

Economics of Industrial Organization

Lecture 10: Collusion and Repeated Games

Explicit vs. Implicit Collusion

- A *cartel* is an organization of firms and countries that openly acts together to control industry prices. *Collusion* is any other action taken by firms to coordinate prices
- In the US and most developed countries, *explicit collusion or cartels is per se illegal*. One of the foundations of competition policy (as in the Sherman Act) is that companies may not act together to effect pricing or quantities.
- So, cartels tend to only exist in international markets (oil, diamonds, shipping), because there is no international law that prevents cartels.
- Explicit collusion is very dangerous, because it is (often) a criminal offence for the executives involved.
- Nonetheless, companies can often follow “implicit” collusion strategies through their pricing policies.

When is collusion most likely?

- Given the illegality of explicit collusion, firms and executives must be careful about attempting any schemes to fix prices. What industry factors might increase the ability of firms to do so?
 1. Small number of firms, fixed number of players (limited entry and exit).
 2. Regular industry meetings where executives from different firms meet
 3. Requirements for shared management of some input or resource
 4. Regular price adjustments.
 5. Product uniformity.
 6. Transparency in price and quantity selections (makes easier to detect and punish defection).

Collusion and Repeated Games

- Many models of oligopoly give at least reasonably competitive outcomes in a one-shot game. Knowing that the game is only played once, players have incentive to increase output or cut prices in order to increase market share. How then can we explain concerns about collusive or cartel behavior?
- In the real world, firms are constantly interacting with each other over time, making pricing decisions on an annual, monthly, weekly daily or even shorter basis (eg electricity market bids every 20 minutes).
- There is much more scope for all kinds of behavior in such a repeated context. In particular, firms may be able to sustain collusive behavior in a repeated setting because they have the ability to punish deviation from a collusive strategy in future periods.
- Now, its not worth always undercutting the other player, because they can punish you in future periods.

Cournot Collusion

- Recall our basic Cournot duopoly game. In the unique Nash equilibrium, players each produce quantities $(\alpha - c)/3$ and earn profits $(\alpha - c)^2/9$, for a total industry quantity of $2(\alpha - c)/3$ and industry profits of $2(\alpha - c)^2/9$.
- Compare this to the quantity and profit of a monopolist with the same demand curve; the monopolist chooses $q_m = (\alpha - c)/2$, and earns profits $(\alpha - c)^2/4$. So, if the duopolists could cooperate, they could each produce half the monopoly output level and gain half the monopoly profits [and note $(\alpha - c)^2/(4 \cdot 2) > (\alpha - c)^2/9$].
- In a one-shot game this kind of cooperation doesn't make sense, because we could always do better by just playing our best response. But in a repeated game we may be able to sustain such behavior.

Repeated Prisoners Dilemma

- Thus far we have studied games, both static and dynamic, which are only played once. We now move to an environment where players interact repeatedly.
- Each player can now condition his action on previous actions by the other players. So a strategy is much more complicated than in previous games, because we must describe how we act as a function of all possible previous histories of the game.
- The first application will be to the prisoner's dilemma.

Repeated Prisoners Dilemma

- Recall the Prisoner's Dilemma in strategic form (positive payoffs):

	Cooperate	Defect
Cooperate	2,2	0, <u>3</u>
Defect	<u>3</u> ,0	<u>1</u> , <u>1</u>

Note in the PD game, Cooperate = Remain Silent and Defect = Confess. Clearly one pure strategy NE at (D,D).

Repeated Prisoners Dilemma

- Suppose now that the game is repeated indefinitely. Are there any strategies we can write down that would sustain (C,C) as the equilibrium strategy in every repetition of the game?
Consider the following “***Grim Trigger Strategy***” for each player:
 - Play C in the first period.
 - Play C in *every* period as long as *all* players in *all* previous periods have played C.
 - If *any* player has deviated (to D) in any previous period, play D in *all* periods forward.
- If the strategy sustains cooperation, players obtain a payoff:
 $2 + 2 + 2 + \dots$
- Starting in any period that a player deviates, the deviating player gets
 $3 + 1 + 1 + \dots$
- So cooperating in all periods is optimal. The short term gains are outweighed by the long term losses.

Repeated Prisoners Dilemma

- Issues:
 - Patience: is \$1000 today worth the same as \$1000 a year from now? We assumed above that players are infinitely patient. What if players weren't so patient?
 - Other NE? Another NE would be both players playing D in all periods.
 - Grim Trigger seems like a strategy with a very severe punishment? Could we sustain cooperation with something less severe?
 - What if there is a final period to the game?

Primer on Arithmetic Series

- Gauss Sum

$$\sum_{i=1}^n i = \frac{1}{2} n(n+1)$$

- Geometric Sum

$$\sum_{i=0}^n a^i = \frac{1-a^{n+1}}{1-a}$$

Limit of a geometric sum. If $|a| < 1$, then as $n \rightarrow \infty$:

$$1 + a^1 + a^2 + \dots = \sum_{i=0}^{\infty} a^i = \frac{1}{1-a}$$

What if the summation index starts at 1 instead?

$$a^1 + a^2 + \dots = \sum_{i=1}^{\infty} a^i = \sum_{i=0}^{\infty} a^i - 1 = \frac{1}{1-a} - 1 = \frac{1-1+a}{1-a} = \frac{a}{1-a}$$

Primer on Arithmetic Series

- Two other useful sums (Still assuming $|a| < 1$).
- Even powers:

$$1 + a^2 + a^4 + a^6 \dots = \sum_{i=0}^{\infty} a^{2i} = \sum_{i=0}^{\infty} (a^2)^i = \frac{1}{1 - a^2}$$

Odd powers:

$$a^1 + a^3 + a^5 \dots = \sum_{i=0}^{\infty} a^{2i+1} = \sum_{i=0}^{\infty} a^{2i} * a = a \sum_{i=0}^{\infty} a^{2i} = \frac{a}{1 - a^2}$$

Most general: For $|r| < 1$,

$$a + ra + r^2a + r^3a + \dots = a/(1-r)$$

Repeated Prisoners Dilemma

- Discounting. Let $\delta_i \in (0,1)$ be the discount factor of player i with utility function $u(\cdot)$
- If player i takes action a^t in period t , his discounted aggregate utility over T periods is:

$$u_i(a^1) + \delta_i u_i(a^2) + \delta_i^2 u_i(a^3) + \dots + \delta_i^{T-1} u_i(a^T) = \sum_{t=1}^T \delta_i^{t-1} u_i(a^t)$$

So if delta is close to 1, the player is very patient. If delta is close to 0, the player discounts the future a lot.

We will usually assume $\delta_i = \delta$ for all players.

What if $T = \infty$ and $a^t = a$ for all t ? Then:

$$\sum_{t=1}^{\infty} \delta^{t-1} u_i(a) = \sum_{t=0}^{\infty} \delta^t u_i(a) = u_i(a) \sum_{t=0}^{\infty} \delta^t = u_i(a) \frac{1}{1-\delta}$$

Repeated Prisoners Dilemma

Nash equilibrium in the finitely repeated Prisoner's Dilemma ? Solve period T and work backwards as usual.

Unique NE is (D,D) in all periods. Cooperation is impossible to sustain.

Nash equilibrium in the infinitely repeated Prisoner's Dilemma ? Most real life situations are not finitely repeated games. Even if they may have a certain ending date, the number of interactions may be uncertain and thus modeling the PD game as an infinitely repeated game seems intuitively pleasing.

As we stated, there may be a strategy in the infinitely repeated PD game that generates cooperation in all periods, for a given level of patience of the players.

Repeated Prisoners Dilemma

Consider the following strategy for player i where j is the other player:

$$s_i(a^1, \dots, a^t) = \left\{ \begin{array}{ll} C, & \text{if } (a_j^1, \dots, a_j^t) = (C, \dots, C) \\ & \text{and } (a_i^1, \dots, a_i^t) = (C, \dots, C) \\ D, & \text{otherwise} \end{array} \right\}$$

Payoff along the equilibrium path:

$$\pi_i^e = 2(1 + \delta + \delta^2 + \dots) = 2 \sum_{t=0}^{\infty} \delta^t = \frac{2}{1-\delta}$$

Payoff following a deviation in the first period:

$$\pi_i^d = 3 + 1(\delta + \delta^2 + \dots) = 3 + \sum_{t=0}^{\infty} \delta^t - 1 = 2 + \frac{1}{1-\delta}$$

Repeated Prisoners Dilemma

- So cooperate is optimal if

$$\frac{2}{1-\delta} > 2 + \frac{1}{1-\delta} \Rightarrow 2 > 2 - 2\delta + 1 \Rightarrow 2\delta > 1 \Rightarrow \delta > \frac{1}{2}$$

So for discount factors greater than 1/2, cooperation in all periods can be sustained as a NE. Players must meet this minimum level of patience.

Repeated Prisoners Dilemma

- We now consider less draconian strategies than the Grim Trigger.
- Tit for Tat Strategy. Play *C* in the first period and then do whatever the other player did in the previous period in all subsequent periods.
- Limited Punishment Strategy. This strategy entails punishing a deviation for a certain number of periods and then reverting to the collusive outcome after the punishment no matter how players have acted during the punishment. For example, Play *D* in periods $t=1$, $t=2$, and $t=3$ if a deviation has occurred in $t=0$ and then play *C* in $t=4$.

Repeated Prisoners Dilemma

- Limited Punishment in the Prisoners Dilemma. Suppose the punishment phase is $k=3$ periods and both players are playing the same strategy.
- Consider the game starting in any period t . If no deviation occurs in periods $t, t+1, t+2$, and $t+3$ then, the payoffs to each player over these periods are:

$$\pi_i^e = 2(1 + \delta + \delta^2 + \delta^3) = 2 \sum_{t=0}^3 \delta^t = \frac{2(1 - \delta^4)}{1 - \delta}$$

If player i deviates in period t , he knows that his opponent will play D for the next 3 periods so he should also play D in those periods. Thus his payoff is:

$$\pi_i^d = 3 + 1(\delta + \delta^2 + \delta^3) = 3 + \sum_{t=0}^3 \delta^t - 1 = 2 + \frac{(1 - \delta^4)}{1 - \delta}$$

Repeated Prisoners Dilemma

- So cooperation is optimal if:

$$\frac{2(1-\delta^4)}{1-\delta} > 2 + \frac{(1-\delta^4)}{1-\delta} \Rightarrow 2(1-\delta^4) > 2 - 2\delta + (1-\delta^4) \Rightarrow 1-\delta^4 > 2-2\delta$$

$$\Rightarrow \delta > 0.55 \text{ (Approx)}$$

What happens as k , the punishment period, gets larger? Delta approaches $1/2$, the grim trigger cutoff.

Repeated Prisoners Dilemma

- Now consider the tit for tat strategy in the Prisoners Dilemma. Suppose player 1 is playing tit for tat and player 2 considers deviating to D in period t . Player 1 will respond with D in all periods until player 2 again chooses C. If player 2 chooses C, we revert to the same situation we started in (and again player 2 should deviate to D).
- So player 2 will either deviate and then play D forever or will alternate between D and C.
- Along the equilibrium path of the game, players earn $2/(1-d)$.

Repeated Prisoners Dilemma

- If player 2 deviates and then always plays D, his payoff is:

$$\pi_2^{d1} = 3 + 1(\delta + \delta^2 + \delta^3 + \dots) = 3 + \sum_{t=0}^{\infty} \delta^t - 1 = 2 + \frac{1}{1-\delta}$$

If player 2 deviates and alternates C and D, his payoff is:

$$\pi_2^{d2} = 3 + 0\delta + 3\delta^2 + 0\delta^3 + 3\delta^4 + 0\delta^5 \dots = 3 * \sum_{t=0}^{\infty} \delta^{2t}$$

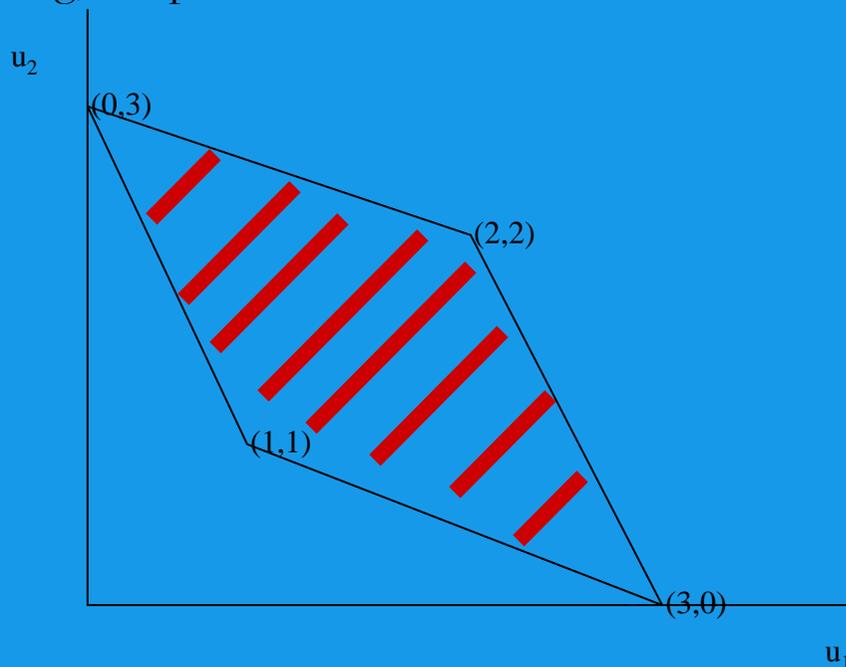
$$\pi_2^{d2} = 3 * \sum_{t=0}^{\infty} (\delta^2)^t = \frac{3}{1-\delta^2}$$

So we need the equilibrium path payoffs to be larger than both of these:

$$\frac{2}{1-\delta} > 2 + \frac{1}{1-\delta} \quad \text{and} \quad \frac{2}{1-\delta} > \frac{3}{1-\delta^2} \quad \longrightarrow \quad \delta > 1/2$$

Repeated Prisoners Dilemma

- So far we have been using trigger type mechanisms to sustain a collusive outcome (ie, Pareto optimal outcome). Can we attain any other payoffs as an equilibrium outcome of the game? Yes!
- Definition. The set of *feasible payoff profiles* of a strategic game is the set of all weighted averages of payoff profiles in the game.
 - Eg, the prisoner's dilemma:

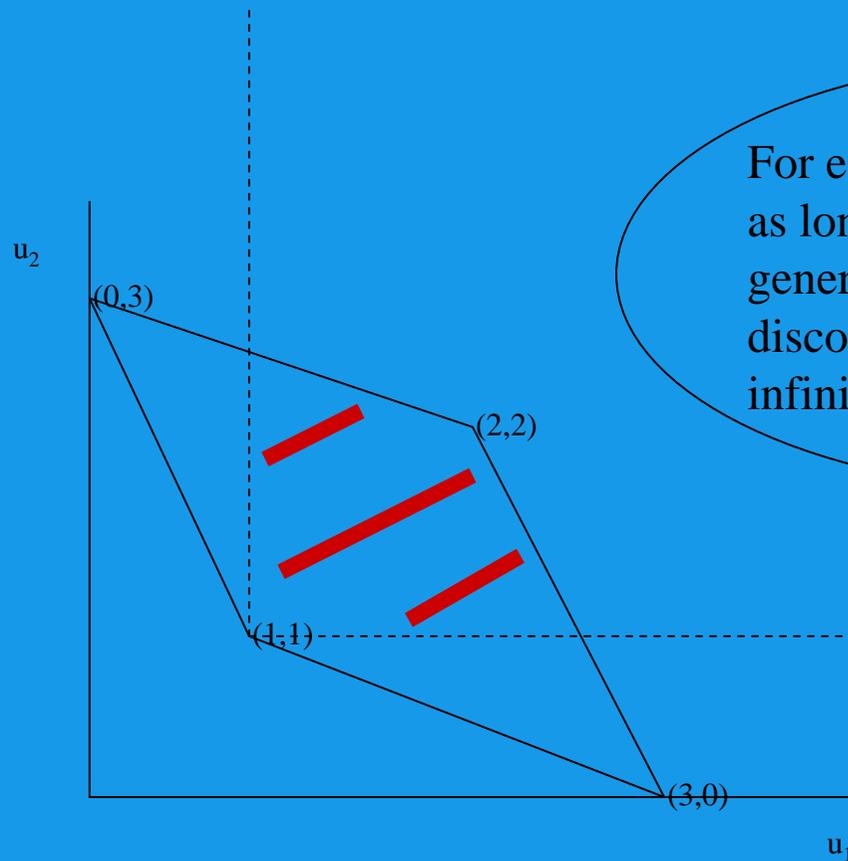


Repeated Prisoners Dilemma

- Folk Theorem for the Prisoners Dilemma:
 - For any discount factor, $0 < d < 1$, the discounted average payoff of each player i in any NE of $G(\infty, d)$ is at least $u_i(D, D)$. I.e., players must at least get the NE payoffs of the static one-shot game.
 - Let (x_1, x_2) be a feasible pair of payoffs in G for which $x_i > u_i(D, D)$ for each player i . Then there is some $\underline{d} < 1$, such that there is a NE of $G(\infty, \underline{d})$ in which the discounted average payoff of each player i is x_i .
 - Note for any discount factor, we can always attain at least $u_i(D, D)$ as a NE of the infinitely repeated game.

Repeated Prisoners Dilemma

- Folk Theorem “Region” (or just the Folk Region) for the PD game:



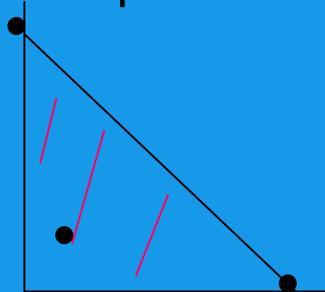
For every point in the shaded region, as long as δ is high enough, we can generate those payoffs as the average discounted payoffs in a NE of the infinitely repeated game.

Repeated Cournot

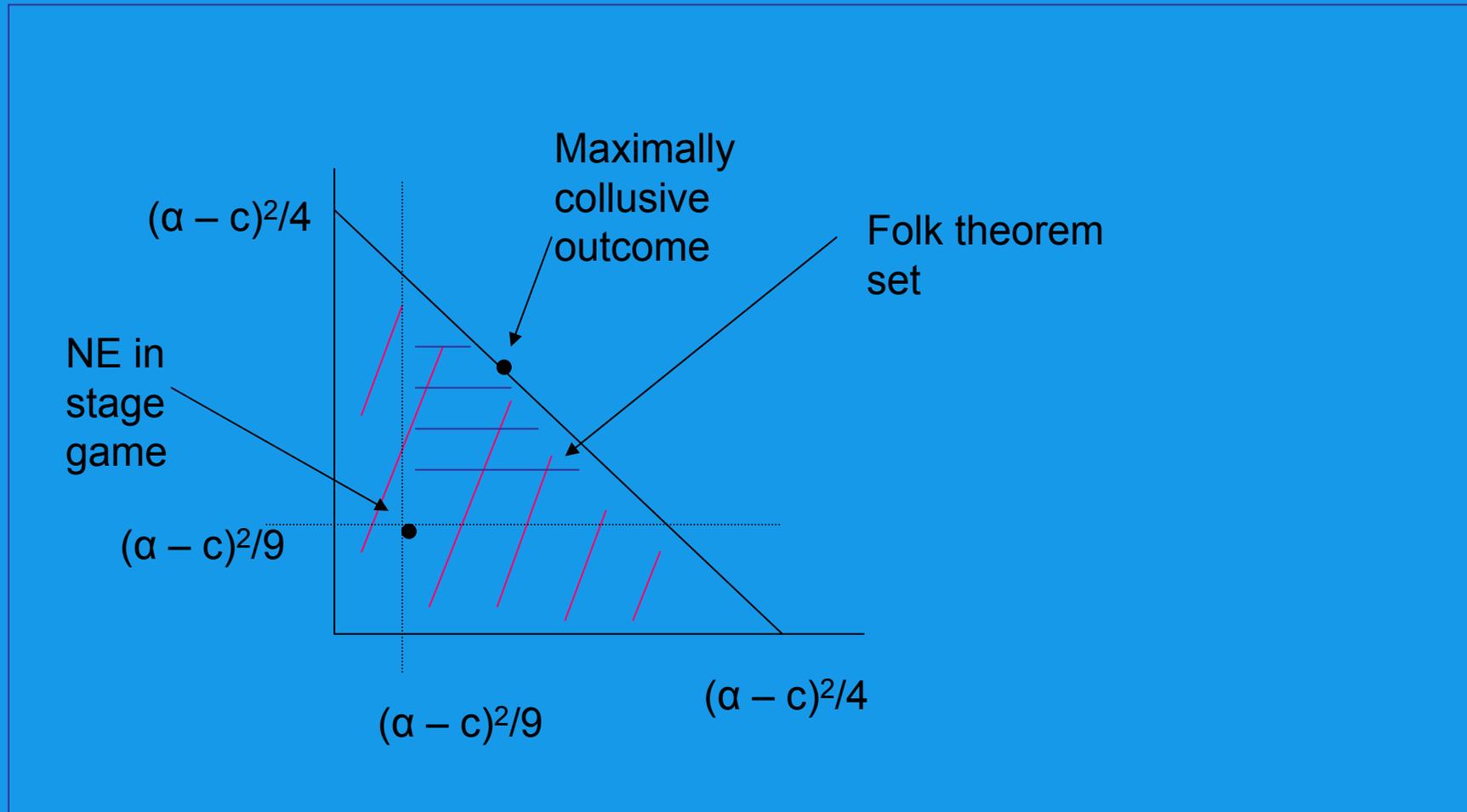
- Same as before.
- Two firms, $i = 1, 2$.
- Market demand $P = \text{Max} [\alpha - Q, 0]$
- Cost function, $C_i(q_i) = cq_i$ (and firms only produce what they sell)
- Player i solves:
$$\text{Max}_{q_i} q_i(\alpha - q_i - q_j - c)$$
$$\text{FOC: } \alpha - 2q_i - q_j - c = 0$$
$$q_i = (\alpha - q_j - c)/2$$
- Applying symmetry gives the equilibrium,
$$q_i = (\alpha - c)/3$$
- This is the unique NE of the one-shot game.

Feasible set

- The unique NE of the stage game is $q_i = (\alpha - c)/3$. This gives payoffs $q_i = (\alpha - c)^2/9$
- The monopolist NE of the stage game is found from solving the monopolist's problem:
 $\max_Q Q(\alpha - Q - c)$
FOC: $\alpha - 2Q - c = 0$
 $Q = (\alpha - c)/2$
Payoff = $(\alpha - c)^2/4$
- No player can get a payoff worse than zero.
- It turns out the frontier is linear (profits are proportional to quantities, and can be spread in any combination between the two firms):



Folk theorem set



Maximally Collusive Equilibrium

- So, could we support an equilibrium with average payoffs of $(\alpha - c)^2/8$ for each player (ie the maximally collusive outcome)?
- Yes, for high enough δ , because this is in the Folk Region.
- Consider the following trigger strategy:
Produce quantity $(\alpha - c)/4$ (half the monopoly output) in the first period. Produce this quantity in every period as long as every player has produced this quantity in all prior periods.
Produce quantity $(\alpha - c)/3$ in every period if any player has produced any quantity other than $(\alpha - c)/4$ in any period.

Maximally Collusive Equilibrium 2

- Find optimal deviation: if other player produces $(\alpha - c)/4$, we can find our optimal output from our best response function.
- Recall $BR_i : q_i = (\alpha - q_j - c)/2$
- So, our best response is to produce $3(\alpha - c)/8$
- This gives an instantaneous payoff of:
$$3(\alpha - c)/8 * (\alpha - c - 5(\alpha - c)/8)$$
$$= 9(\alpha - c)^2/64$$
- But gives only payoffs of $(\alpha - c)^2/9$ forever after.
- Payoff on the equilibrium path:
$$(\alpha - c)^2/8 + \delta(\alpha - c)^2/8 + \delta^2(\alpha - c)^2/8 + \dots$$
- Payoff from deviating:
$$9(\alpha - c)^2/64 + \delta(\alpha - c)^2/9 + \delta^2(\alpha - c)^2/9 + \dots$$

Maximally Collusive Equilibrium 3

- So we find our critical δ by solving this.
- $[(\alpha-c)^2/8]/(1-\delta) \geq 9(\alpha-c)^2/64 + [\delta(\alpha-c)^2/9]/(1-\delta)$
- $(\alpha-c)^2/8 \geq 9(1-\delta)(\alpha-c)^2/64 + \delta(\alpha-c)^2/9$
- $0 \geq (\alpha-c)^2/64 - 17\delta(\alpha-c)^2/576$
- $\delta \geq 9(\alpha-c)^2/17$

Another Example

- Could we sustain an outcome (approximately) halfway between the NE and the maximally collusive outcome?
- Yes, for high enough δ , because this is in the Folk Theorem Region.
- How would we support this?
- Consider the following strategy: produce half the monopolistic quantity in the first round. Produce the NE quantity in the second round, and in every even round. Produce half the monopolistic quantity in every odd round, as long as in every prior odd round no player has produced anything other than the monopoly quantity, otherwise produce the NE amount forever.

Repeated Bertrand

- Consider our standard Bertrand duopoly model, with $Q = a - \min(p_i, p_j)$, $C(q) = cq$, and $q_i = 0, Q/2$ or Q depending on relative p_i and p_j .
- Suppose now that this game is infinitely repeated, where players play the following trigger strategies; play $p_i = p_m$ (the monopoly price) as long as every player has played p_m in all prior periods, play $p_i = c$ forever otherwise.
Recall that $p_m = (a+c)/2$, and $\pi_m = (a - c)^2/4$
- Payoffs on the equilibrium path = $\pi_m/2 + \delta\pi_m/2 + \delta^2\pi_m/2 + \dots$
 $= (\pi_m/2)/(1 - \delta)$
- “Optimal deviation” not defined (with continuous prices), but we would like to just undercut the monopoly price by some ε . This leads to us capturing the entire market at (effectively) the monopoly price and (and quantity).

Repeated Bertrand 2

- So, payoffs from optimal deviation = $\pi_m + \delta 0 + \delta^2 0 + \dots = \pi_m$

- Collusion can be sustained when:

$$(\pi_m/2)/(1 - \delta) \geq \pi_m$$

$$(1/2)/(1 - \delta) \geq 1$$

$$\delta \geq 1/2$$

Cartel Enforcement and Antitrust

- The Folk theorem shows us that collusive outcomes are potentially attainable by firms, so we cannot rely on defection by firms to prevent price fixing.
- Explicit intervention by policy makers is needed to prevent collusion.
- Suppose that a cartel exists and that it is self-sustaining. Now suppose that there exists an antitrust authority which is looking for and prosecuting cartels.
- Assume that in any given period, there is a probability a that the authority will investigate the cartel. If there is an investigation, assume there is a probability s that it leads to successful prosecution, which leads to a fine of F to cartel members and the cartel breaks down (forever). If the prosecution is unsuccessful, the cartel continues.

- Suppose that under the cartel agreement, firms earn π_M . Suppose that from optimal deviation, it gets an instantaneous profit of π_D . Suppose that Nash equilibrium payoffs are π_N .
- Denote V_C to be the present value of profits under the cartel, with this model of antitrust intervention.
- We need to consider three terms to evaluate V_C :
 1. No investigation in period 0. Occurs with prob $(1 - \alpha)$.

$$V_1 = (1 - \alpha)(\pi_M + \delta V_C).$$
 2. Unsuccessful investigation in period 0, prob $\alpha(1 - s)$

$$V_2 = \alpha(1 - s)(\pi_M + \delta V_C).$$
 3. Successful prosecution. Probability αs

$$V_3 = \alpha s[\pi^M - F + \delta \pi_N / (1 - \delta)]$$
- Expected present value of profits for a cartel member is thus:

$$V_C = V_1 + V_2 + V_3$$
- Solving for V_C gives:

$$V_C = [\pi^M - \alpha s F + (\alpha s \delta) \pi^N / (1 - \delta)] / [1 - \delta(1 - \alpha s)]$$

 Compare to V_C with no antitrust = $\pi_M / (1 - \delta)$

Fines, Detection, and Prosecution

- Clearly, profits are decreasing (and so collusive outcomes will be harder to maintain) in the size of the fine, the probability of detection and the success of prosecution.
- Fines are generally the most cost-effective form of punishment (increasing the fine size is cheap, whereas increasing cartel detection or prosecution success is very expensive), but fines still require positive values of α and s .
- Also, we cannot increase the size of fines forever, because we cannot fine a company a larger value than its assets (the “judgement-proof” problem). So we cannot rely on large fines with low probabilities alone.

Factors that facilitate collusion: High Industry concentration

- Recall that the sustainability of collusion depends on the payoffs from cooperation relative the payoffs from the Nash equilibrium.
- For example: Bertrand. With n player Bertrand, payoffs along the equilibrium path are π_m/n per period, but payoffs from deviation remain unchanged.
- Collusion can be sustained when:
$$(\pi_m/n)/(1 - \delta) \geq \pi_m$$
$$(1/n)/(1 - \delta) \geq 1$$
$$\delta \geq (n-1)/n$$
- With larger n , collusion is harder to sustain. So collusion is much more likely to occur in a highly concentrated industry.

Significant entry barriers

- Easy entry undermines collusion. A new entrant increases the number of players in the industry. A collusive equilibrium where incumbents earn positive profits are more likely to facilitate entry. New entrants that do not play by the cartel agreement will also undermine a collusive strategy.
- So we are more likely to observe collusion in industries with large entry barriers. Cartels are likely to be unsustainable if members cannot prevent entry.

Frequent price changes

- The more rapidly firms face price changes, the shorter is each “period” and so the higher is the value of δ .
- Frequent price changes effectively make it possible to rapidly punish deviations from the collusive agreement, so make collusion easier to sustain.

Rapid market growth

- In a market where profits are increasing over time, deviation now gains you only today's (relatively small) profits and means that you miss out on increasing future collusive profits.
- Similarly, if profits are decreasing over time, deviation gets you today's (relatively large) profits, while you miss out on a stream of profits that is declining.
- Consequently, collusion is easier to sustain in markets where profits are growing, and are harder to sustain in markets where profits are declining.

Technology or cost symmetry

- When firms are of similar size and have similar cost structures it becomes easier to “share” profits or output between firms.

Product homogeneity

- Collusion is easier to sustain when firms are producing a small range of products, and where products are very similar across firms. With fewer products there are fewer prices to monitor to test for deviation.
- With product differentiation, the collusive agreement may have different prices for different products. When products are similar, it is easier to determine the “fair” collusive price for each product, and easier to detect deviation.
- When products are highly differentiated, it is more difficult to punish deviation since having rivals cut their prices has a smaller impact on a deviator’s market, and since it can be hard to determine which cartel members should cut their prices in order to punish the deviator.

Observability

- If price or output decisions are hard to observe, it is harder to detect defection and so harder to sustain collusion.
- “Meet the competition” clauses

Stable market conditions

- In unstable markets where demand or costs are fluctuating, the optimal collusive agreement can be changing over time.
- In such circumstances, it can be difficult to determine whether a price cut by a rival firm is a deviation from the cartel, or is merely a response to changing market conditions.
- In such markets, sometimes the best feasible collusive agreement is one that sometimes institutes price war punishment phases even when no deviation has occurred.
- Consider a differentiated product environment where a firm observes only the (residual) demand for its own product. If a firm observes that its demand has fallen, it can't tell whether this is from a market shock, or from a rival defecting from the cartel. So in order to deter defection, firms have to implement some (temporary) punishment whenever they suffer lower prices.
- Thus, we can have an equilibrium where we periodically have price wars even when no deviation actually occurs.