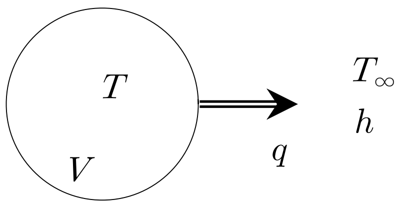
제 4장 비정상 열전도

4-1 집중계 해석



$$\dot{Q} = hA(T_\infty - T)$$
$$= mC_p \frac{dT}{dt}$$

$$\frac{d(T - T_\infty)}{T - T_\infty} = -\frac{hA}{\rho V C_p} dt$$

$$\frac{T - T_\infty}{T_i - T_\infty} = e^{-\frac{hA}{\rho V C_p} t} \qquad T_i = T(t=0)$$

대류 열전달율

$$\dot{Q} = hA(T(t) - T_\infty)$$

열 전달량

$$Q = mC_p(T(t) - T_i)$$

최대 열전달량

$$Q_{\max} = mC_p(T_\infty - T_i)$$

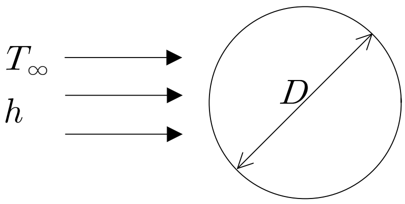
집중계 해석의 기준

$$B_i = \frac{hL_c}{k} \le 0.1 \qquad B_i : Biot\,Nmber$$

$$L_c = \frac{V}{A} \qquad L_c : \text{특성길이}$$

$$B_i = \frac{L_c/k}{1/h} = \frac{\text{물체 내부의 전도 저항}}{\text{물체 표면의 대류저항}}$$

(Ex 4-1)



$$L_c = \frac{V}{A} = \frac{\frac{1}{6}\pi D^3}{\pi D^2} = \frac{1}{6}D$$

$$B_i = \frac{hL_c}{k} < 0.1$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = 0.01 = e^{-bt}$$

$$b = \frac{hA_s}{\rho C_p V} = \frac{h}{\rho C_p L_c}$$

4-2 대형 평면벽, 긴원통, 구에서의 비 정상 열전도

무차원화

온도 : $\theta(x,t) = \dfrac{T(x,t) - T_\infty}{T_i - T_\infty}$

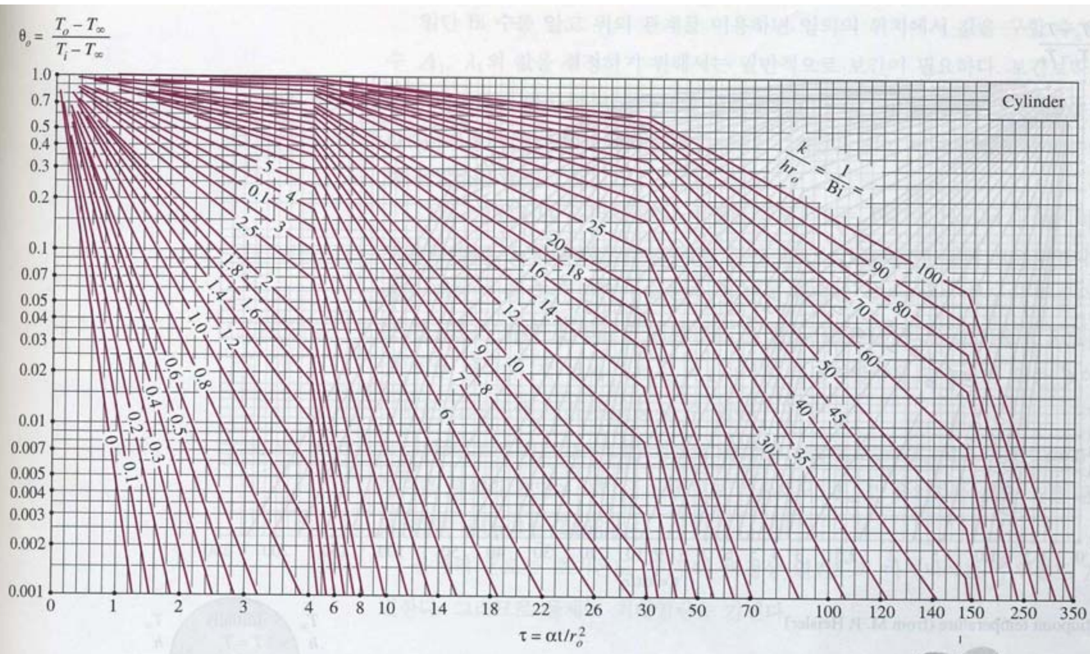
거리 : $X = \dfrac{x}{L}$

열전달계수 : $B_i = \dfrac{hk}{L}$

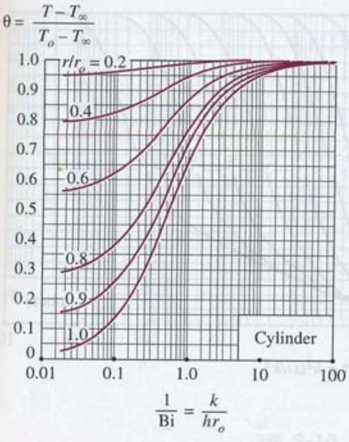
시간 : $\tau = \dfrac{\alpha t}{L^2}$ $\quad$ $Fouier\ Nmber$

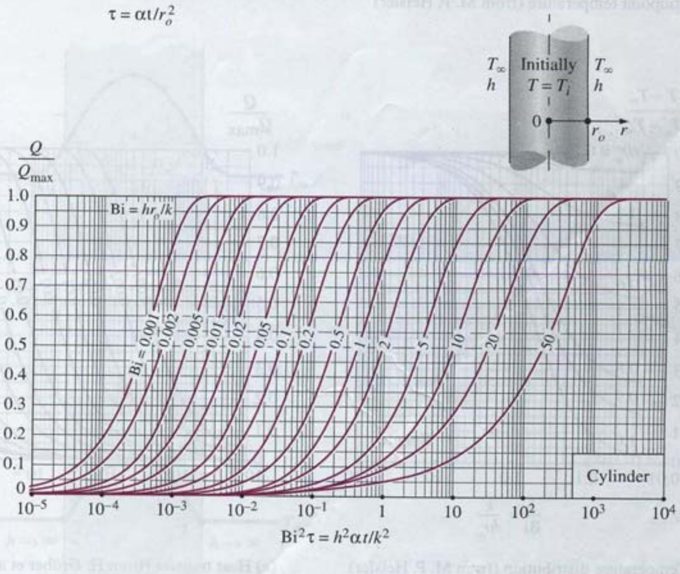
α : 열확산계수 $\dfrac{k}{\rho C_p}$

집중계 해석을 사용 할 수 없는 경우 비정상 온도 차트를 사용하여 온도를 구할수 있다. (그림 4-13 ~ 4-23)



(*a*) Centerline temperature (from M. P. Heisler)





(*b*) Temperature distribution (from M. P. Heisler)

(*c*) Heat transfer (from H. Gröber et al.)

그림 2-14

열전달계수 h로 모든 면이 주위온도 T_∞에 노출되고, 초기온도 T_i에서 지름 r_o인 원통에 대한 비정상 온도와 열전달 차트

표 2-1

평면벽, 원통, 구에 대하여 비정상 1차원 열전도에 대한 단항근사해법에서 사용된 계수

(두께 $2L$인 평면벽에 대하여는 Bi=hL/k, 반지름 r_o인 원통과 구에 대하여는 Bi=hr_o/k)

	Plane slab		Cylinder		Sphere	
Bi	λ_1	A_1	λ_1	A_1	λ_1	A_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.1	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.2	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.3	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0931	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0918	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5995	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2403	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	2.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2880	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2732	2.4048	1.6021	3.1416	2.0000

표 2-2

0차 및 1차의 1종 Bessel 함수

ξ	$J_0(\xi)$	$J_1(\xi)$
0.0	1.0000	0.0000
0.1	0.9975	0.0499
0.2	0.9900	0.0995
0.3	0.9776	0.1483
0.4	0.9604	0.1960
0.5	0.9385	0.2423
0.6	0.9120	0.2867
0.7	0.8812	0.3290
0.8	0.8463	0.3688
0.9	0.8075	0.4059
1.0	0.7652	0.4400
1.1	0.7196	0.4709
1.2	0.6711	0.4983
1.3	0.6201	0.5220
1.4	0.5669	0.5419
1.5	0.5118	0.5579
1.6	0.4554	0.5699
1.7	0.3970	0.5778
1.8	0.3400	0.5815
1.9	0.2818	0.5812
2.0	0.2239	0.5767
2.1	0.1666	0.5683
2.2	0.1104	0.5560
2.3	0.0555	0.5399
2.4	0.0025	0.5202

단항 근사해법을 사용하면 $(\tau > 0.2)$

$$\text{평면벽} : \theta_{wall} = A_1 e^{-\lambda_1 \tau} \cos(\lambda_1 x / L)$$

$$\text{원통} : \theta_{cyl} = A_1 e^{-\lambda_1 \tau} J_0(\lambda_1 r / r_0)$$

$$\text{구} : \theta_{cyl} = A_1 e^{-\lambda_1 \tau} \frac{\sin(\lambda_1 r / r_0)}{\lambda_1 r / r_0}$$

B_i 값에 따른 A_1과 λ_1은 표 4-1 참조

최대 열전달

$$Q_{\max} = m C_p (T_\infty - T_i) = \rho V C_p (T_\infty - T_i)$$

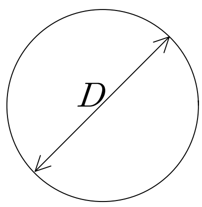
열전달비

$$\text{평면벽} : \frac{Q}{Q_{\max}})_{wall} = 1 - \theta_{o,wall} \frac{\sin \lambda_1}{\lambda_1}$$

$$\text{원통} : \frac{Q}{Q_{\max}})_{cyl} = 1 - 2\theta_{o,cyl} \frac{J_1(\lambda_1)}{\lambda_1}$$

$$\text{구} : \frac{Q}{Q_{\max}})_{sph} = 1 - 3\theta_{o,sph} \frac{\sin \lambda_1 - \lambda_1 \cos \lambda_1}{\lambda_1^3}$$

(Ex 4-3) 달걀 삶기



$D = 5 cm$ $T_i = 5\ ^oC$

$T_\infty = 95^o C$

$h = 1200\, W/m^o C$

$T_o = 70^o C \quad \rightarrow \quad t = ?$

(해)

$\dfrac{5+70}{2} = 37.5^oC$에서 물로부터 열전도도와 열확산계수 α

$k = 0.627\, W/m^oC$, $\alpha = \dfrac{k}{\rho C_p} = 0.151 \times 10^{-6}\, m^2/s$ (표A)

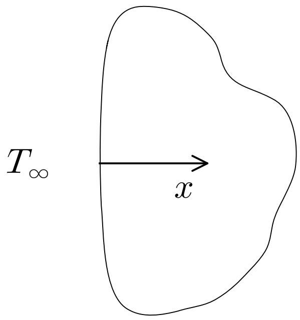
$B_i = \dfrac{hr_o}{k} = 47.8 > 0.1$

표 4－1에서 $\lambda_1 = 3.0753$, $A_1 = 1.9958$

$\dfrac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$ $\therefore \tau = 0.209$

$t = \dfrac{\tau r_o^2}{\alpha} = 865\ s$

4-3 반무한 고체에서의 비정상 열전도



$$1 - \theta(x,t) = \frac{T(x,t) - T_i}{T_\infty - T_i}$$

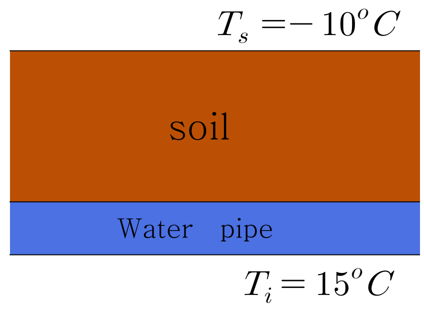
$$erfc(x) = 1 - \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$$

$$\frac{T - T_i}{T_\infty - T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)\right]$$

$h \rightarrow \infty$

$$\frac{T - T_i}{T_s - T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

(Ex 4-6) 동파 방지를 위한 수도관의 최소 매설 깊이



$k = 0.4\ W/m^oC$

$\alpha = 0.15 \times 10^{-6}\ m^2/s$

(해) $$\frac{T(x,t) - T_i}{T_s - T_i} = erfc(\frac{x}{2\sqrt{\alpha t}})$$

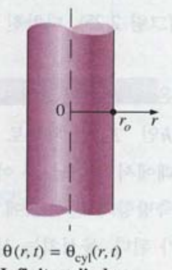
$$\frac{0-15}{-10-15} = erfc(\frac{x}{2\sqrt{\alpha t}}) = 0.6$$

표 4-3에서 $\zeta = 0.37$

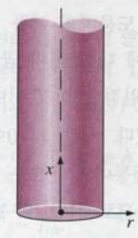
$t = 3$개월 $= 90$일 $\times 24$시간 $\times 3600$초 $= 7.78 \times 10^6$(sec)

표 2-4

조기온도 T_i이고 모든 표면이 주위온도 T_∞에 노출된 1차원 해의 곱셈으로 표현된 다차원 해

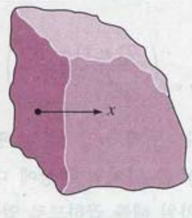


$\theta(r,t) = \theta_{cyl}(r,t)$
Infinite cylinder

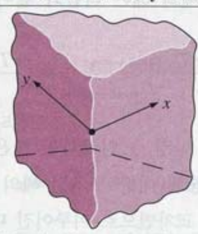


$\theta(x,r,t) = \theta_{cyl}(r,t)\,\theta_{semi\text{-}inf}(x,t)$
Semi-infinite cylinder

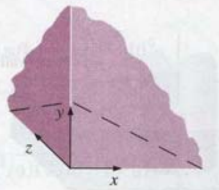
$\theta(x,r,t) = \theta_{cyl}(r,t)\,\theta_{wall}(x,t)$
Short cylinder



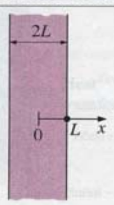
$\theta(x,t) = \theta_{semi\text{-}inf}(x,t)$
Semi-infinite medium



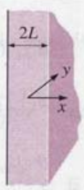
$\theta(x,y,t) = \theta_{semi\text{-}inf}(x,t)\,\theta_{semi\text{-}inf}(y,t)$
Quarter-infinite medium



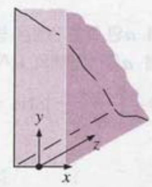
$\theta(x,y,z,t) =$
$\theta_{semi\text{-}inf}(x,t)\theta_{semi\text{-}inf}(y,t)\,\theta_{semi\text{-}inf}(z,t)$
Corner region of a large medium



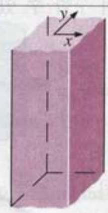
$\theta(x,t) = \theta_{wall}(x,t)$
Infinite plate (or plane wall)



$\theta(x,y,t) = \theta_{wall}(x,t)\,\theta_{semi\text{-}inf}(y,t)$
Semi-infinite plate

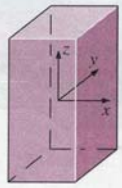


$\theta(x,y,z,t) =$
$\theta_{wall}(x,t)\theta_{semi\text{-}inf}(y,t)\,\theta_{semi\text{-}inf}(z,t)$
Quarter-infinite plate



$\theta(x,y,t) = \theta_{wall}(x,t)\theta_{wall}(y,t)$
Infinite rectangular bar

$\theta(x,y,z,t) =$
$\theta_{wall}(x,t)\,\theta_{wall}(y,t)\,\theta_{semi\text{-}inf}(z,t)$
Semi-infinite rectangular bar



$\theta(x,y,z,t) =$
$\theta_{wall}(x,t)\,\theta_{wall}(y,t)\,\theta_{wall}(z,t)$
Rectangular parallelepiped

4-4 다차원에서의 비정상 열전도

Product Solution 방법

$$\left[\theta_{(r,x,t)}\right]_{\substack{short\\cylinder}} = \left[\theta_{(x,t)}\right]_{\substack{Plane\\wall}} \cdot \left[\theta_{(r,t)}\right]_{\substack{Infinite\\cylinder}}$$

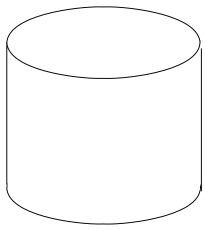
$$\left[\theta_{(x,y,t)}\right]_{\substack{rectanglar\\bar}} = \left[\theta_{(x,t)}\right]_{wall} \cdot \left[\theta_{(y,t)}\right]_{wall}$$

열 전달량

$$\left[\frac{Q}{Q_{\max}}\right]_{total,2D} = \left[\frac{Q}{Q_{\max}}\right]_1 + \left[\frac{Q}{Q_{\max}}\right]_2\left(1-\left[\frac{Q}{Q_{\max}}\right]_1\right)$$

$$\left[\frac{Q}{Q_{\max}}\right]_{total,3D} = \left[\frac{Q}{Q_{\max}}\right]_1 + \left[\frac{Q}{Q_{\max}}\right]_2\left(1-\left[\frac{Q}{Q_{\max}}\right]_1\right) + \left[\frac{Q}{Q_{\max}}\right]_3\left(1-\left[\frac{Q}{Q_{\max}}\right]_1\right)\left(1-\left[\frac{Q}{Q_{\max}}\right]_2\right)$$

(Ex 4-7) 짧은 황동 원통의 냉각



$D = 0.1\,cm$, $L = 0.6$
$T_i = 120\,^oC$, $T_\infty = 25^oC$
$h = 60$, $t = 15 \times 60$ 후의 온도

(해) $k = 110\,W/m^oC$, $\alpha = 33.9 \times 10^{-6}\ m^2/s$ (표 $A-3$)

평판

$$\tau = \frac{\alpha t}{L^2} = 8.48 > 0.2$$

$$\frac{1}{B_i} = \frac{k}{hL} = 30.6$$

$\Rightarrow \theta_{wall}(0,t) = 0.8$

원통

$$\tau = \frac{\alpha t}{r_o^2} = 12.2 \qquad \frac{1}{B_i} = \frac{k}{hr_o} = 36.7 \quad \Bigg\} \quad \theta_{cyl}(0,t) = 0.5$$

$$\theta = \theta_{wall} \times \theta_{cyl} = 0.4$$

b) 원통의 윗면 중심은 $r=0$, $x=L$

평면 $x=L$

$$\frac{x}{L} = 1 \qquad \frac{1}{B_i} = \frac{k}{hL} = 30.6 \quad \Bigg\} \quad \frac{T(L,t)-T_\infty}{T_o-T_\infty} = 0.98$$

$$\theta_{wall}(L,t) = \frac{T(L,t)-T_\infty}{T_i-T_\infty} = \underbrace{\frac{T(L,t)-T_\infty}{T_o-T_\infty}}_{0.98}\underbrace{\frac{T_o-T_\infty}{T_i-T_\infty}}_{0.8} = 0.784$$

$$\frac{T(L,V,t)-T_\infty}{T_i-T_\infty} = \theta_{wall}(L,t)\ \theta_{cyl}(0,t) = 0.784 \times 0.5 = 0.392$$

c) $\rho = 8530$, $C_p = 0.380$

$$m = \rho V = \rho \pi r_o^2 (2L) = 8.04kg$$

$$Q_{\max} = m C_p (T_i - T_\infty) = 290.2\ kJ$$

평면

$$B_i = \frac{1}{30.6} = 0.0327 \qquad \frac{h^2 \alpha t}{k^2} = B_i^2 \tau = 0.0091 \quad \Bigg\} \quad \left(\frac{Q}{Q_{\max}}\right)_{wall} = 0.23$$

원통

$$B_i = \frac{1}{36.7} = 0.0272 \qquad \frac{h^2 \alpha t}{k^2} = B_i^2 \tau = 0.0090 \quad \Bigg\} \quad \left(\frac{Q}{Q_{\max}}\right)_{\substack{\text{i}finite \\ cylinder}} = 0.47$$

$$\left(\frac{Q}{Q_{\max}}\right)_{\substack{short\\cyl}} = \left(\frac{Q}{Q_{\max}}\right)_1 + \left(\frac{Q}{Q_{\max}}\right)_2 \left[1 - \left(\frac{Q}{Q_{\max}}\right)_1\right]$$

$$= 0.23 + 0.47(1 - 0.23) = 0.592$$

$$\therefore \quad Q = 0.592 \quad Q_{\max} = 0.592 \times (290.2) = 171.8(kJ)$$