volume → **Crazing and other hole-forming process do not take place** because the pressure component of the stress matrix tends to decrease rather than to increase the volume**.**

▷Above this line**: Crazing** is the principal mechanism for fracture.

<u>For multiaxial loading</u>

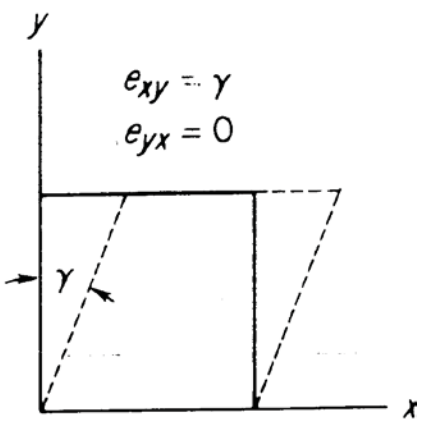
▶Craze envelop is asymptotic to the pure shear.

▶Within the tensile quadrant (1st) and over part of 2nd, 4th quadrants,

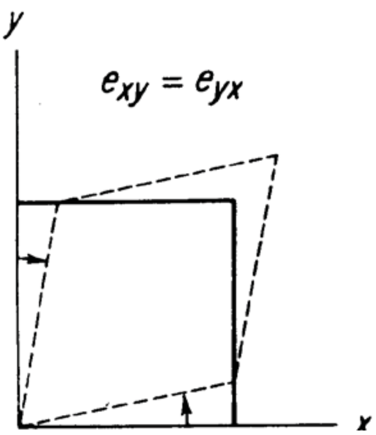
crazing envelop < shear yielding envelop →Material fractures without yielding.

▶Below the pure shear line PMMA failures by a pressure-dependent von Mises criterion (Eq 14) where C increasing with p.

Notes types of shear



Simple shear (du/dy≠0, dv/dx=0)



Pure shear (du/dy= dv/dx≠0,)

Solvent crazing, environmental stress crack

- ▸Crazing and fracture by absorbed liquid (water, solvent...) are most serious limitation on the use of plastics (solvent crazing, environmental stress crack) provided they plasticize the polymers.

ASTM D 1693 for ESCR

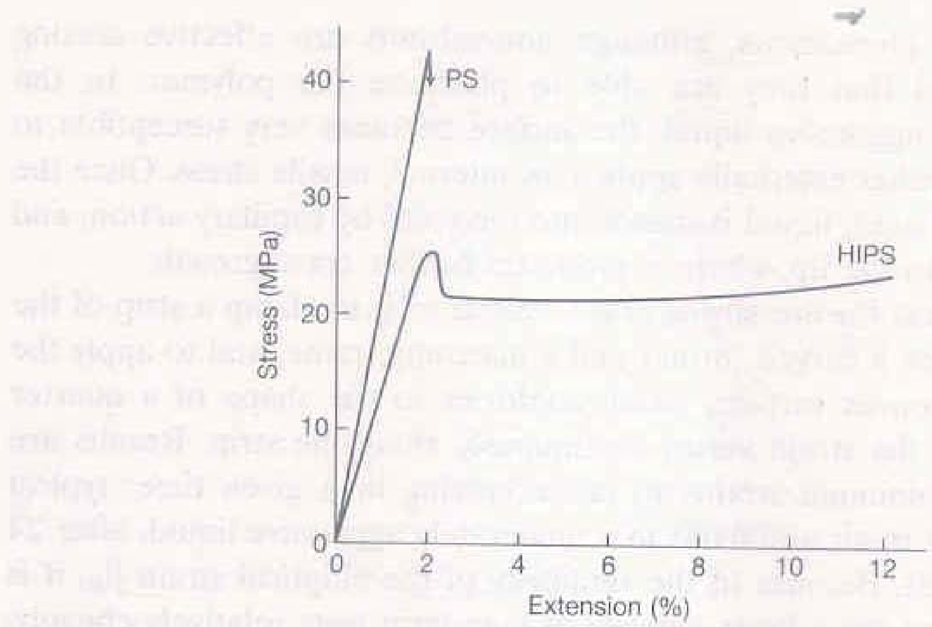
A surface cut of a specified length and depth is made on the sample parallel to the long axis with a mounted razor blade. The specimens are stressed by bending them 180 degrees. The specimens are then placed in a stress cracking agent at 50°C. Periodically, the specimens are inspected for visual cracks perpendicular to the cuts. A PE is failed when 5 of 10 specimens show cracks perpendicular to the cuts.

Rubber particles in plastics (ABS, HIPS and any rubber toughened plastics)

- ▸As **stress concentrator, not only initiate multiple crazing at low applied stress, but also extend and deform with the crazed matrix providing stability against premature fracture.** (Elastic deformation consumes mechanical energy elastically)
- ▸Crazing itself is of minor importance in engineering, but significant in the sense that it is the **precursor to fracture**.

See Fig 8 for stress-strain behavior for HIPS

- ▸PS, SAN foam only a few crazes before fracture at low strain, 0.02..
- ▸But, rubber toughened grades form large numbers of craze (**multiple crazes**) allowing them to yield and extend strain up to 0.12 before fracturing
- ▸Numerous reflecting planes → Yielded materials whitened

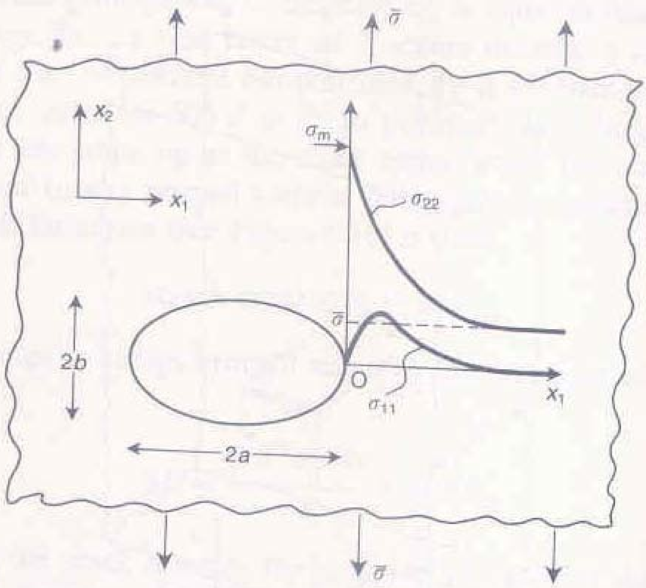


5.8 Stress–strain curves for polystyrene (PS) and high-impact polystyrene (HIPS) which is a mixture of polystyrene with extremely small rubber particles.

5.4 Linear Elastic Fracture Mechanics

Brittle solids fracture because the applied stress is amplified by minute cracks, O(1μ) in size which occur naturally as a result of fabrication, solidification, fatigue damage, etc. (called **Griffith crack**)

Fig. 5.9: An elliptic through-thickness crack



5.9 An elliptical, through-thickness crack in an elastic sheet subject to a stress $\bar{\sigma}$ in the x_2-direction causes a stress distribution along Ox_1 as shown; σ_{22} is amplified from $\bar{\sigma}$ to σ_m at both tips of the crack.

Thickness = B

Width = W >> 2a (called wide or infinite width)

Axes of ellipse = 2a, 2b

A force F is applied to the end surface to develop the stress,

$$\bar{\sigma} = \frac{F}{WB} \qquad (15)$$

The amplification of the applied stress is the greatest at the crack surface (x_1=0) and is given by

$$\frac{\sigma_m}{\bar{\sigma}} = [1 + \frac{2a}{b}] \qquad (16)$$

For a circular crack

$$\sigma_m = 3\bar{\sigma}$$

For a/b=500 (sharp crack)

$$\sigma_m = 1000\bar{\sigma}$$

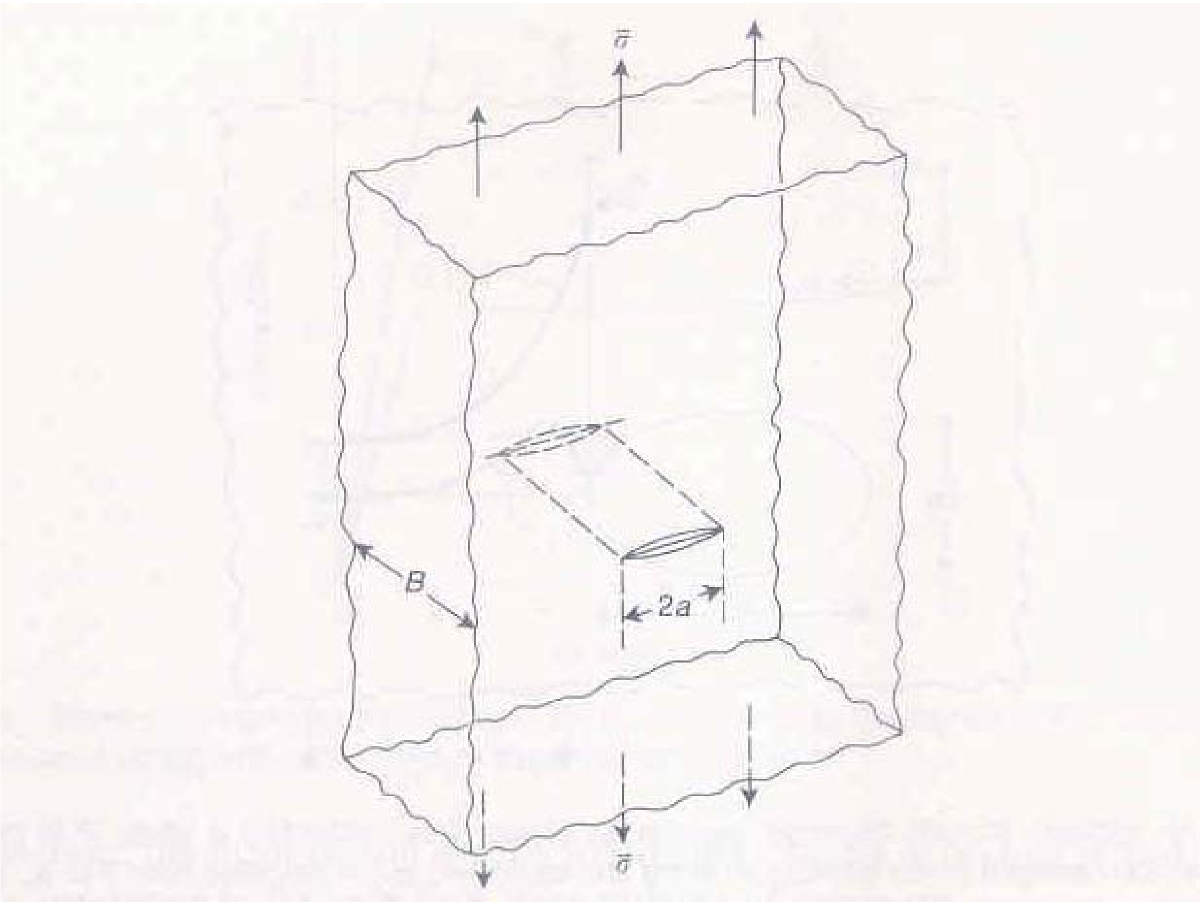
→Even at low applied stress the stress at the crack tip may approach the theoretical strength and the material fracture, if the materials do not relieve the stress concentration by plastic flow (yield) or other mechanism of crack blunting..

But failure is inhibited by energy absorbing process around the crack tip:

A crack will spread only if the total energy of the system is lowered thereby.

● Consider a through thickness (B) sharp crack with a crack length 2a(Fig 10)

Total energy of the system= elastic strain energy+work of crack formation

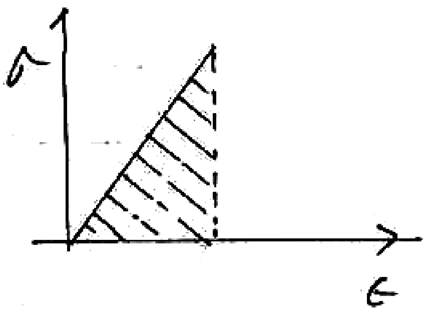


5.10 A narrow, through-thickness crack in a thin, wide sheet subject to a stress $\bar{\sigma}$.

● Elastic strain energy per unit volume:

$$u = \int_0^{\epsilon} \sigma d\epsilon = \int_0^{\epsilon} E\epsilon d\epsilon = E\frac{\epsilon^2}{2} = \frac{\sigma\epsilon}{2} = \frac{\bar{\sigma}^2}{2E} \leftarrow \left[\frac{\text{Energy}}{L^3} = \frac{\text{Force}}{L^2}\right]$$

Assume brittle fracture



Area = Elastic strain energy

= Toughness

▶The total strain energy prior to the introduction of crack $= V\frac{\bar{\sigma}^2}{2E}$ (a)

(V= Specimen volume)

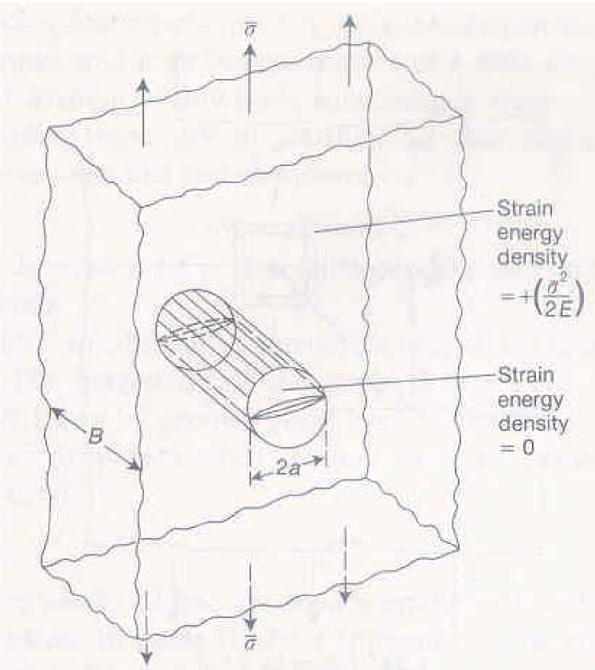
▶Introduction of crack modifies the stress field around the crack (Fig 9). Then, using the stress field, total strain energy can be calculated as

Total strain energy $= \int_V \frac{\sigma^2(\text{local})}{2E} dV$ (b)여기에 수식을 입력하십시오.

The net effect of crack insertion= (b)-(a)

$= -\frac{\bar{\sigma}^2 \pi a^2 B}{2E}$ (rough approximation) (**Decreased**) (See Fig 5.23 in Note 5N4)

$= -\frac{\bar{\sigma}^2 \pi a^2 B}{E}$ (better agreed with computation)



5.23 Figure for 5.N.4.

Interpretation: The effect of crack is to drain off completely the strain from a cylinder of diameter 2a centered on the crack, and to leave the strain energy outside the cylinder at $\frac{\bar{\sigma}^2}{2E}$ per volume$(= \pi a^2 B)$ (Strain energy release).

● Work of crack formation

As crack spreads, two new surfaces are prised by mechanical force. The work done for unit area =G_c: =Work to form new surface + Dissipation to heat.

Griffith assumed

No heat dissipation + Crack surfaces are normal (identical to a surface not formed by crack) →

$G_c = 2\gamma$ ← Normal surface energy (measured from contact angle)

(1 m^2 of crack is composed of 2 m^2 of surface)

↑ Work done for unit area of crack surface

For polymers

$2\gamma = 0 \ (1\ J/m^2)$

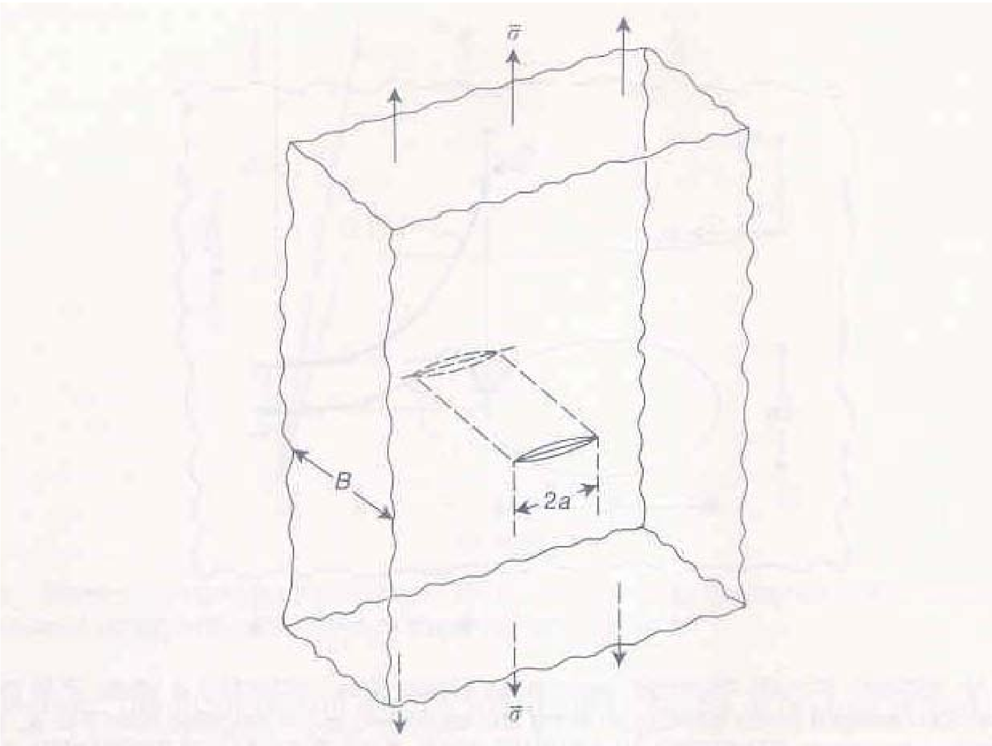
$G_c = 0 \ (500\ J/m^2)$

$\Rightarrow G_c \gg 2\gamma$: Much heat is generated and dissipated at the crack tip

<u>Total work of crack formation</u>

Total work = G_c x Crack area

= $2aB\,G_c$ (See Fig 5.10)



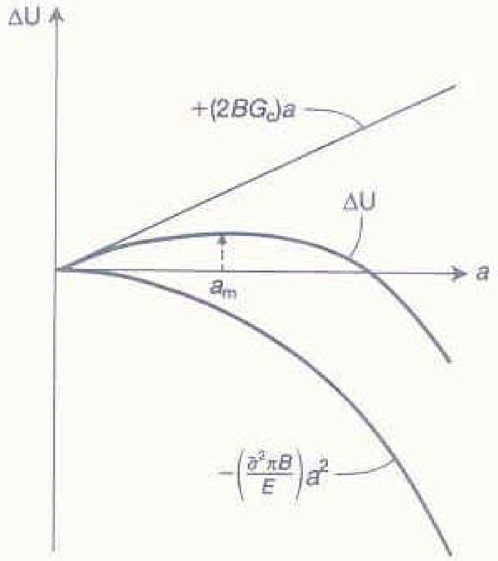
5.10 A narrow, through-thickness crack in a thin, wide sheet subject to a stress $\bar{\sigma}$.

The change in energy brought about by the introduction of the crack is

$$\Delta U = -\frac{\bar{\sigma}^2 \pi a^2 B}{E} + 2aB\,G_c \qquad (17)$$

Increases as crack spreads (↑ $+2aB\,G_c$)

Elastic strain energy decrease as the crack spreads. (↑ $-\frac{\bar{\sigma}^2 \pi a^2 B}{E}$)



5.11 Dependence of ΔU (the change in energy of a wide sheet) on a (the length of the crack is $2a$) at constant $\bar{\sigma}$. For $a < a_m$ the crack is stable; for $a > a_m$ the crack propagates catastrophically. (Fixed-grip conditions.)

For small a (a<1): ($2aB\,G_c$) term dominates.

ΔU increases as crack spreads.

For large a (a>1): a^2 term dominates.

ΔU decreases as crack spreads.

∴ With crack propagation, an "**a**" is obtained where

"Work of crack propagation = Decrease in strain energy"

$$\Rightarrow \frac{dU}{da} = 0 \quad @ \text{ maximum U}$$

$$\frac{d}{da}\left(\frac{\bar{\sigma}^2 \pi a^2 B}{E}\right) = \frac{d}{da}(2aB\,G_c) \qquad (18)$$

$$\frac{\bar{\sigma}^2 \pi (2a) B}{E} = 2B\,G_c$$

$$EG_c = \pi a \bar{\sigma}^2 \qquad (19)$$

$$a_m = \frac{EG_c}{\pi \bar{\sigma}^2} \qquad (20)$$

↑Maximum value of "**a**" which is stable under given $\bar{\sigma}$

For $a < a_m$

$$\frac{du}{da} > 0\ ;\ as\ a \uparrow \Rightarrow \Delta U \uparrow$$

Crack will not spread.

For $a > a_m$

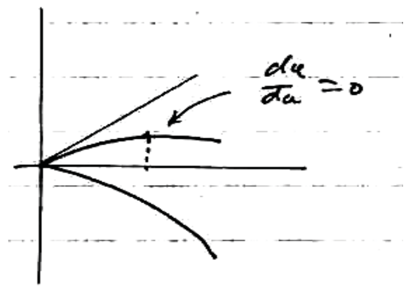
$$\frac{dU}{da} < 0\ ;\ as\ a \uparrow \Rightarrow \Delta U \downarrow$$

Crack will spread catastrophically.

Fracture stress

From (19) $\quad EG_c = \pi a \bar{\sigma}^2 \quad$ (19)

$$\bar{\sigma}_F = \left(\frac{EG_c}{\pi a}\right)^{\frac{1}{2}} \qquad (21)$$



Griffith Eq $(G_c = 2\gamma)$

$$\bar{\sigma}_F = \left(\frac{E(2\gamma)}{\pi a}\right)^{\frac{1}{2}} \qquad (21)'$$

(21), (21)': Valid for plane stress conditions at the crack tip, which are most likely to occur for thin sheet. ($\mathbf{G_c} =$ **fracture energy**, applicable for plane stress)
$\mathbf{G_c} =$ **plane stress fracture energy**

For plane stress more useful parameter = **Critical stress intensity factor** ($\mathbf{K_c}$) **which is defined in the case of wide sheet, by**

$$K_c \equiv \bar{\sigma}_F (\pi a)^{\frac{1}{2}} \qquad \text{(Plane stress)} \quad (22)$$

$$= \left(\frac{EG_c}{\pi a}\right)^{\frac{1}{2}} (\pi a)^{\frac{1}{2}} = (EG_c)^{\frac{1}{2}} \qquad (23)$$

K_c or G_c is fracture parameter for plane stress since the two are related by (23).
<$G_c=K_c^2/E$>

Determination of Kc

Measure the value of $\bar{\sigma}$ at which a crack of length 2a in a thin, wide plate starts to begin, and determine Kc using (23).

Use of Kc

For the largest crack

if K < Kc ⇒ Crack does not spread.

K > Kc ⇒ Crack will spread.

where $K = \bar{\sigma}(\pi a)^{\frac{1}{2}}$

Scan Ex 5.4 here

Example 5.4

A sharp, central crack of length 60 mm in a wide, thin sheet of a glassy plastic commences to propagate at $\bar{\sigma}_F = 3.26$ MPa. (i) Find K_c; (ii) find G_c given that $E = 3$ GPa; and (iii) will a crack of length 2 mm in a similar sheet fracture under $\bar{\sigma} = 10$ MPa?

(i) $$K_c = \bar{\sigma}_F(\pi a)^{\frac{1}{2}} = 3.26\left(\pi \times \tfrac{1}{2} \times 60 \times 10^{-3}\right)^{\frac{1}{2}}$$
$$= 1.00 \text{ MPa m}^{0.5}.$$

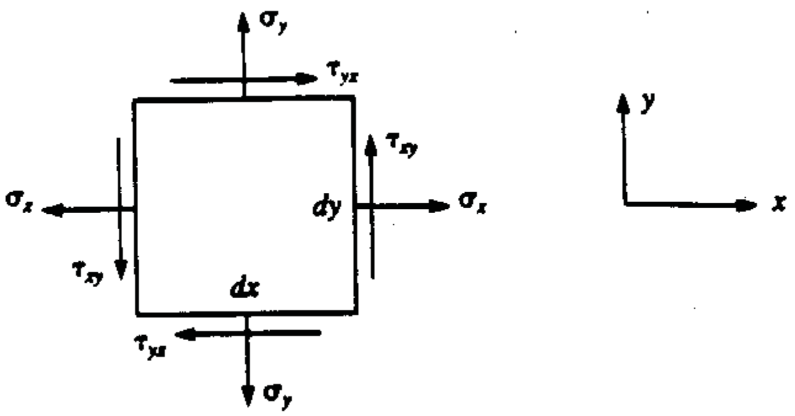
(ii) $$G_c = K_c^2/E = (10^6)^2/3 \times 10^9$$
$$= 333 \text{ J m}^{-2}.$$

(iii) $$K = \bar{\sigma}(\pi a)^{\frac{1}{2}} = 10(\pi \times 1 \times 10^{-3})^{\frac{1}{2}}$$
$$= 0.56 \text{ MPa m}^{0.5}.$$

K is 56% of K_c: the sheet will not fracture.

Plane stress vs Plane strain

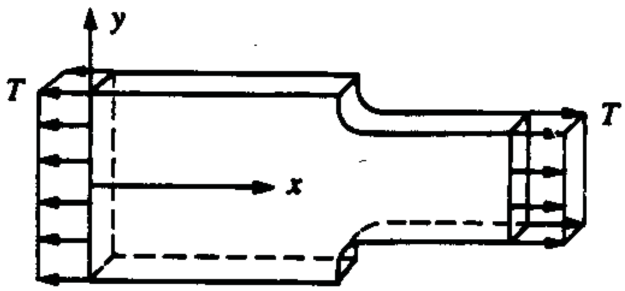
Two dimensional state of stress



σ_{xx}, σ_{yy} = Normal stress

τ_{xy}, τ_{yx} = Shear stress

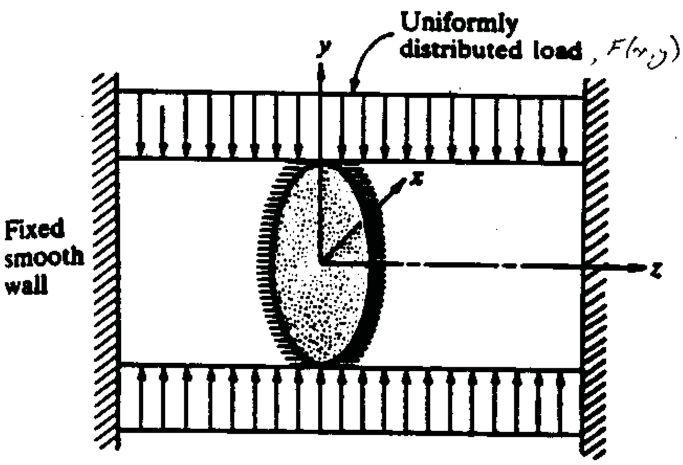
Stress is confined on the xy plane, i.e., as normal stress (σ_{zz}) and shear stress (τ_{xz}, τ_{yz}) directed perpendicular to the xy plane are zero. One dimension (z) is very thin as comparedwith the other two dimensions (x and y) (Thin plate, sheet, film). The loadds are applied uniformly over the thickness in the plane of the plate (See below)



Plane strain

Strain is confined on xy plane, i.e., $\varepsilon_z = 0$, $\gamma_{xz} = 0$, $\gamma_{yz} = 0$.

Z dimension is very large as compared with x and y dimemsion. Applied forces act on xy plane and do not vary in z direction, the forces are uniformly distributed wrt z and act perpendicular to it (Prismatic cylinder, tunnel).



In the thin sheet, thckness decrease at the crack tip due to the Poisson contraction; **plane stress conditions occur at the crack tip**. In thick plate, at the crack tip thickness does not decreases since through thickness stresses are generated which offset Poisson contraction. **Plane strain conditions occur at the crack tip** for thick specimen. Note in this case, the outer surfaces of the plate a thin skin of materials deforms in plane stress because there, at the surface, can not be through thickness stress. Plane strain conditions reduce shear yielding since all three principal stresses are tensile the shear stress generated is small- Yielding is driven by shear stress can not occur in pure three axial tension.

Parameters describing plane strain fracture : $\mathbf{K_{IC}, G_{IC}}$

Prising open by tensile force

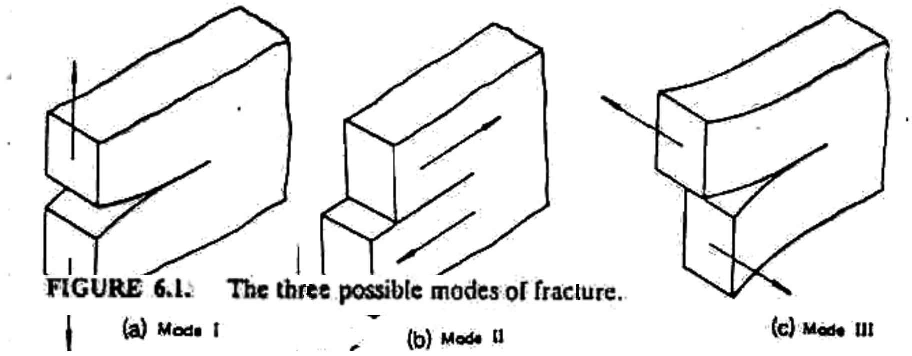


FIGURE 6.1. The three possible modes of fracture.

Modes of fracture

Ⅰ: opening mode of fracture

Ⅱ: sliding mode of fracture

Ⅲ: tearing mode of fracture

$$\bar{\sigma}_F = \left(\frac{EG_{IC}}{\pi(1-\nu^2)a}\right)^{\frac{1}{2}} \qquad \text{(plane strain)} \quad (24)$$

$$K_{IC} = \bar{\sigma}_F(\pi a)^{\frac{1}{2}} \qquad \text{(plane strain)} \quad (25)$$

$$= \left(\frac{EG_{IC}}{(1-\nu^2)}\right)^{\frac{1}{2}} \qquad \text{(plane strain)} \quad (26)$$

Note

G_C, K_C : Critical plane stress parameters

G_{IC}, K_{IC} : Critical plane strain parameters

ν is the Poisson's ratio defined by (for 1D in x-direction)

$\nu = -\frac{\epsilon_{yy}}{\epsilon_{xx}} = -\frac{\epsilon_{zz}}{\epsilon_{xx}} \sim 0.5$ for polymer

The stress intensity factor in a plane strain under $\bar{\sigma}$ for a wide plate is

$$K_I = \bar{\sigma}\ (\pi a)^{\frac{1}{2}} \qquad (27)$$

For $K_I < K_{IC}$ the plate will not fracture.

Scan Ex 5.5 here

Example 5.5

A thick, wide plate of polystyrene contains a central, sharp crack of length $2a = 40$ mm. The crack is found to propagate at $\bar{\sigma}_F = 4.20$ MPa. (i) Find K_{IC}; (ii) find G_{IC} given that $E = 3.0$ GPa and $\nu = 0.40$; and (iii) will a crack of length 2 mm in a similar sheet fracture if $\bar{\sigma} = 10$ MPa?

Solution

(i) $$K_{IC} = \bar{\sigma}_F(\pi a)^{\frac{1}{2}} = 4.20\left(\pi \times \tfrac{1}{2} \times 40 \times 10^{-3}\right)^{\frac{1}{2}}$$
$$= 1.05 \text{ MPa m}^{0.5}.$$

(ii) $$G_{IC} = \frac{(1-\nu^2)K_{IC}^2}{E} = \frac{(1-0.4^2)(1.053 \times 10^6)^2}{3.0 \times 10^9}$$
$$= 310 \text{ J m}^{-2}.$$

(iii) $$K_I = 10 \times 10^6(\pi \times 1 \times 10^{-3})^{\frac{1}{2}}$$
$$= 0.56 \text{ MPa m}^{0.5}$$

K_I is 53% of K_{IC}: the plate will not fracture.

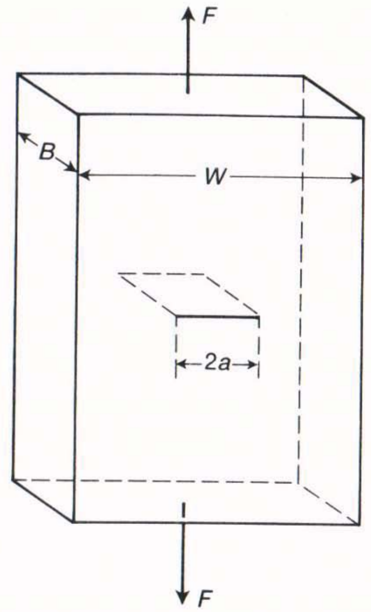
5.4.1 Measurement and Application of K_{IC}

For a thick plate of infinite width (W>>2a) (Fig 12), the applied force F develops $\bar{\sigma}$ as

$$\bar{\sigma} = \frac{F}{BW}$$

Scan Fig 12 here

Scan Ex 5.6 here



5.12 A crack of length $2a$ in a sheet of width W; $2a$ is of the same order of magnitude as W.

At $\bar{\sigma} = \bar{\sigma}_F$, crack starts to grow, and $K_I = K_{IC}$

$$K_{IC} = \bar{\sigma}_F \left[W \tan\frac{\pi a}{W}\right]^{\frac{1}{2}} \quad (28) \qquad K_{IC} = \bar{\sigma}_F(\pi a)^{\frac{1}{2}} \quad (25)$$

where

$$K_I = \bar{\sigma}\left[W \tan\frac{\pi a}{W}\right]^{\frac{1}{2}} \quad (29) \qquad K_I = \bar{\sigma}\ (\pi a)^{\frac{1}{2}} \quad (27)$$

For $\frac{W}{a} = \infty \Rightarrow$ (28) → (25), (29) → (27)

<Note Taylor series expansion for $\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 \ldots\ldots$>

In general, **plane strain toughness** < **plane stress toughness**

So, $\mathbf{G_{IC} < G_C}$ **and** $\mathbf{K_{IC} < K_C}$

∴ Materials not fractured byplane structue do not fracture by plane stress.

Scan Ex 5.6 here

Example 5.6

A plate of polystyrene of width $W = 100$ mm contains a central sharp crack of length $2a = 40$ mm. The crack is found to propagate at $\bar{\sigma}_F = 3.91$ MPa. (1) Find K_{IC}. (2) Will a central crack of length 14 mm in an identical plate propagate under $\bar{\sigma} = 9$ MPa? (3) Will a crack of length 3 mm in an infinitely wide polystyrene plate propagate under a stress of 10 MPa?

Solution

(1)
$$K_{IC} = \bar{\sigma}_F \left[W \tan\left(\frac{\pi a}{W}\right)\right]^{\frac{1}{2}}$$
$$= 3.91\left[100 \times 10^{-3} \times \tan\left(180° \times \frac{20}{100}\right)\right]^{\frac{1}{2}}$$
$$= 1.05 \text{ MPa m}^{0.5}.$$

(2)
$$K_I = \bar{\sigma} \left[W \tan\left(\frac{\pi a}{W}\right)\right]^{\frac{1}{2}}$$
$$= 9\left[100 \times 10^{-3} \times \tan\left(180° \times \frac{7}{100}\right)\right]^{\frac{1}{2}}$$
$$= 1.35 \text{ MPa m}^{0.5}.$$

$K_I > K_{IC}$, hence plate will fracture.

(3) For a plane of infinite width use eqn 5.27 for K_I,

$$K_I = \bar{\sigma}(\pi a)^{\frac{1}{2}} = 10\left(\pi \times \tfrac{3}{2} \times 10^{-3}\right)^{\frac{1}{2}}$$
$$= 0.686 \text{ MPa m}^{0.5}.$$

$K_I < K_{IC}$, hence plate will not fracture.

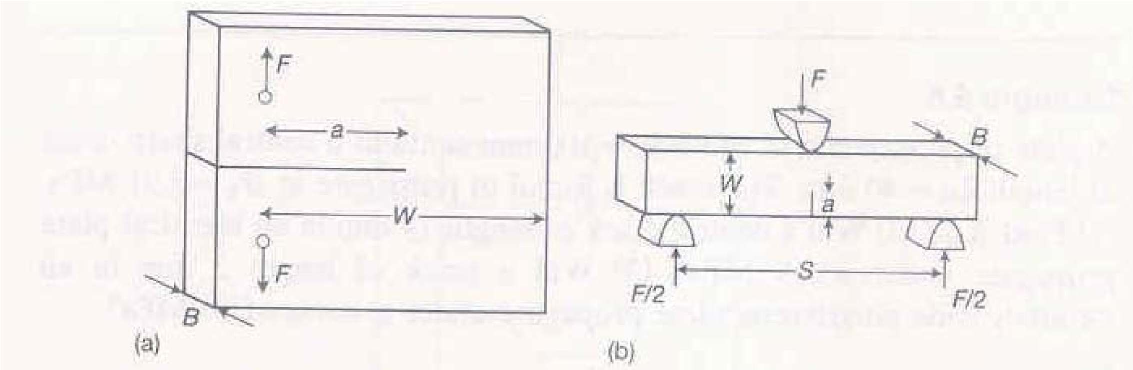
$$\frac{(\text{Pa}\cdot\text{m}^{\frac{1}{2}})^2}{\text{Pa}} = \text{Pa}\cdot\text{m} = \frac{\text{N}}{\text{m}^2}\cdot\text{m} = \frac{\text{J}}{\text{m}^2}$$

Most widely used test pieces for plastics are:

Compact tension (**CT**) specimen (**edge crack**) (See Fig 5.13a) and

Three point bending (See Fig 5.13b)

Fig 5.13 here



5.13 Specimen geometries used to determine K_{IC}: (a) compact tension (CT) specimen; and (b) single-edge-notched bend (SENB) specimen.

See the effective width, W and effective crack length, a from the figure.

For **compact tension**;

$$K_{IC} = Y\overline{\sigma}_F(\pi a)^{\frac{1}{2}} \qquad (30)$$

$$Y = 16.7 - 104.7\ (a/W) + 369.9\ (a/W)^2$$

$$\overline{\sigma} = F/BW \qquad (31)$$

Y = Geometry factor

For **3 point bending** (**flexural modulus**)

$$K_{IC} = Y\overline{\sigma}_F(\pi a)^{\frac{1}{2}} \qquad (30)$$

$$Y = 1.11 - 1.55\left(\frac{a}{w}\right) + 7.71\left(\frac{a}{w}\right)^2 - 13.5\left(\frac{a}{w}\right)^3 + 14.2\left(\frac{a}{w}\right)^4 \qquad (31)$$

$\overline{\sigma}$ =F/BW

Scan Ex 5.7 here

Example 5.7

A CT specimen machined from PMMA plate has thickness $B = 6$ mm, effective width $W = 50$ mm and effective crack length $a = 25$ mm. The force F at the loading pins increases linearly with deflection until fracture occurs at $F = 225$ N. (1) Find K_{IC}; (2) find G_{IC} given $E = 3.2$ GPa and $\nu = 0.42$; and (3) estimate the critical length of crack for a wide plate stressed at $\bar{\sigma} = 15$ MPa (see Figure 5.10).

Solution

(1) For $(a/W) = 25/50 = 0.5$, $Y = 7.63$.

$$K_{IC} = Y\bar{\sigma}_F(\pi a)^{\frac{1}{2}} = 7.63\left(\frac{225}{0.006 \times 0.050}\right)(\pi \times 0.025)^{\frac{1}{2}}$$

$$K_{IC} = 1.60 \text{ MPa m}^{0.5}.$$

(2)
$$G_{IC} = \frac{K_{IC}^2(1-\nu^2)}{E} = \frac{(1.60 \times 10^6)^2(1-0.42^2)}{3.2 \times 10^9}$$

$$G_{IC} = 659 \text{ J m}^{-2}.$$

(3) For the wide (infinite) plate the crack length is $2a$ and

$$K_{IC} = \bar{\sigma}_F(\pi a)^{\frac{1}{2}}$$

$$a = \frac{K_{IC}^2}{\pi\bar{\sigma}_F^2} = \frac{(1.60 \times 10^6)^2}{\pi(15 \times 10^6)^2}$$

$$= 3.62 \text{ mm}$$

and
$$2a = 7.24 \text{ mm}.$$

Scan Ex 5.8 here

Example 5.8

A rectangular bar of PMMA in the form of an SENB specimen, of thickness $B = 6$ mm and width $W = 10$ mm, contains a central edge crack of length $a = 1$ mm (see Figure 5.13(b)). Calculate the force F required to fracture the bar in single-edge-notched bending with span $S = 80$ mm. For this geometry, with $S/W = 80/10 = 8.0$, Y is given by

$$Y = 1.11 - 1.55(a/W) + 7.71(a/W)^2 - 13.5(a/W)^3 + 14.2(a/W)^4.$$

For $(a/W) = 1/10$ we obtain $Y = 1.02$.

$$K_{IC} = Y\sigma_{max}(\pi a)^{\frac{1}{2}}$$

$$1.60 \times 10^6 = 1.02 \times \sigma_{max} \times (\pi \times 0.001)^{\frac{1}{2}}$$

$$\sigma_{max} = 28.0 \text{ MPa}.$$

From elementary beam theory, the outer fibre stress at the site of the crack is

$$\sigma = \frac{3S}{2BW^2} F$$

$$28.0 \times 10^6 = \frac{3 \times 0.08}{2 \times 0.006 \times (0.01)^2} \times F$$

$$F = 140 \text{ N}.$$

So far the discussion was limited to linear elasticity, i.e.,

$\sigma = E\epsilon$ (Hookean), $\sigma \propto$ deflection

Yielding is negligible near the crack tip

This conditions are satisfied if

B, (W-a), a $> 2.5(K_{IC}/\sigma_y)^2$ (Plane strain region)

→ Plastic deformation (yielding) < 2% → K_{IC} is the true material parameter.

See Ex 5.9 for minimum specimen dimension for plane-strain fracture.