

Lecture 05 – Cost Concepts for Decision Making

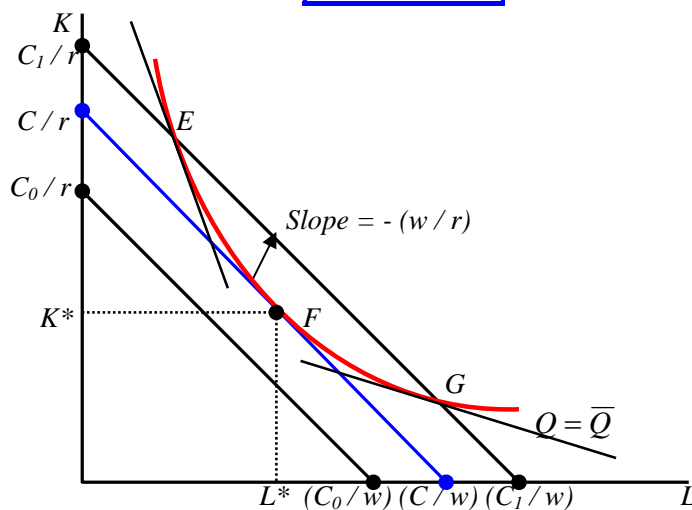
1. Costs

- a) Explicit costs which involve a direct monetary outlay.
Indirect costs that do not involve outlays of cash.
- b) Accounting Costs (total sum of explicit costs)
Economic Costs (sum of explicit and implicit costs)
- c) Opportunity Costs
- d) Sunk (unavoidable) costs vs. Non-sunk (avoidable) costs, Initial set-up costs.

2. Cost-Minimization Problem

- a) iso-cost curve

$$wL + rK = TC = C, \quad K = -\frac{w}{r}L + \frac{C}{r}$$



- b) Cost minimization

At point F, (slope of iso-cost curve) = (slope of isoquant curve). So, $\frac{w}{r} = MRTS_{L,K}$

And we already know that $MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{w}{r}$ or $\frac{MP_L}{w} = \frac{MP_K}{r}$.

The last expression implies that the additional output per dollar spent on labor services equals the additional output per dollar spent on capital services (“equal bang for the buck”).

You can check the condition at points E and G.

How about the corner point solutions? Does the above equation hold at the corner solution?

Generally, if we have m units of inputs, the cost minimization condition (FOC) is

$$\frac{MP_1}{w_1} = \frac{MP_2}{w_2} = \dots = \frac{MP_m}{w_m}$$

- c) Expansion Path

$$Q_0 < Q_1 < Q_2$$

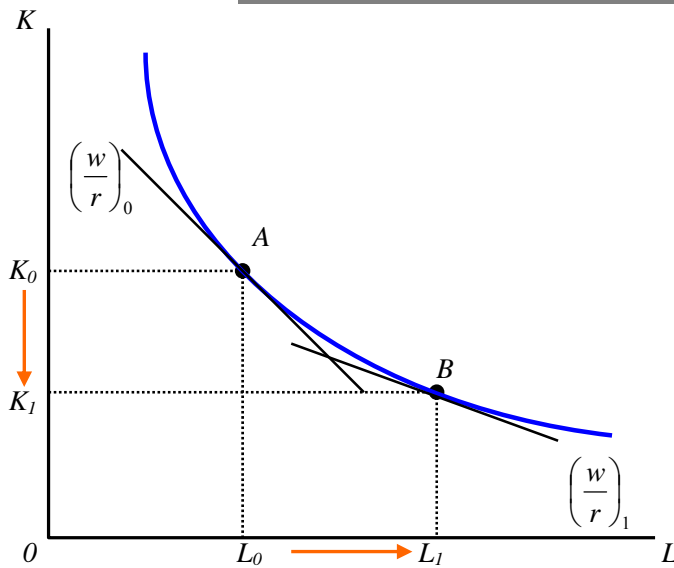
d) Comparative Statics Analysis of Changes in Input Prices

Demand for inputs is *Derived Demand* depending upon change in quantities of final products.

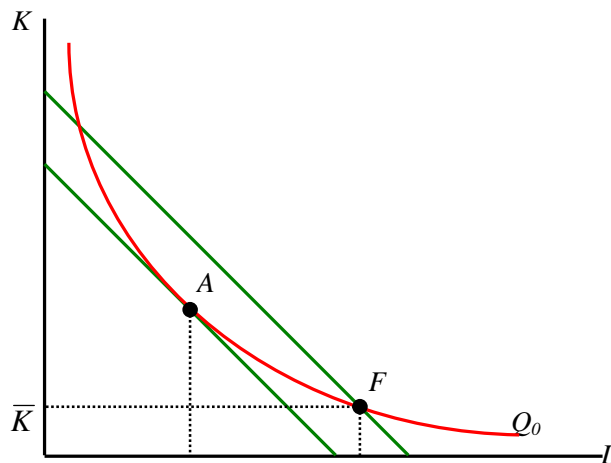
So, we can't use the method in consumption theory to derive input demand curves.

To analyze the effect of change in input prices without considering change in quantities, we need to confine our analysis to change in input combinations that can produce same amount of output (analogous to consumption theory of only substitution effect!).

$$\sigma = \frac{\Delta(K/L)/(K/L)}{\Delta MRTS / MRTS} = \frac{\Delta(K/L)/(K/L)}{\Delta(w/r)/(w/r)}$$



e) Cost Minimization in SR



When the firm's capital is fixed at \bar{K} , the short-run cost-minimizing input combination is at point F. If the firm were free to adjust all of its inputs, the cost-minimizing combination would be at point A.

$$\begin{cases} \min_{(L,K)} wL + rK \\ \text{subject to: } Q = f(L, K) \end{cases}$$

We proceed by defining a Lagrangean function

$$\Lambda(L, K, \lambda) = wL + rK + \lambda[f(L, K) - Q]$$

,where λ is a Lagrange multiplier.

The conditions for an interior optimal solution ($L > 0, K > 0$) to this problem are

$$\begin{cases} \frac{\partial \Lambda}{\partial L} = 0 \Rightarrow w = \lambda \frac{\partial f(L, K)}{\partial L} = \lambda MP_L & (1) \\ \frac{\partial \Lambda}{\partial K} = 0 \Rightarrow r = \lambda \frac{\partial f(L, K)}{\partial K} = \lambda MP_K & (2) \\ \frac{\partial \Lambda}{\partial \lambda} = 0 \Rightarrow f(L, K) = Q & (3) \end{cases}$$

From (1) and (2), we can get $\frac{MP_L}{MP_K} = \frac{w}{r}$ (4)

Equations (3) and (4) are two equations with two unknowns, L and K. They are identical to the conditions that we derived for an interior solution to the cost-minimization problem using graphical arguments. The solution to these conditions are the long-run input demand functions, $L^*(w, r, Q)$ and $K^*(w, r, Q)$.

Ex) Production function is $Q = 50\sqrt{LK}$. What are the demand curves for labor and capital?

f) Duality: “Backing Out”

The above analysis shows how we can start with a production function and derive the input demand function. But we can also reverse directions: If we start with input demand functions, we can characterize the properties of a production function and sometimes even write down the equation of the production function. This is because of duality, which refers to the correspondence between the production function and the input demand function.

Ex) Suppose we are given respective labor demand function and capital demand function

$$L = \frac{Q}{50} \sqrt{\frac{r}{w}} \text{ and } K = \frac{Q}{50} \sqrt{\frac{w}{r}}. \text{ Solving for } w \text{ in terms of } Q, r, \text{ and } L: w = \left(\frac{Q}{50L}\right)^2 r$$

Plugging the last expression in capital demand function,

$$K = \frac{Q}{50} \left(\frac{(Q/50L)^2 r}{r}\right)^{1/2} = \frac{Q^2}{2,500L}. \text{ Finally, } Q^2 = 2500LK, Q = 50\sqrt{LK} = 50L^{0.5}K^{0.5}$$

3. Production Cost in the Short-Run

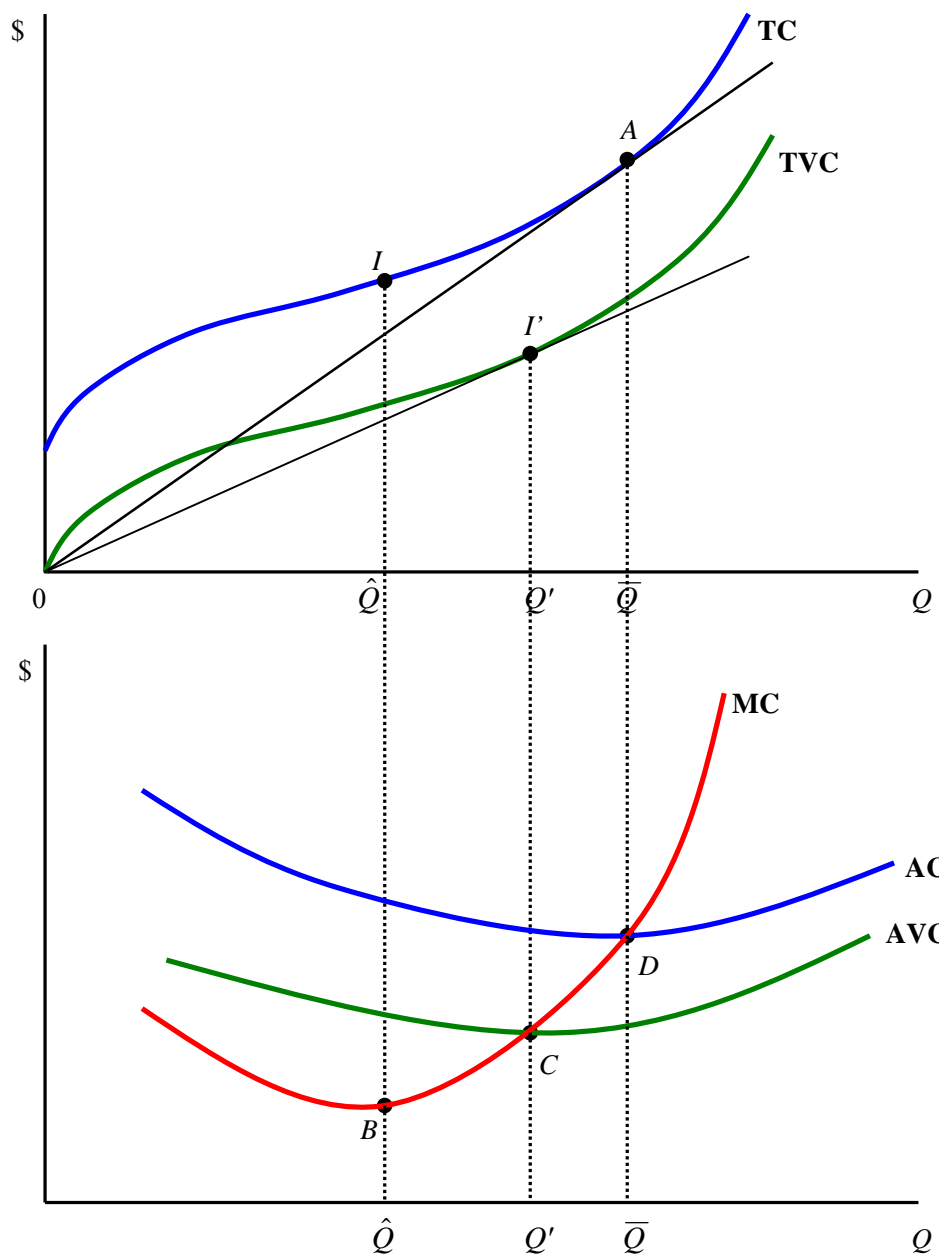
The function $TC = TC(w, r, Q)$ simplifies to $TC = TC(Q)$

$$TC = TFC + TVC = Q \cdot AC = Q(AFC + AVC)$$

$$AC = \frac{TC}{Q} = \frac{TFC}{Q} + \frac{TVC}{Q} = AFC + AVC$$

$$MC = \frac{\Delta TC}{\Delta Q} = \frac{\Delta TFC}{\Delta Q} + \frac{\Delta TVC}{\Delta Q} = MFC + MVC = MVC$$

Marginal Cost in SR is Marginal Variable Cost ($\because MFC = 0$)



$TC(Q) = F + VC(Q)$. The necessary condition of minimum AVC (at point C) is

$$\frac{d[VC(Q)/Q]}{dQ} = \frac{VC'(Q)Q - VC(Q)}{Q^2} = \frac{VC'(Q)}{Q} - \frac{VC(Q)}{Q^2}$$

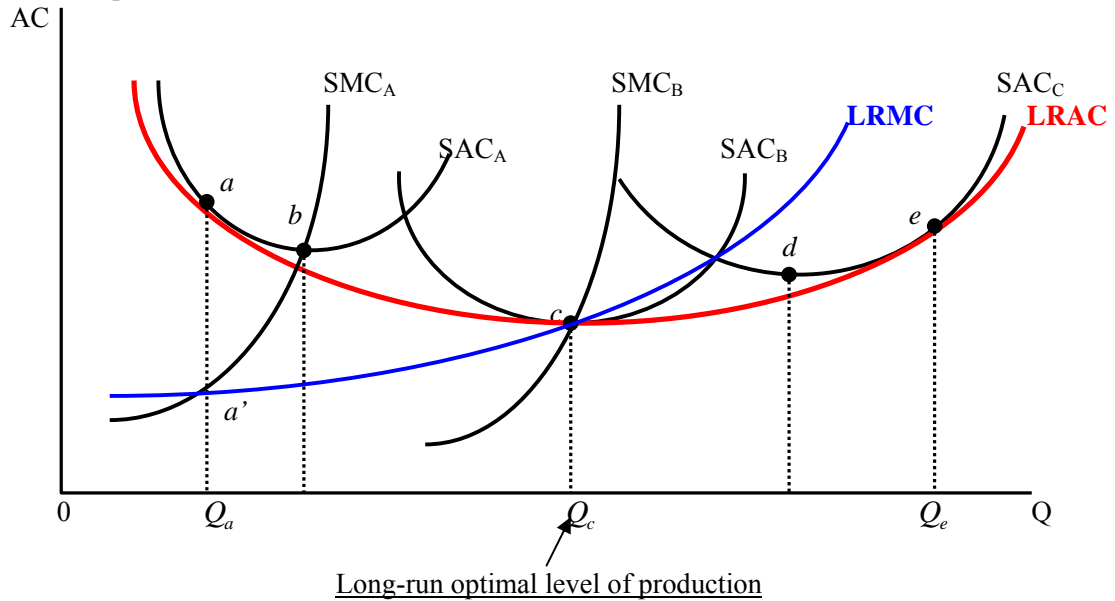
$$\frac{dVC(Q)}{dQ} \cdot \frac{1}{Q} - \frac{VC(Q)}{Q^2} = \left[\frac{dVC(Q)}{dQ} - \frac{VC(Q)}{Q} \right] \frac{1}{Q} = 0. \text{ So,}$$

$$\frac{dVC(Q)}{dQ} = \frac{VC(Q)}{Q}, \text{ MVC} = \text{AVC}$$

Likewise, can you prove the condition at point D?

4. Production Cost in the Long-Run

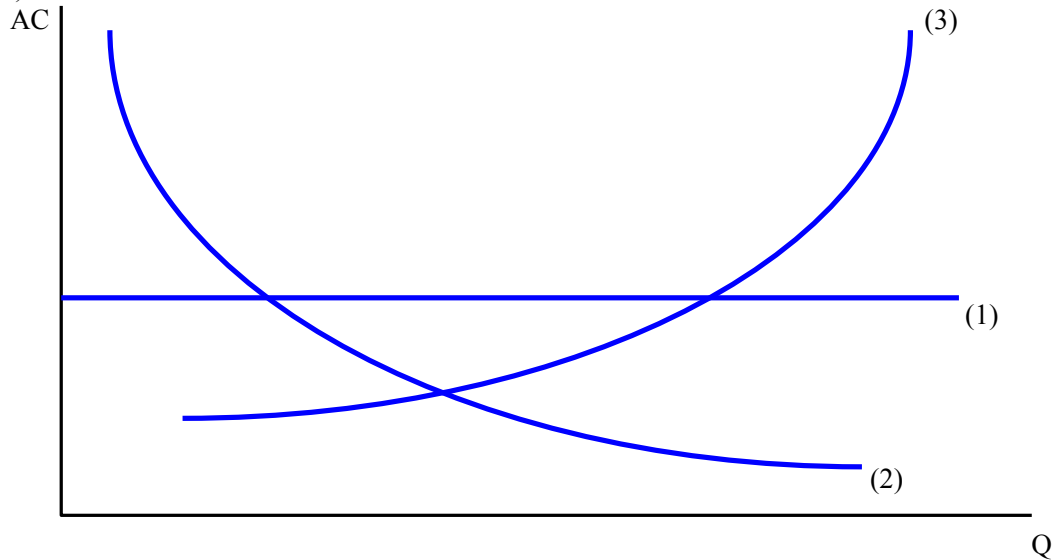
a) Envelope Curve



b) LRMC Curve

At point c, $LRAC = LRMC = SAC_B = SMC_B$

c) Returns to Scale and LRAC Curve



- (1) Constant Returns to Scale
- (2) Increasing Returns to Scale (Economies of Scale or Scale Economy)
→ Natural Monopoly
- (3) Decreasing Returns to Scale (Diseconomies of Scale)

d) Economies of Scope

$$TC(Q_1, Q_2) < TC(Q_1, 0) + TC(0, Q_2)$$

$$ES = \frac{TC(Q_1) + TC(Q_2) - TC(Q_1, Q_2)}{TC(Q_1, Q_2)}$$

Sources of Scope Economies: Shared common inputs, cost complementation (oil and benzene, oil and natural gas, computer software and computer support etc.)

e) Economies of Experience (Learning by Doing)

Economies of scale (EOS) and Economies of Experience (EOE) are different. EOS refer to the ability to perform activities at a lower unit cost when those activities are performed on a larger scale at a given point in time. EOE refer to reductions in unit costs due to accumulating experience over time. EOS may be substantial even when EOE are minimal. This is likely to be the case in mature, capital-intensive production processes, such as aluminum can manufacturing. Likewise, EOE may be substantial even when EOS are minimal as in such complex labor-intensive activities as the production of handmade watches.

Firms that do not correctly distinguish between EOS and EOE draw incorrect inferences about the benefits of size in a market. For example, if a firm has low average costs because of EOS, reductions in the current volume of production will increase unit costs. If the low average costs are the results of cumulative experience, the firm may be able to cut back current production volumes without raising its average costs.