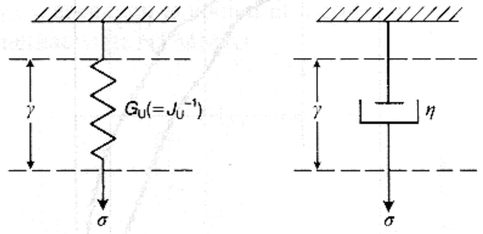
More realistic model is given below.

4.3 THEORY OF LINEAR VISCOELASTICITY



4.13 The spring and dashpot. The strain in the spring is $\gamma = J_U\sigma$; the strain rate $(d\gamma/dt) = J_U(d\sigma/dt)$. The strain in the dashpot cannot be related simply to the stress (it depends on the stress history). The strain rate in the dashpot is proportional to the stress and is $(d\gamma/dt) = \sigma/\eta$.

Fig 4.13 here

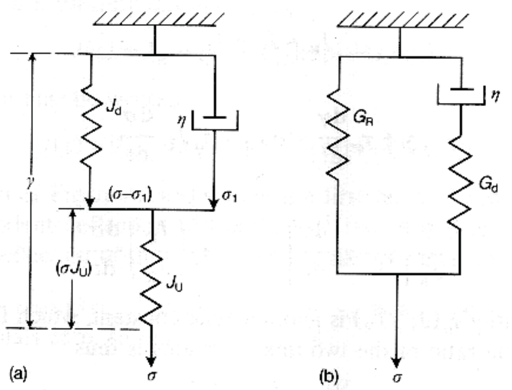
Spring : Elasticity

$$\gamma = J_U\,\sigma = \frac{\sigma}{G_U} \qquad (42,\ 43)$$

Dashpot : Viscosity

$$\sigma = \eta\frac{d\gamma}{dt} \qquad (44)$$

Zener model



4.14 The Zener model (or standard linear solid). The model may be represented as a spring in series with a Kelvin model, as in (a), or as a spring in parallel with a Maxwell model, as in (b). The significant properties inherent in the Zener model include: (i) two time constants, one for constant stress τ_σ and one for constant strain τ_γ; (ii) an instantaneous strain at $t=0$ when subject to a step-function stress; and (iii) full recovery following removal of the stress. For the Kelvin and Maxwell models, see Problem 4.8.

(a) have the same strain. We now derive the differential equation describing the relationship between σ, γ, $\dot{\sigma}$, and $\dot{\gamma}$ for (a). Let the stress in the dashpot be σ_1. The stress in J_d is then $\sigma - \sigma_1$, so that

Fig 4.14 here

Total strain (deformation)

$$\gamma = \sigma J_u + (\sigma - \sigma_1)J_d$$

$$\rightarrow \gamma - \sigma J_u = J_d(\sigma - \sigma_1) \quad \text{and} \quad \sigma_1 = \eta \frac{d}{dt}(\gamma - \sigma J_u) = \eta(\frac{d\gamma}{dt} - J_u \frac{d\sigma}{dt})$$

Then

$$\gamma - \sigma J_u = J_d\, \sigma - J_d \eta(\frac{d\gamma}{dt} - J_u \frac{d\sigma}{dt})$$

Define

$$\tau_\sigma \equiv J_d \eta \qquad (= \frac{\eta}{G_d} [=] \frac{m}{l \cdot t} \cdot \frac{1}{\frac{m \cdot l / t^2}{l^2}} = t \quad : \text{rleaxation time})$$

Then

$$\gamma = \sigma(J_u + J_d) - \tau_\sigma \frac{d\gamma}{dt} + \tau_\sigma J_u \frac{d\sigma}{dt}$$

Define $J_R \equiv J_u + J_d$

Then

$$\gamma + \tau_\sigma \frac{d\gamma}{dt} = \sigma J_R + \tau_\sigma J_u \frac{d\sigma}{dt}$$

$$\Rightarrow \frac{1}{J_R}\left(\gamma + \tau_\sigma \frac{d\gamma}{dt}\right) = \sigma + \tau_\sigma \frac{J_u}{J_R} \frac{d\sigma}{dt} \qquad (45)$$

Define $\tau_\gamma = \tau_\sigma \frac{J_u}{J_R}$ (45)'

Then

$$\frac{\tau_\sigma}{\tau_\gamma} = \frac{J_R}{J_u} (= \frac{G_u}{G_R}) \qquad (46)$$

(45)'→(45)

$$\frac{1}{J_R}\left(\gamma + \tau_\sigma \frac{d\gamma}{dt}\right) = \sigma + \tau_\gamma \frac{d\sigma}{dt}$$ DE for Zener model (47)

4.3.1.1 Creep Analysis

@t=0, $\sigma = \sigma_0 \quad \Rightarrow \frac{d\sigma}{dt} = 0$

(47) →

$$\frac{1}{J_R}[\gamma + \tau_\sigma \frac{d\gamma}{dt}] = \sigma_0 \qquad \rightarrow \gamma + \tau_\sigma \frac{d\gamma}{dt} = J_R \sigma_0$$

$$\rightarrow \frac{d\gamma}{dt} + \frac{1}{\tau_\sigma}\gamma = \frac{J_R}{\tau_\sigma}\sigma_0$$

Solution for 1st order linear differential (Formula)

$$y' + P(x)y = Q(x)$$

General solution

$$y = e^{-\int P\,dx}\left[\int Qe^{\int P\,dx}dx + c\right]$$

Then $P = \frac{1}{\tau_\sigma}$, $Q = \frac{J_R}{\tau_\sigma}\sigma_0$

$$\therefore\ \gamma = e^{-\int \frac{1}{\tau_\sigma}dt}\left[\frac{J_R}{\tau_\sigma}\sigma_0 \int e^{\int \frac{1}{\tau_\sigma}dt}dt + c\right]$$

$$= e^{-\frac{t}{\tau_\sigma}}\left[\frac{J_R\sigma_0}{\tau_\sigma}\tau_\sigma e^{\frac{t}{\tau_\sigma}} + c\right] = J_R\sigma_0 + ce^{-\frac{t}{\tau_\sigma}}$$

Apply I.C. @t=0, $\sigma = \sigma_0 \quad \Rightarrow \gamma = \gamma_0 = J_u\sigma_0$

So, $J_u\sigma_0 = J_R\sigma_0 + c \quad \rightarrow \quad c = \sigma_0(J_u - J_R)$

$$\therefore\ \gamma = J_R\sigma_0 + \sigma_0(J_u - J_R)e^{-\frac{t}{\tau_\sigma}}$$ ← notice t !!

$$= J_R\sigma_0 - \sigma_0(J_R - J_u)e^{-\frac{t}{\tau_\sigma}}$$

$$\rightarrow \frac{\gamma}{\sigma_0} = J(t) = J_R - (J_R - J_u)e^{-\frac{t}{\tau_\sigma}} \qquad (49)$$

or

$$J(t) = J_u + \underline{(J_R - J_u)}(1 - e^{-\frac{t}{\tau_\sigma}}) \qquad (50)$$

↑ J_d

4.3.1.2 Stress Relaxation

I.C. @t=0, $\gamma = \gamma_0$ $\Rightarrow$ $\sigma(t)$?

$\rightarrow \frac{d\gamma}{dt} = 0$ @ (47) $\frac{1}{J_R}\left(\gamma + \tau_\sigma \frac{d\gamma}{dt}\right) = \sigma + \tau_\gamma \frac{d\sigma}{dt}$

$$\frac{1}{J_R}\gamma_0 = \sigma + \tau_\gamma \frac{d\sigma}{dt}$$

$$\rightarrow \frac{d\sigma}{dt} + \frac{1}{\tau_\gamma}\sigma = \frac{1}{\tau_\gamma}\frac{\gamma_0}{J_R}$$

Solution

$$\sigma = e^{-\int \frac{1}{\tau_\gamma} dt}\left[\frac{1}{\tau_\gamma}\frac{\gamma_0}{J_R}\int e^{\int \frac{1}{\tau_\gamma} dt} dt + c\right]$$

$$\sigma = e^{-\frac{t}{\tau_\gamma}}\left[\frac{1}{\tau_\gamma}\frac{\gamma_0}{J_R}\tau_\gamma e^{\frac{t}{\tau_\gamma}} + c\right]$$

$$\sigma = \frac{\gamma_0}{J_R} + ce^{-\frac{t}{\tau_\gamma}} = \gamma_0 G_R + ce^{-\frac{t}{\tau_\gamma}}$$

Apply I.C. @ t=0, $\gamma = \gamma_0$ $\Rightarrow$ $\sigma = \frac{\gamma_0}{J_U} = G_u\gamma_0 \; (= \sigma_0)$

So,

$$G_u\gamma_0 = G_R\gamma_0 + c$$

$$\rightarrow c = (G_u - G_R)\gamma_0$$

$$\therefore \sigma = G_R\gamma_0 + (G_u - G_R)\gamma_0 e^{-\frac{t}{\tau_\gamma}}$$

$$\sigma(t) = G_R\gamma_0 + (G_u - G_R)\gamma_0 e^{-\frac{t}{\tau_\gamma}} \quad (52)$$

$G_R\,\gamma_0$: Time independent stress in the element G_R.

The time dependent stress in the other arm starts off at t=0 with value $G_d\gamma_0$ because the total deflection is instantaneously in the spring G_d.

Note @t=0, $\sigma = G_R\gamma_0 + (G_u - G_R)\gamma_0 = G_u\gamma_0$

In stress relaxation

$$G(t) = \frac{\sigma(t)}{\gamma} = \frac{\sigma(t)}{\gamma_0} \qquad (4\text{-}11)$$

(11→52)

$$G(t) = \frac{\sigma(t)}{\gamma_0} = G_R + (G_u - G_R)e^{-\frac{t}{\tau_\gamma}} \qquad (53)$$

$$\Rightarrow G(t) = G_u - (G_u - G_R)[1 - e^{-\frac{t}{\tau_\gamma}}] \qquad (54)$$

Example calculation

$$\text{With } J_u = 1\text{GPa}^{-1},\ \frac{J_R}{J_u} = 10,\ \tau_\sigma = 100\text{s} \xrightarrow{\text{Plot } \begin{smallmatrix} J(t)\ (50) \\ G(t)\ (54) \end{smallmatrix}} \text{Fig. 4.15}$$

$$\tau_\gamma = \frac{J_u}{J_R}\tau_\sigma = \left(\frac{1}{10}\right)(100) = 10\text{s}$$

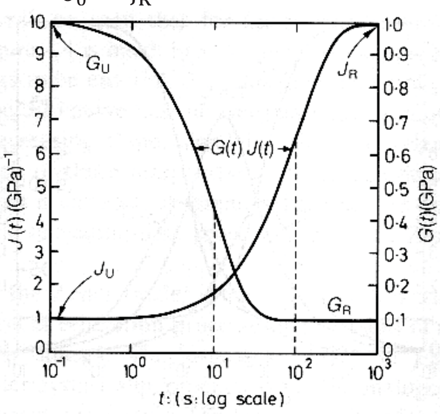
Note that in both creep and relaxation the solutions are identical

$$@t \ll \tau_\sigma \Rightarrow J(t) \quad (50) \quad \frac{\gamma}{\sigma_0} = J_u \qquad G(t) = G_u$$

$$\Rightarrow \frac{\gamma}{\sigma_0} = \frac{1}{J_u} = G_u$$

$$@t \gg \tau_\sigma \Rightarrow J(t) = \frac{\gamma}{\sigma_0} = J_R \qquad G(t) = G_R$$

$$\Rightarrow \frac{\gamma}{\sigma_0} = \frac{1}{J_R} = G_R$$



4.15 Solution to the Zener model for a creep experiment ($J(t)$) or for a stress-relaxation experiment ($G(t)$). Model with $J_U = 1$ GPa^{-1}, $J_R/J_U = 10$, and $\tau_\sigma = 100$ s. Note that from eqn 4.46 $\tau_\gamma = 10$ s.

Fig. 4.15 here

4.3.1.3 Frequency response

$$\gamma^* = \gamma_0 e^{i\omega t} \qquad (20)$$

$$\sigma^* = \sigma_0 e^{i(\omega t + \delta)} \qquad (21)$$

Work with Zener differential model:

$$\frac{1}{J_R}\left[\gamma + \tau_\sigma \frac{d\gamma}{dt}\right] = \sigma + \tau_\gamma \frac{d\sigma}{dt} \qquad (47)$$

(20),(21) ⇒ (47)

$$\frac{1}{J_R}\left[\cancel{\gamma_0} e^{i\cancel{\omega t}} + \tau_\sigma \cancel{\gamma_0} i\omega \cancel{e^{i\omega t}}\right] = \frac{\sigma_0}{\gamma_0} e^{i(\cancel{\omega}t+\delta)} + \frac{\tau_\gamma}{\gamma_0}\sigma_0 i\omega e^{i(\cancel{\omega}t+\delta)}$$

$$\Rightarrow \frac{1}{J_R}[1+\tau_\sigma i\omega] = \frac{1}{J^*}[1+i\omega\tau_\gamma]$$

Note $J^* = \frac{\gamma^*}{\sigma^*} = \frac{\gamma_0}{\sigma_0}e^{-i\delta}$

$\frac{1}{J^*} = \frac{\sigma_0}{\gamma_0}e^{i\delta}$

$$\therefore J^* = \frac{(1+i\omega\tau_\gamma)J_R}{1+i\omega\tau_\sigma}$$

$$= \frac{J_R + i\omega J_R\tau_\gamma}{1+i\omega\tau_\sigma} \qquad \leftarrow \tau_\gamma = \tau_\sigma\frac{J_u}{J_R}$$

$$= \frac{J_R + i\omega J_R\,\tau_\sigma\frac{J_u}{J_R} = J_R + i\omega\tau_\sigma J_u}{1+i\omega\tau_\sigma}$$

$$= \frac{J_R + i\omega\tau_\sigma J_u + (1+i\omega\tau_\sigma)J_u - (1+i\omega\tau_\sigma)J_u}{1+i\omega\tau_\sigma}$$

$$= J_u + \frac{J_R - J_u}{1+i\omega\tau_\sigma} \qquad (55)$$

$$J^* = J' - iJ''$$

$$= J_u + \frac{J_R - J_u}{1+i\omega\tau_\sigma}$$

$$= J_u + \frac{(J_R - J_u)(1-i\omega\tau_\sigma)}{(1+i\omega\tau_\sigma)(1-i\omega\tau_\sigma)}$$

$$= J_u + \frac{J_R - i\omega J_R\tau_\sigma - J_u + i\omega J_u\tau_\sigma}{1+\omega^2\tau_\sigma{}^2}$$

$$= J_u + \frac{(J_R - J_u)}{1+\omega^2\tau_\sigma{}^2} - i\frac{(J_R\tau_\sigma - J_u\tau_\sigma)\omega}{1+\omega^2\tau_\sigma{}^2}$$

So,

$$J'(\omega) = J_u + \frac{(J_R - J_u)}{1+\omega^2\tau_\sigma{}^2} \qquad (56)$$
$$J''(\omega) = \frac{(J_R - J_u)\tau_\sigma\omega}{1+\omega^2\tau_\sigma{}^2}$$

For G^*

$$\frac{1}{J_R}[1 + i\omega\tau_\sigma] = \frac{1}{J^*}[1 + i\omega\tau_\gamma] = G^*[1 + i\omega\tau_\gamma]$$

So, $$G^* = \frac{1+i\omega\tau_\sigma}{1+i\omega\tau_\gamma}\frac{1}{J_R}$$

$$= \frac{1+i\omega\tau_\sigma}{1+i\omega\tau_\gamma} G_R$$

$$= \frac{G_R + i\omega\tau_\sigma G_R + (1+i\omega\tau_\gamma)G_u - (1+i\omega\tau_\gamma)G_u}{1+i\omega\tau_\gamma} \quad \leftarrow \tau_\sigma = \tau_\gamma \frac{G_u}{G_R}$$

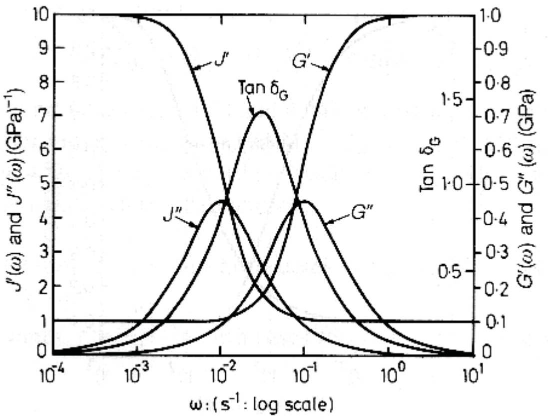
$$= \frac{G_R + i\omega\tau_\gamma \frac{G_u}{G_R} \cdot G_R + (1+i\omega\tau_\gamma)G_u - (1+i\omega\tau_\gamma)G_u}{1+i\omega\tau_\gamma}$$

$$G^* = \frac{G_R + i\omega\tau_\gamma G_u + (1+i\omega\tau_\gamma)G_u - (G_u + i\omega\tau_\gamma G_u)}{1+i\omega\tau_\gamma}$$
$$= G_U - \frac{(G_U - G_R)}{1+i\omega\tau_\gamma} \qquad (57)$$

Since

$$G^* = G' + iG''$$
$$= G_u - \frac{(G_u - G_R)}{1+i\omega\tau_\gamma} = G_u - \frac{(G_u - G_R)(1-i\omega\tau_\gamma)}{(1+i\omega\tau_\gamma)(1-i\omega\tau_\gamma)}$$
$$= G_u - \frac{(G_u - i\omega\tau_\gamma G_u - G_R + i\omega\tau_\gamma G_R)}{1+\omega^2\tau_\gamma{}^2}$$
$$= \frac{G_u + G_u\omega^2\tau_\gamma{}^2 - G_u + i\omega\tau_\gamma G_u + G_R - i\omega\tau_\gamma G_R}{1+\omega^2\tau_\gamma{}^2}$$
$$= \frac{\cancel{G_u} + G_u\omega^2\tau_\gamma{}^2 - \cancel{G_u} + i\omega\tau_\gamma G_u + G_R - i\omega\tau_\gamma G_R}{1+\omega^2\tau_\gamma{}^2}$$
$$= \frac{(1+\omega^2\tau_\gamma{}^2)G_R - (1+\omega^2\tau_\gamma{}^2)G_R + G_u\omega^2\tau_\gamma{}^2 + i\omega\tau_\gamma G_u + G_R - i\omega\tau_\gamma G_R}{1+\omega^2\tau_\gamma{}^2}$$
$$= G_R + \frac{-\cancel{G_R} - \omega^2\tau_\gamma{}^2 G_R + G_u\omega^2\tau_\gamma{}^2 + i\omega\tau_\gamma G_u + \cancel{G_R} - i\omega\tau_\gamma G_R}{1+\omega^2\tau_\gamma{}^2}$$
$$= G_R + \frac{(G_u - G_R)\omega^2\tau_\gamma{}^2}{1+\omega^2\tau_\gamma{}^2} + i\frac{(G_u - G_R)\omega\tau_\gamma}{1+\omega^2\tau_\gamma{}^2} = G' + iG'' \qquad (58)$$

Note $G' = f(\omega)$: Frequency response



4.16 Solution to the Zener model for a dynamic experiment. Model with $J_U = 1$ GPa^{-1}, $J_R/J_U = 10$ and $\tau_\sigma = 100$ s. Note that maxima occur: in J'' at $\omega = \tau_\sigma^{-1} = 0.01$ rad s^{-1}; in G'' at $\omega = \tau_\gamma^{-1} = 0.1$ rad s^{-1}; and in tan δ at $\omega = (\tau_\sigma \tau_\gamma)^{-\frac{1}{2}} = 0.0316$ rad s^{-1}.

Fig. 4.16 here

Fig. 4.16 G', G'', $\tan\delta_G$, J', J'' vs ω

With $J_u = 1 \text{GPa}^{-1}$

$J_R/J_u = 10$

$T_\sigma = 100\text{s}$

Noth the inflection point (in J' and G') and the maximum (in J" and G") occur @

$$\omega = \frac{1}{\tau_\sigma} \quad (J' \text{ and } J") \qquad \tau_\sigma : \text{ Constant stress (For creep)} \tag{59}$$

$$\omega = \frac{1}{\tau_\gamma} \quad (\text{in } G' \text{ and } G") \ \tau_\gamma : \text{ Constant strain (For stress relaxation)} \tag{60}$$

Note: $\tau_\gamma = \tau_\sigma \left(\frac{J_R}{J_u}\right) = (100)\left(\frac{1}{10}\right) = 10\text{s}$

Maximum of $\tan\delta$ (see in the Fig 4.16) occurs @

$$\omega = \frac{1}{(\tau_\sigma \tau_\gamma)^{\frac{1}{2}}} \tag{61}$$

Peak points are obtained as for example for (60).

$$G'' = \frac{(G_u - G_R)\omega\tau_\gamma}{1+\omega^2{\tau_\gamma}^2} \tag{58}$$

$$\frac{dG''}{d\omega} = \frac{(G_u - G_R)\{\tau_\gamma(1+\omega^2{\tau_\gamma}^2) - \omega\tau_\gamma(2\omega{\tau_\gamma}^2)\}}{(1+\omega^2{\tau_\gamma}^2)^2} = 0$$

$$\tau_\gamma + \omega^2\tau_\gamma^3 = 2\omega^2\tau_\gamma^3,\ 1 + \omega^2\tau_\gamma^2 = 2\omega^2\tau_\gamma^2$$

$$1 = \omega^2\tau_\gamma^3$$

for $\tan\delta = \frac{G''}{G'}$

$\frac{d}{d\omega}\left(\frac{G'}{G''}\right) = 0$ try!

$$\therefore \omega = \frac{1}{\tau_\gamma}$$

4.3.2 Distribution of relaxation time

Three adjustable parameters of Zener model to fit the data = J_u, J_R, τ_σ

Zener model predicts narrower distribution of relaxation time than experimental data.

For example:

Fig. 4.15 (Zener) : 3 decades of time is okay.

Fig. 4. 4(LDPE data) : Much broader than 3 decades of time.

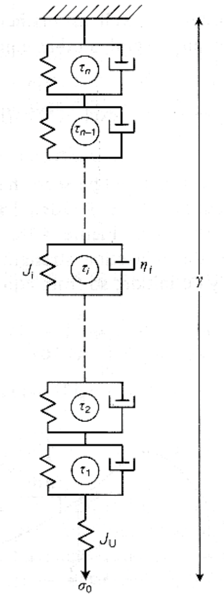
→ Introduce a set of relaxation process, rather than one into Zener model (called generalized Zener model)

Zener model : 1 J_u(spring), 1 J_d, 1 η

Generalized Zener : 1 J_u(spring), n J_d, n η(n parallel elements)

→ with J_i and η_i for each (Fig. 4.17)

Fig. 4.17 here



4.17 Distribution of n Kelvin relaxation elements $\tau_1, \tau_2, \ldots, \tau_i, \ldots, \tau_{n-1}, \tau_n$. Each element is stressed by the stress σ_0 which also acts on the instantaneous compliance J_U. The strains in the elements all add to give the total strain.

@ $t = 0, \sigma = \sigma_0$ is applied

Then instantaneous deformation=$\sigma_0 J_u$

For bottom 3 elements (Zener)

$$\frac{1}{J_R}\left[\gamma + \tau_\sigma \frac{d\gamma}{dt}\right] = \sigma + \tau_\gamma \frac{d\sigma}{dt} \qquad (47)$$

For creep (solution)

$$J(t) = J_u + [J_R - J_u][1 - e^{-t/\tau_\sigma}] \qquad (50)$$

$$J_R - J_u = J_d \;\Rightarrow\; J_i$$

$$\tau_\sigma = J_d \eta \text{ (Zener)} \;\Rightarrow\; \tau_i = J_i \eta_i$$

For generalized Zener

$$J(t) = J_u + \sum_{i=1}^{n} J_i (1 - e^{-t/\tau_i}) \qquad (61) \qquad (\text{Prob. 4.11})$$

$$= J_u + J_1\left(1 - e^{-\frac{t}{\tau_1}}\right) + J_2\left(1 - e^{-\frac{t}{\tau_2}}\right) + \cdots$$

↓ Replace the summation by integral

$$= J_u + \int_0^\infty \left(1 - e^{-\frac{t}{\tau}}\right) j(\tau) d\tau \quad (62, 63) \text{ where, } J_i = j(\tau)d\tau \text{ being used}$$

@ $t = \infty$ ↓ (63)

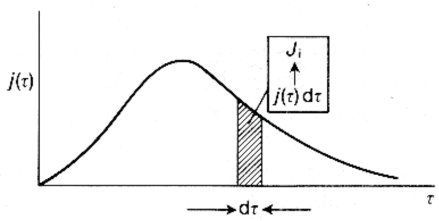
$$J(\infty) = J_u + \int_0^\infty j(\tau)d\tau \equiv J_R$$

$$\rightarrow J_R - J_u = \underline{\int_0^\infty j(\tau)d\tau} \qquad (64) \text{ (Adjust } J_u, J_R, j(\tau) \text{ to fit the creep data)}$$

↑ Area under the curve (See Fig 18)

Area= $j(\tau)d\tau = J_i$ (Relaxation strength of the relaxation at τ)

See Fig 18 here



4.18 The distribution of relaxation times plotted against τ from the model of Figure 4.17. The element of area at τ, $j(\tau)\,d\tau$, represents the strength of the relaxation at τ. The integrated area (from eqn 4.64) equals $J_R - J_U$.

Likewise,

$$G(t) = G_R + \sum G_i e^{-t/\tau_i} \qquad (\text{Prob.12}) \qquad (65)$$

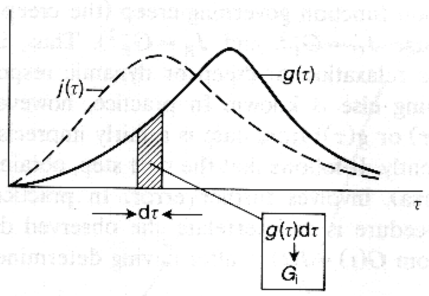
$$= G_R + \underline{\int_0^\infty e^{-t/\tau_\sigma} g(\tau)d\tau} \qquad (66)$$

↑ area under the curve

Area under the $g(\tau)d\tau$ curve = $\int_0^\infty g(\tau)d\tau = G_U - G_R$ (67) (Fig 4.19)

Adjust $G_U, G_R, g(i) \Rightarrow$ Fit relaxation data.

Fig 4.19 here



4.19 The distribution of relaxation times $g(\tau)$ for the model of Problem 4.12; a continuous distribution of Maxwell elements. The element of area at τ, $g(\tau)\,d\tau$, represents the strength of the relaxation at τ. The integrated area (from eqn 4.67) equals $G_U - G_R$.

4.3.3 Origin of temperature dependence

3 parameters of Zener model= $J_u,\ J_R,\ \tau_\sigma = f(\mathrm{Temp})$

Among these, τ_σ is most temperature dependent. So we work with this.

Let relaxation time (τ_σ) at temperature $T \equiv \tau_T$

Let relaxation time (τ_σ) at temperature $T_0 \equiv \tau_{T_0}$

Then

$$\tau_T = a_T \tau_{T_0} \qquad (68)$$

,where $$a_T = e^{\frac{\Delta H}{R}\left(\frac{1}{T}-\frac{1}{T_0}\right)} \qquad (69)$$ a_T : Shift Factor

Following the Arrhenius's temperature dependency of relaxation time

$$a_T = \frac{\eta_T = Ae^{\frac{\Delta H}{RT}}}{\eta_{T_0} = Ae^{\frac{\Delta H}{RT_0}}} = e^{\frac{\Delta H}{R}\left(\frac{1}{T}-\frac{1}{T_0}\right)}$$ ΔH: Activation energy of flow

For $T < T_0 \Rightarrow a_T > 1$

$T = T_0 \Rightarrow a_T = 1$

$T > T_0 \Rightarrow a_T < 1$: Relaxation at high T is fast.

Let's work this out with creep compliance'

Creep compliance (Eq. 4.50)

$$J(t) = J_u + (J_R - J_u)(1 - e^{-\frac{t}{\tau_\sigma}}) \qquad (50)$$

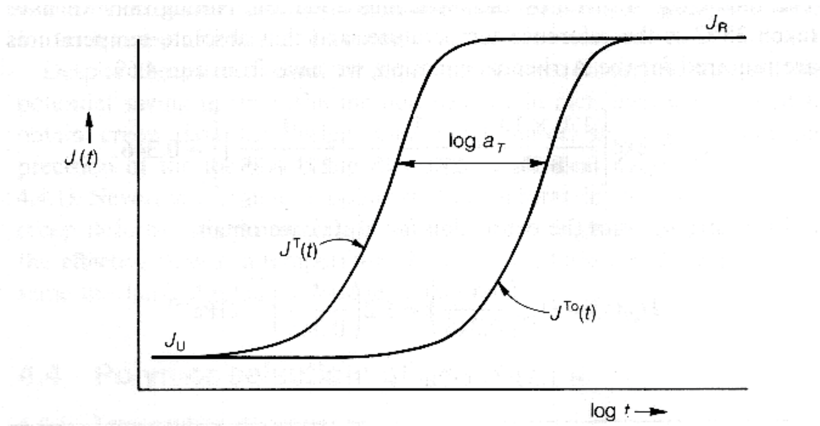
Creep compliance at time t and Temperature T_0:

$$J^{T_0}(t) = J_u + (J_R - J_u)(1 - e^{-\frac{t}{\tau_{T_0}}}) \qquad (70)$$

Creep compliance at time $a_T t$ 및 Temp T:

$$J^{T}(a_T t) = J_u + (J_R - J_u)(1 - e^{-\frac{a_T t}{a_T \tau_{T_0}}}) \qquad (71)$$

i.e. $J^{T_0}(t) = J^{T}(a_T t)$ (72)



4.20 Illustration of the time–temperature shift (eqn 4.72). The shear compliance curves at T and T_0 when plotted against log t are simply displaced horizontally by log a_T: a_T is the shift factor (eqn 4.68). The small temperature dependence of J_U and J_R is neglected here.

Fig 4.20 here

Ex 4.4

PP @ 35℃

$D(t) = 1.2t^{0.1}\,\mathrm{GPa}^{-1}$ $\qquad t[=]s,\ \Delta H = 170\,\mathrm{kJ/mole}$

Calculate $D(t)$ @ 40℃

Solution

$$T_0 = 35℃ = 273 + 35 = 308K$$
$$T = 40℃ = 273 + 40 = 313K$$

$$J^{T_0}(t) = J^{T}(a_T t) \qquad (72)$$

So, $D_{35}(t) = D_{40}(a_T t)$

$$\therefore D_{40}(t) = D_{35}\left(\frac{t}{a_{40}}\right)$$

Now $a_{40} = e^{\frac{\Delta H}{R}\left(\frac{1}{T}-\frac{1}{T_0}\right)} = e^{\frac{1.70\times10^3}{8.31}\left(\frac{1}{313}-\frac{1}{308}\right)} = 0.346$

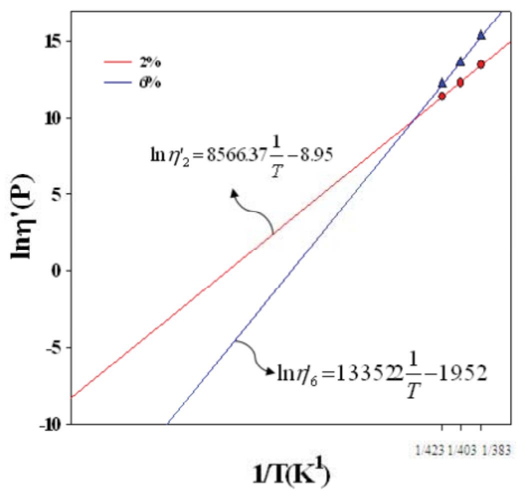
$$\therefore D_{40}(t) = D_{35}\left(\frac{t}{0.346}\right) = 1.2\left(\frac{t}{0.346}\right)^{0.1} = 1.33t^{0.1}\text{GPa}^{-1}$$

Conclusionㄴ

@ 35℃ $D(t) = 1.2t^{0.1}$

@ 40℃ $P(t) = 1.33t^{0.1}$

Arrhenius Plot (UHMWPE/PP Blends)



$$\eta = Ae^{\frac{E}{RT}}$$

$$\ln\eta = \ln A + \frac{E}{R}\cdot\frac{1}{T}$$

	lnA (P)	E/R (K)	E(cal/mol)
2%	-8.95	8566.4	17.02×10^3
6%	-19.52	13351.2	26.53×10^4
10%	-	-	-

Shift Factor (α_T)

Base temperature = 130℃

	2%	6%
110℃	3.03	5.64
150℃	0.36	0.20

$$\alpha_T = \frac{\eta'}{\eta'_0} = Ae^{\frac{E}{RT}} \Big/ Ae^{\frac{E}{RT_0}} = e^{\frac{E}{R}\left(\frac{1}{T}-\frac{1}{T_0}\right)}$$

T_0 = 130℃ (403.15K)

T = 110℃, 150℃ (383.15K, 423.15K)