Lecture 3

Discounted Cash Flow Valuation

Discounted Cash Flow Valuation

- *PV* (*현가*, 현재가치 또는 현금가치), *FV* (*미래가* 또는 미래가치)
  - Be able to compute the *FV* (future value) and/or *PV* (present value) of a single cash flow or series of cash flows
- the return on an investment 투자수익율
  - Be able to compute the return on an investment
  - Be able to use a financial calculator and/or spreadsheet to solve time value problems
- perpetuities 영구연금 and annuities 연금
  - Understand perpetuities and annuities

The One-Period Case

- Future Value
  - In the one-period case, the formula for *FV* can be written as:

$$FV = C_0 \times (1 + r)$$

, where C_0 is cash flow today (time zero), and r is the appropriate interest rate.

- Present Value
  - In the one-period case, the formula for *PV* can be written as:

$$PV = \frac{C_1}{1+r}$$

, where C_1 is cash flow at date 1

Future Value
FV (미래가 또는 미래가치)

$$FV = C_0 \times (1 + r)$$

- If you were to invest \$10,000 at 5-percent interest for one year, your investment would grow to \$10,500.

\$500 would be interest (\$10,000 × .05)
<u>\$10,000</u> is the principal repayment (\$10,000 × 1)
\$10,500 is the total due. It can be calculated as:

$$\$10{,}500 = \$10{,}000 \times (1.05)$$

- The total amount due at the end of the investment is call the *Future Value* (*FV*).

Present Value
PV (현가, 현재가치 또는 현금가치)

$$PV = \frac{C_1}{1+r}$$

- If you were to be promised \$10,000 due in one year when interest rates are 5-percent, your investment would be worth \$9,523.81 in today's dollars.

$$\$9,523.81 = \frac{\$10,000}{1.05}$$

- The amount that a borrower would need to set aside today to be able to meet the promised payment of \$10,000 in one year is called the *Present Value* (*PV*). Note that

$$\$10,000 = \$9,523.81 \times (1.05).$$

PV and FV

- PV or P, FV or F
- $F = P \times (1 + r)$ and $P = F/(1+r)$

PV and FV

- *PV or P, FV or F*
- $F = P \times (1 + r)$ and

  $P = F/(1+r)$ or

$$P = \frac{F}{1+r} \quad , P = F\frac{1}{1+r} \quad , P = Fd \ with \ d = \frac{1}{1+r}$$

Net Present Value

- The Net Present Value (*NPV*) of an investment is the present value of the expected cash flows, less the cost of the investment.
- Suppose an investment that promises to pay \$10,000 in one year is offered for sale for \$9,500. Your interest rate is 5%. Should you buy?

Net Present Value

$$NPV = -\$9{,}500 + \frac{\$10{,}000}{1.05}$$

$$NPV = -\$9{,}500 + \$9{,}523.81$$

$$NPV = \$23.81$$

The present value of the cash inflow is greater than the cost. In other words, the Net Present Value is positive, so the investment should be purchased.

Net Present Value

In the one-period case, the formula for *NPV* can be written as:

$$NPV = -Cost + PV$$

If we had *not* undertaken the positive *NPV* project considered on the last slide, and instead invested our $9,500 elsewhere at 5 percent, our *FV* would be less than the $10,000 the investment promised, and we would be worse off in *FV* terms :

$$\$9{,}500 \times (1.05) = \$9{,}975 < \$10{,}000$$

The Multiperiod Case

- The general formula for the future value of an investment over many periods can be written as:

$$FV = C_0 \times (1 + r)^T$$

Where

C_0 is cash flow at date 0,

r is the appropriate interest rate, and

T is the number of periods over which the cash is invested.

Future Value

- Suppose a stock currently pays a dividend of \$1.10, which is expected to grow at 40% per year for the next five years.
- What will the dividend be in five years?

$$FV = C_0 \times (1 + r)^T$$

$$\$5.92 = \$1.10 \times (1.40)^5$$

Future Value and Compounding

- Notice that the dividend in year five, \$5.92, is considerably higher than the sum of the original dividend plus five increases of 40-percent on the original \$1.10 dividend:

$$\$5.92 > \$1.10 + 5 \times [\$1.10 \times .40] = \$3.30$$

This is due to *compounding 복리계산*.

Future Value and Compounding

$\$1.10 \times (1.40)^5$

$\$1.10 \times (1.40)^4$

$\$1.10 \times (1.40)^3$

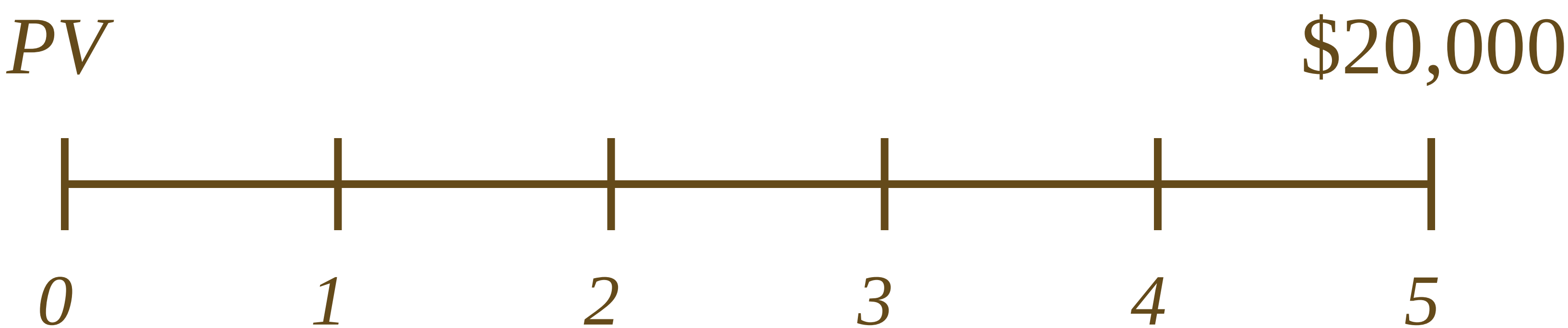
$\$1.10 \times (1.40)^2$

$\$1.10 \times (1.40)$

\$1.10	\$1.54	\$2.16	\$3.02	\$4.23	\$5.92
0	*1*	*2*	*3*	*4*	*5*

Present Value and Discounting 할인

- How much would an investor have to set aside today in order to have $20,000 five years from now if the current rate is 15%?



$$\$9{,}943.53 = \frac{\$20{,}000}{(1.15)^5}$$

Present Value and Discounting 할인

- $P = C_T / (1 + r)^T$ *or* $P = C_T (1 + r)^{-T}$

$$P = \frac{C_T}{(1+r)^T} \quad , P = C_T \frac{1}{(1+r)^T} \quad , P = Fd \ \ with \ d = \frac{1}{(1+r)^T}$$

How Long is the Wait?

If we deposit \$5,000 today in an account paying 10%, how long does it take to grow to \$10,000?

$$FV = C_0 \times (1+r)^T \qquad \$10{,}000 = \$5{,}000 \times (1.10)^T$$

$$(1.10)^T = \frac{\$10{,}000}{\$5{,}000} = 2$$

$$\ln(1.10)^T = \ln(2)$$

$$T = \frac{\ln(2)}{\ln(1.10)} = \frac{0.6931}{0.0953} = 7.27 \text{ years}$$

What Rate Is Enough?

Assume the total cost of a college education will be $50,000 when your child enters college in 12 years. You have $5,000 to invest today. What rate of interest must you earn on your investment to cover the cost of your child's education?

About 21.15%.

$$FV = C_0 \times (1+r)^T$$

$$\$50{,}000 = \$5{,}000 \times (1+r)^{12}$$

$$(1+r)^{12} = \frac{\$50{,}000}{\$5{,}000} = 10$$

$$(1+r) = 10^{1/12}$$

$$r = 10^{1/12} - 1 = 1.2115 - 1 = .2115$$

PV and FV with multiple cash flow stream in general

- Valuation Basics

DCF, Discounted Cash Flow 현금할인

DCF formula and its variations

What Is a Firm Worth?

Valuation of Stocks and Bonds

PV and FV with multiple cash flow stream in generalValuation

- Cash flows, present value, and future value
  - Cash Flows

$$C_0, C_1, C_2, \ldots, C_t, \ldots, C_T$$

  - Time and uncertainty (risk) must be considered in valuation

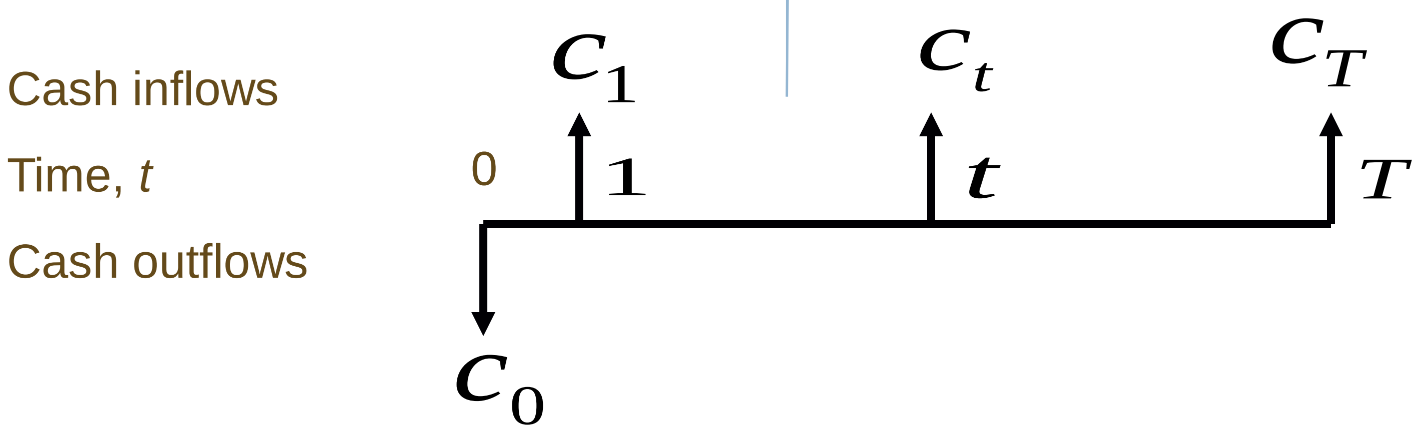
$$P \rightarrow F, F \rightarrow P$$
$$P(F), F(P)$$

- Mostly value in the present cash form

Cash Flows Diagram현금흐름표

- Cash Flows

$$C_0, C_1, C_2, \ldots, C_t, \ldots, C_T$$



Discounted Cash Flow Valuation

- Cash Value and Cash Equivalent
  - Present (Cash) value of a future cash flow is its cash equivalent value
- $P = dF$ or $F = (1/d)P$
  - *d $(1/d)$ is a factor to convert F (P) to P (F)*
  - *It is essentially a present (or cash) value of \$1 in the future*
  - *It is closely associated with opportunity cost of \$1 today not used, and therefore an interest rate*

Discounted Cash Flow Valuation

- Additivity Principle
  - PV of a sequence of future cash flows the sum of the present value of each of individual cash flow

$$P(F_1, F_2, \ldots, F_T) = \sum_{t=1}^{T} P(F_t)$$

DCF (Discounted Cash Flow) Valuation

- DCF formula
- DCF, discount factor and interest rates

$$P = \sum_{t=0}^{T} d_t C_t = \sum_{t=0}^{T} (1+r)^{-t} C_t$$

4.3 Compounding Periods복리계산기간

Compounding an investment *m* times a year for *T* years provides for future value of wealth:

$$FV = C_0 \times \left(1 + \frac{r}{m}\right)^{m \times T}$$

Compounding Periods

- For example, if you invest \$50 for 3 years at 12% compounded semi-annually, your investment will grow to

$$FV = \$50 \times \left(1 + \frac{.12}{2}\right)^{2\times 3} = \$50 \times (1.06)^6 = \$70.93$$

Effective Annual Rates of Interest
실효연간이자율(금리)

A reasonable question to ask in the above example is "what is the effective *annual* rate of interest on that investment?"

$$FV = \$50 \times (1 + \frac{.12}{2})^{2\times 3} = \$50 \times (1.06)^6 = \$70.93$$

The Effective Annual Rate (EAR) of interest is the annual rate that would give us the same end-of-investment wealth after 3 years:

$$\$50 \times (1 + EAR)^3 = \$70.93$$

Effective Annual Rates of Interest

$$FV = \$50 \times (1 + EAR)^3 = \$70.93$$

$$(1 + EAR)^3 = \frac{\$70.93}{\$50}$$

$$EAR = \left(\frac{\$70.93}{\$50}\right)^{1/3} - 1 = .1236$$

So, investing at 12.36% compounded annually is the same as investing at 12% compounded semi-annually.

Effective Annual Rates of Interest

- Find the Effective Annual Rate (EAR) of an 18% APR loan that is compounded monthly.
- What we have is a loan with a monthly interest rate rate of 1½%.
- This is equivalent to a loan with an annual interest rate of 19.56%.

$$\left(1+\frac{r}{m}\right)^{n\times m} = \left(1+\frac{.18}{12}\right)^{12} = (1.015)^{12} = 1.1956$$

Continuous Compounding 연속복리

- The general formula for the future value of an investment compounded continuously over many periods can be written as:

$$FV = C_0 \times e^{rT}$$

Where

C_0 is cash flow at date 0,

r is the stated annual interest rate,

T is the number of years, and

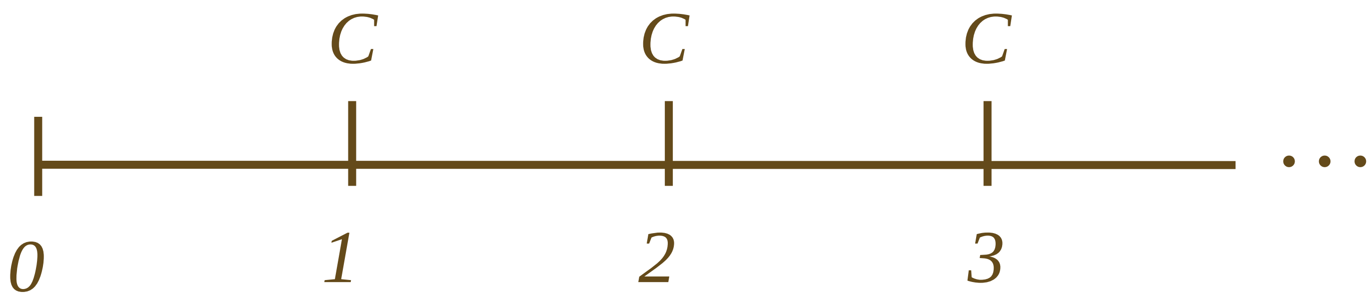
e is a transcendental number approximately equal to 2.718. e^x is a key on your calculator.

Simplifications

- Perpetuity
  - A constant stream of cash flows that lasts forever
- Growing perpetuity
  - A stream of cash flows that grows at a constant rate forever
- Annuity
  - A stream of constant cash flows that lasts for a fixed number of periods
- Growing annuity
  - A stream of cash flows that grows at a constant rate for a fixed number of periods

Perpetuity 영구연금

A constant stream of cash flows that lasts forever



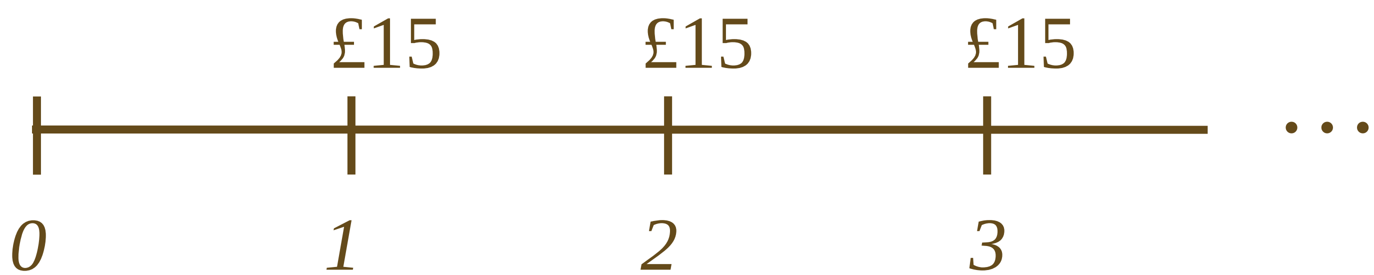
$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots$$

$$PV = \frac{C}{r}$$

Perpetuity: Example

What is the value of a British consol that promises to pay £15 every year for ever?

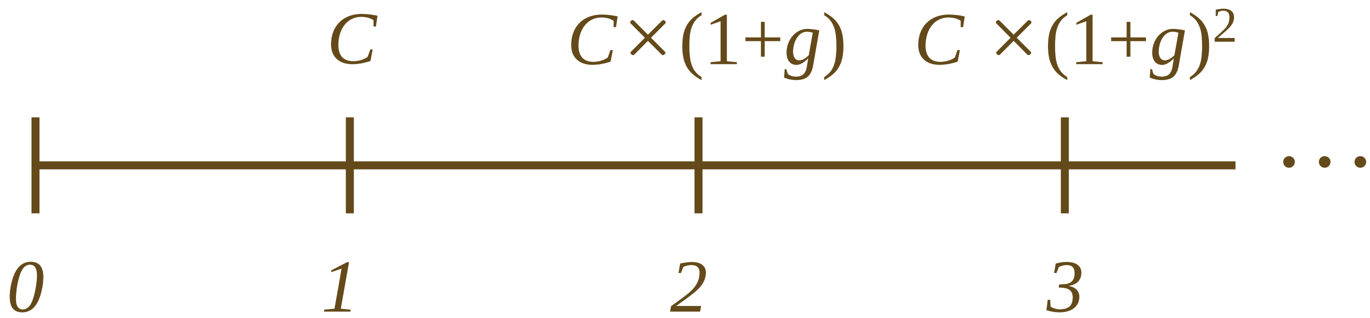
The interest rate is 10-percent.



$$PV = \frac{?5}{.10} = ?50$$

Growing Perpetuity 성장영구연금

A growing stream of cash flows that lasts forever



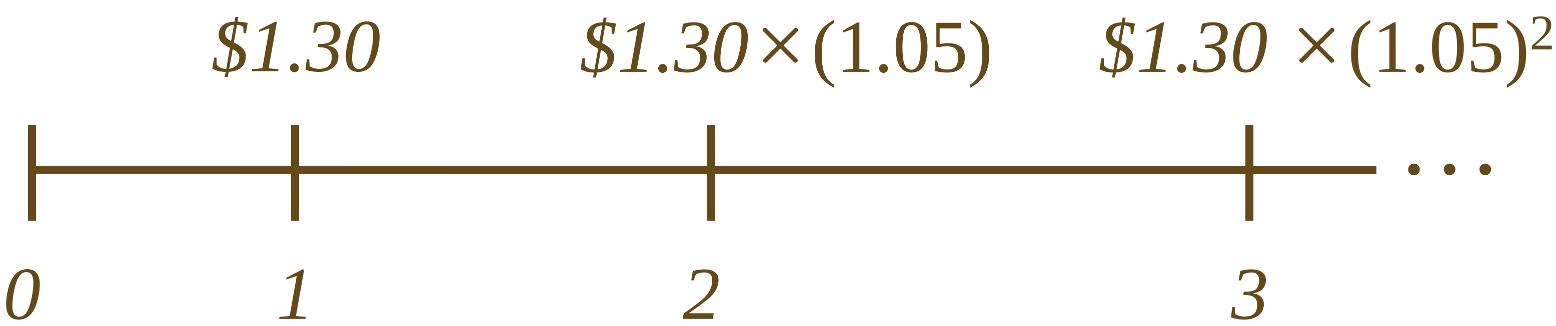
$$PV = \frac{C}{(1+r)} + \frac{C \times (1+g)}{(1+r)^2} + \frac{C \times (1+g)^2}{(1+r)^3} + \cdots$$

$$PV = \frac{C}{r-g}$$

Growing Perpetuity: Example

The expected dividend next year is \$1.30, and dividends are expected to grow at 5% forever.

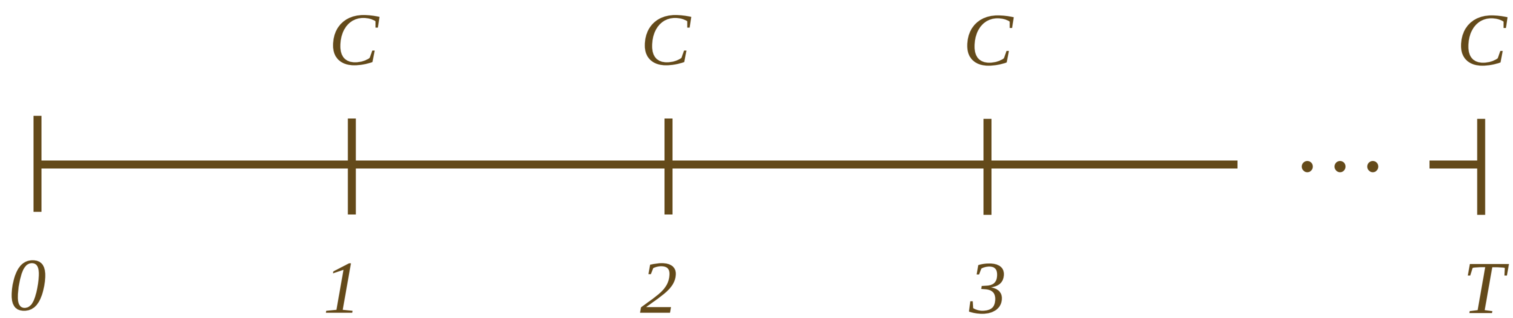
If the discount rate is 10%, what is the value of this promised dividend stream?



$$PV = \frac{\$1.30}{.10 - .05} = \$26.00$$

Annuity 연금

A constant stream of cash flows with a fixed maturity

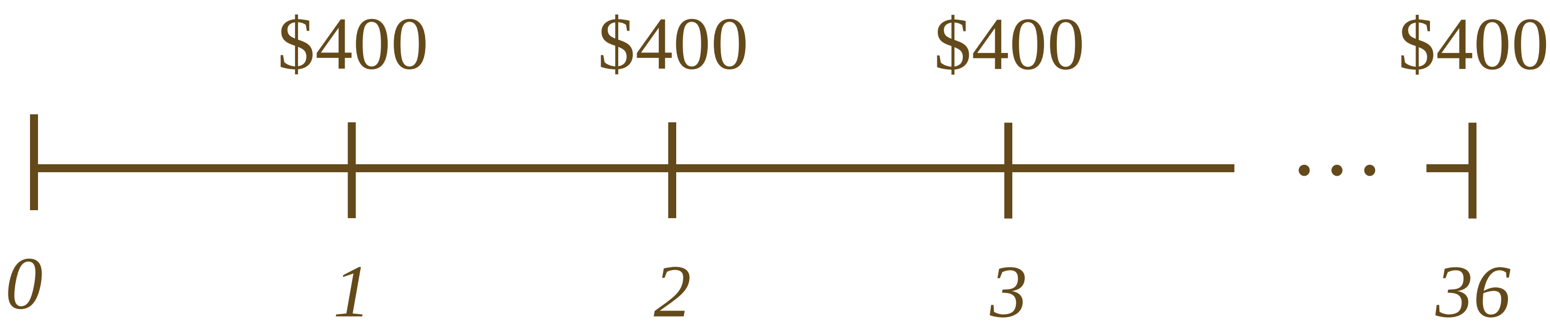


$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots \frac{C}{(1+r)^T}$$

$$PV = \frac{C}{r}\left[1 - \frac{1}{(1+r)^T}\right]$$

Annuity: Example

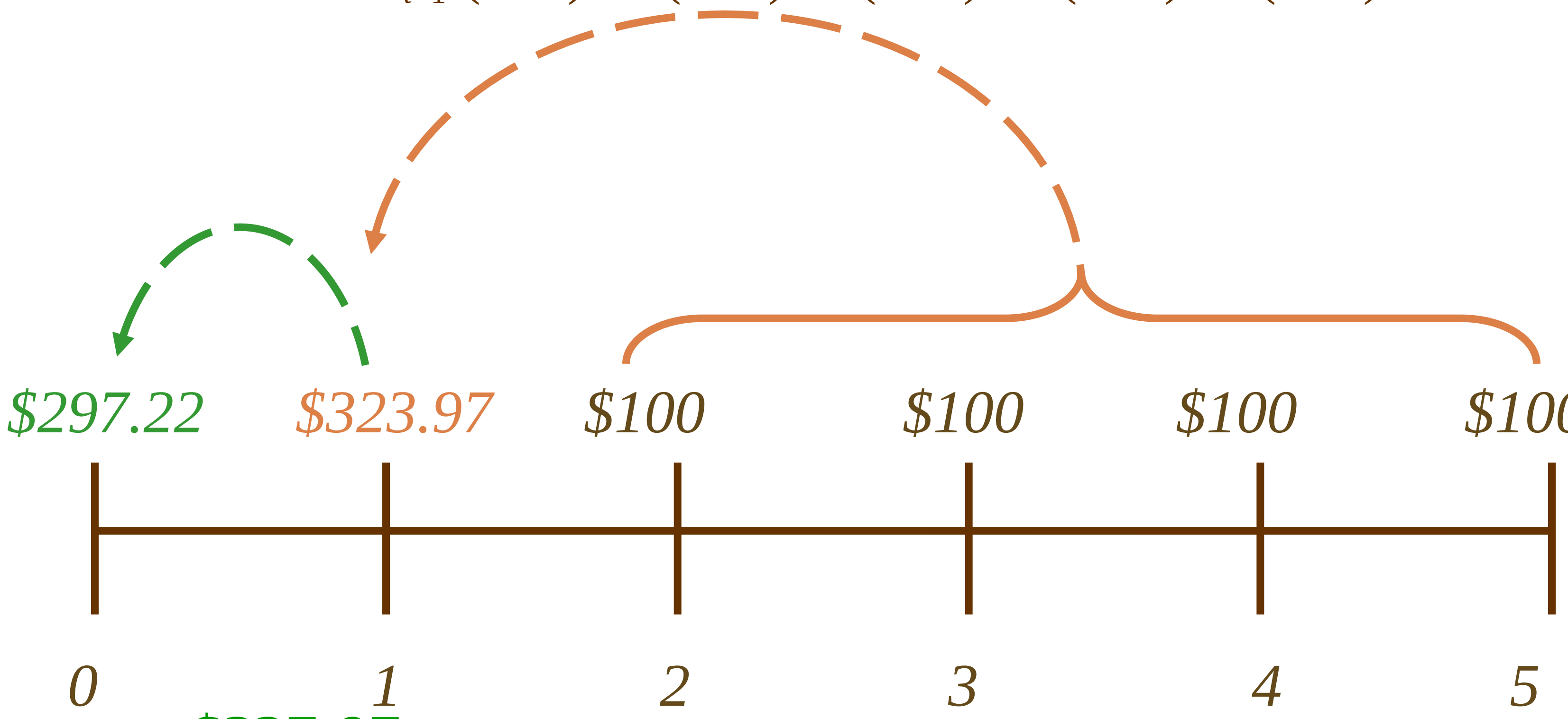
If you can afford a $400 monthly car payment, how much car can you afford if interest rates are 7% on 36-month loans?



$$PV = \frac{\$400}{.07/12}\left[1 - \frac{1}{(1+.07/12)^{36}}\right] = \$12{,}954.59$$

What is the present value of a four-year annuity of $100 per year that makes its first payment two years from today if the discount rate is 9%?

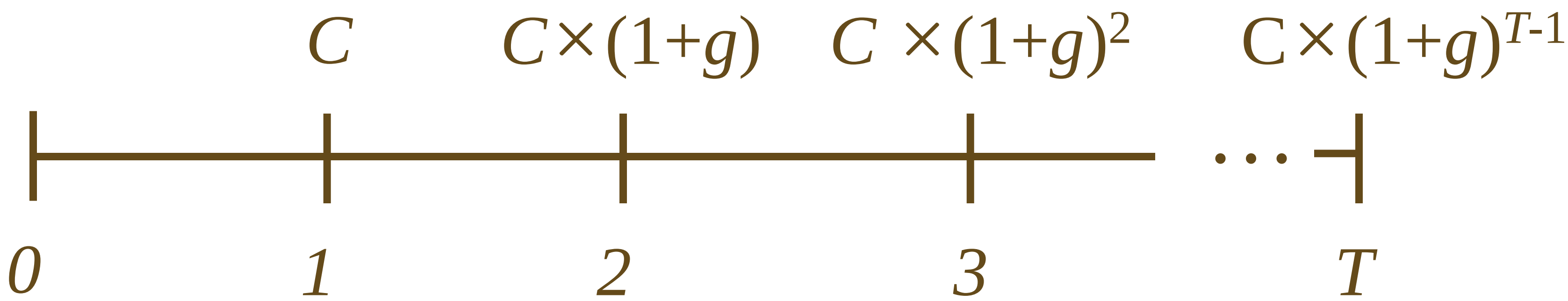
$$PV_1 = \sum_{t=1}^{4} \frac{\$100}{(1.09)^t} = \frac{\$100}{(1.09)^1} + \frac{\$100}{(1.09)^2} + \frac{\$100}{(1.09)^3} + \frac{\$100}{(1.09)^4} = \$323.97$$



$$PV_0 = \frac{\$327.97}{1.09} = \$297.22$$

Growing Annuity 성장연금

A growing stream of cash flows with a fixed maturity



$$PV = \frac{C}{(1+r)} + \frac{C\times(1+g)}{(1+r)^2} + \cdots + \frac{C\times(1+g)^{T-1}}{(1+r)^T}$$

$$PV = \frac{C}{r-g}\left[1-\left(\frac{1+g}{(1+r)}\right)^T\right]$$

Growing Annuity: Example

A defined-benefit retirement plan offers to pay $20,000 per year for 40 years and increase the annual payment by 3% each year. What is the present value at retirement if the discount rate is 10%?

$$\$20{,}000 \qquad \$20{,}000\times(1.03) \qquad \$20{,}000\times(1.03)^{39}$$

$$0 \qquad 1 \qquad 2 \qquad \cdots \qquad 40$$

$$PV = \frac{\$20{,}000}{.10-.03}\left[1-\left(\frac{1.03}{1.10}\right)^{40}\right] = \$265{,}121.57$$

Growing Annuity: Example

You are evaluating an income generating property. Net rent is received at the end of each year. The first year's rent is expected to be \$8,500, and rent is expected to increase 7% each year. What is the present value of the estimated income stream over the first 5 years if the discount rate is 12%?

$\$8{,}500 \times (1.07) =$ $\quad \$8{,}500 \times (1.07)^2 =$ $\quad \$8{,}500 \times (1.07)^3 =$ $\quad \$8{,}500 \times (1.07)^4 =$

\$8,500 \$9,095 \$9,731.65 \$10,412.87 \$11,141.77

\$34,706.26

What Is a Firm Worth?

- Conceptually, a firm should be worth the present value of the firm's cash flows.
- The tricky part is determining the size, timing, and *risk* of those cash flows.

Net Present Value: First Principles of Finance

- Understand the theoretical foundations of the Net Present Value (NPV) rule
  - Making Consumption Choices over Time
  - Making Investment Choices
  - Illustrating the Investment Decision

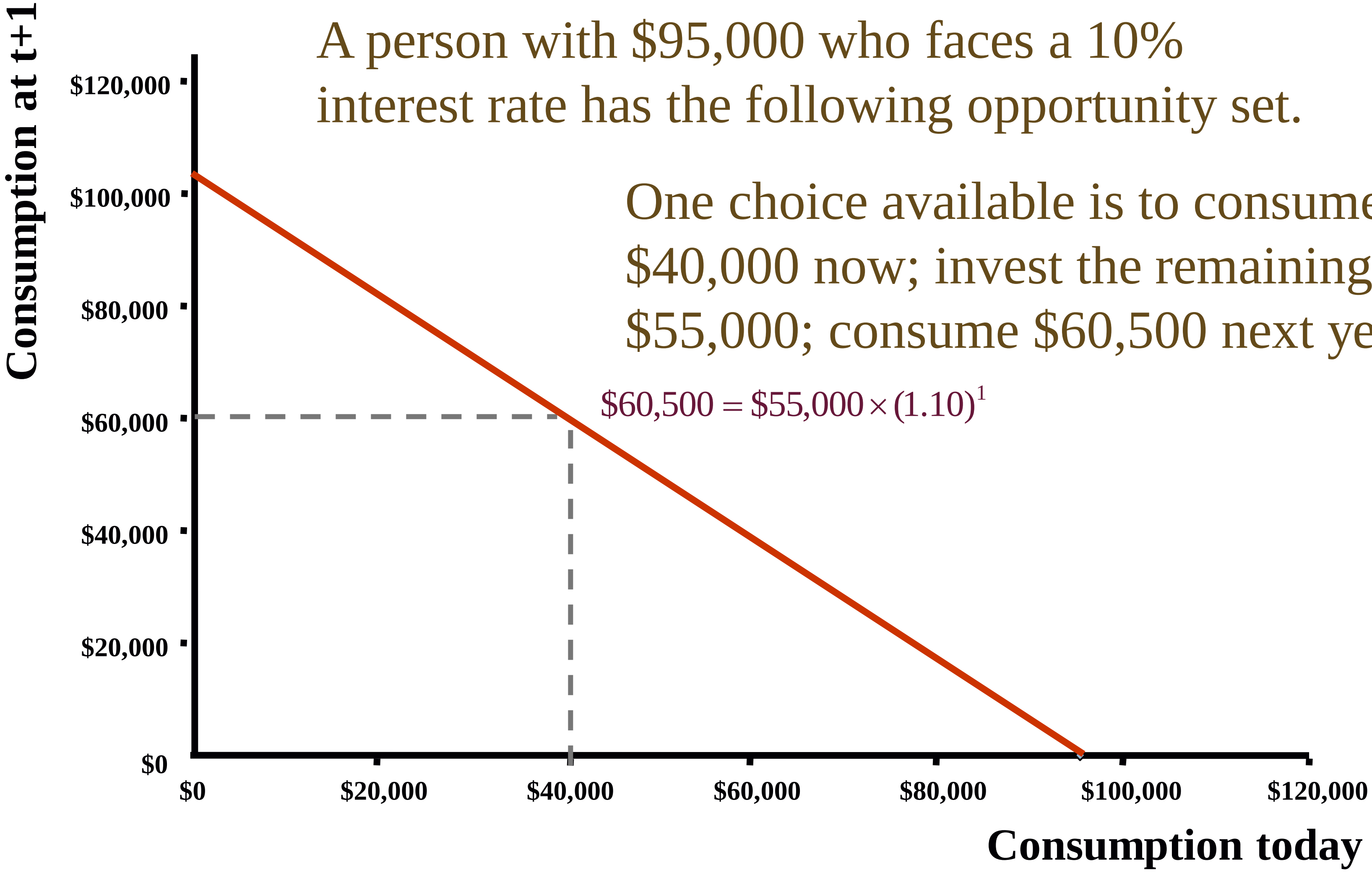
Making Consumption Choices over Time

- An individual can alter his consumption across time periods through borrowing and lending.
- We can illustrate this by graphing consumption today versus consumption in the future.
- This graph will show intertemporal consumption opportunities.

Intertemporal Consumption Opportunity Set

A person with \$95,000 who faces a 10% interest rate has the following opportunity set.

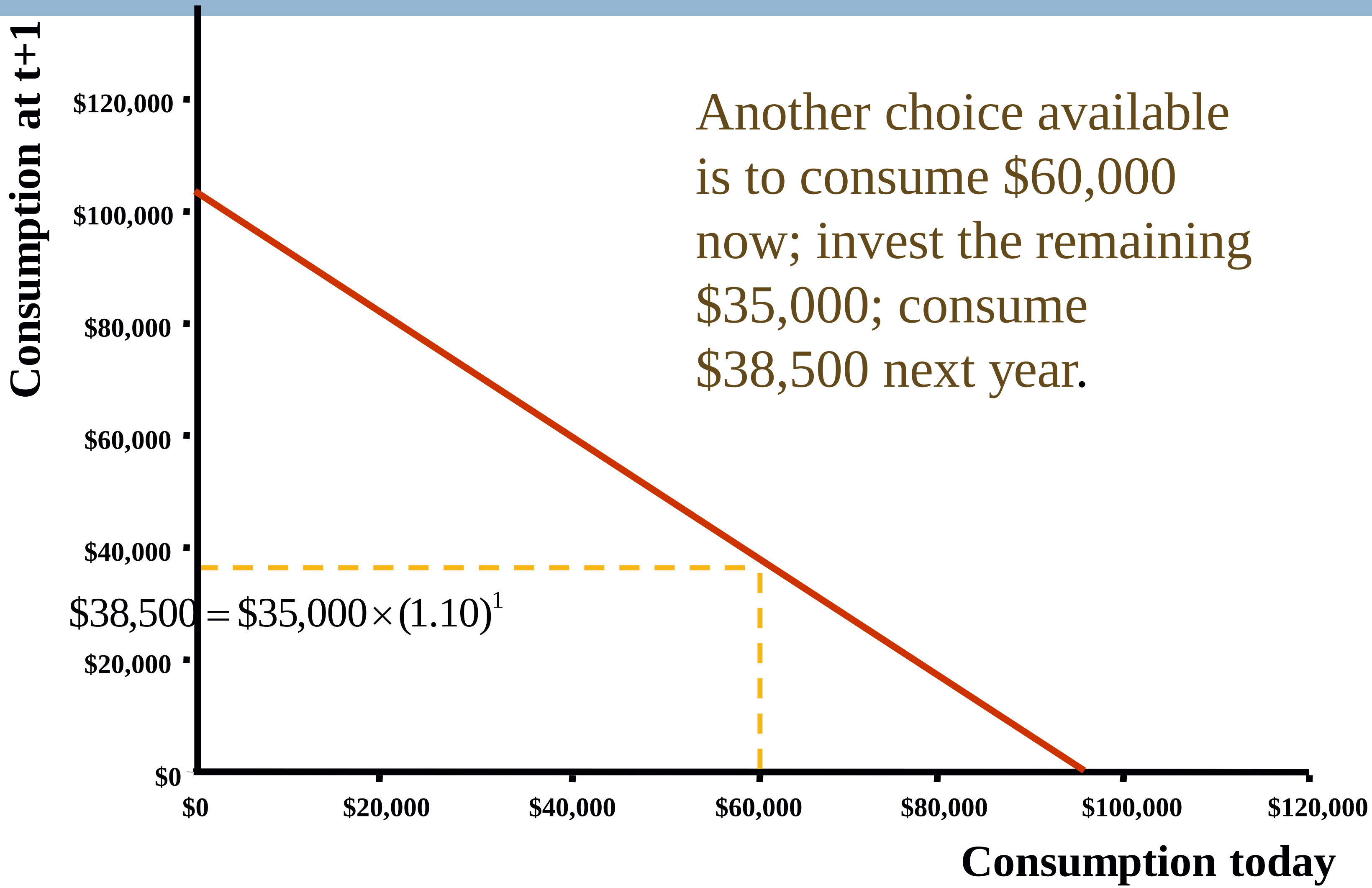
One choice available is to consume \$40,000 now; invest the remaining \$55,000; consume \$60,500 next year.



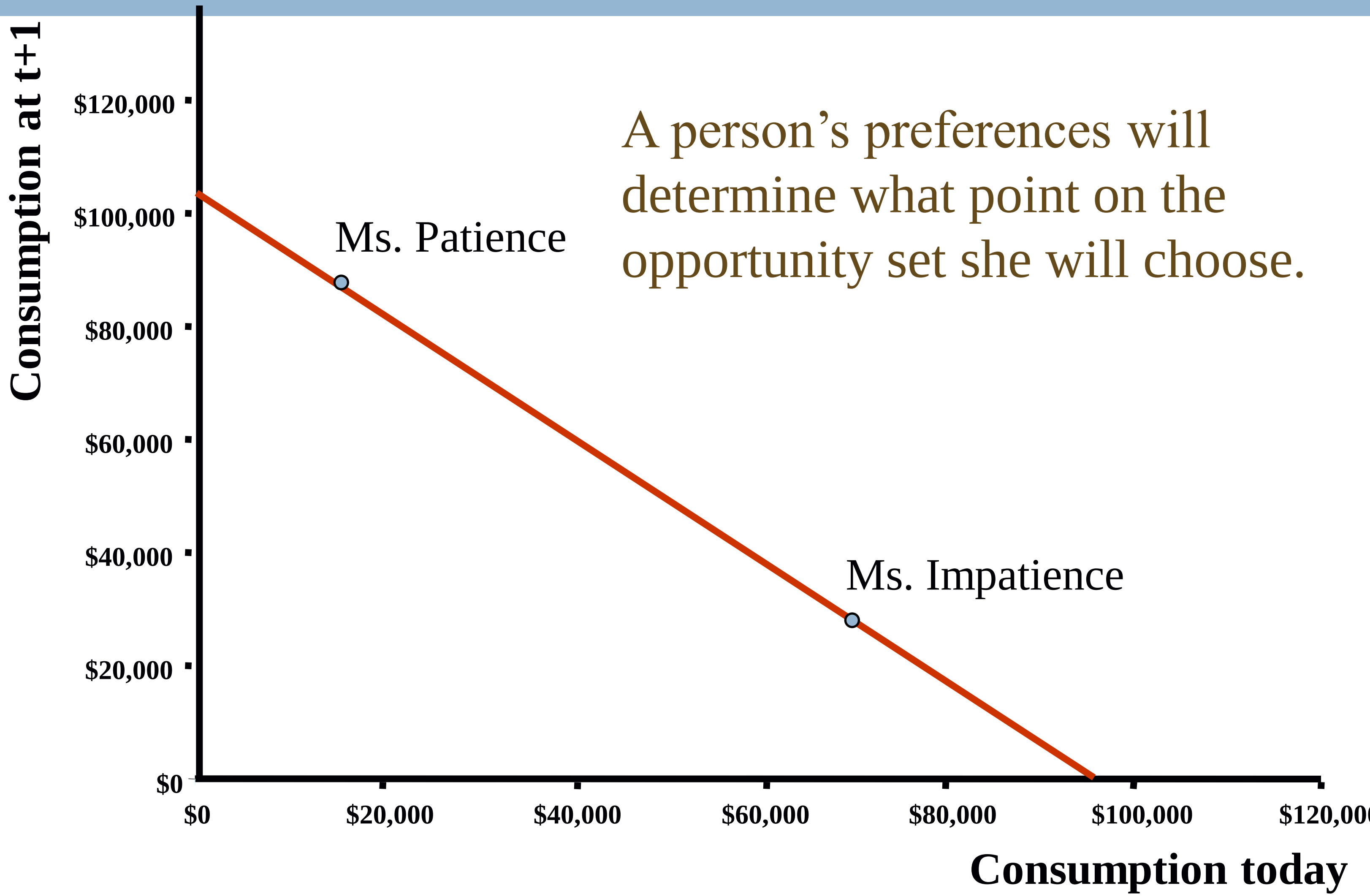
$$\$60{,}500 = \$55{,}000 \times (1.10)^1$$

Intertemporal Consumption Opportunity Set

Another choice available is to consume $60,000 now; invest the remaining $35,000; consume $38,500 next year.

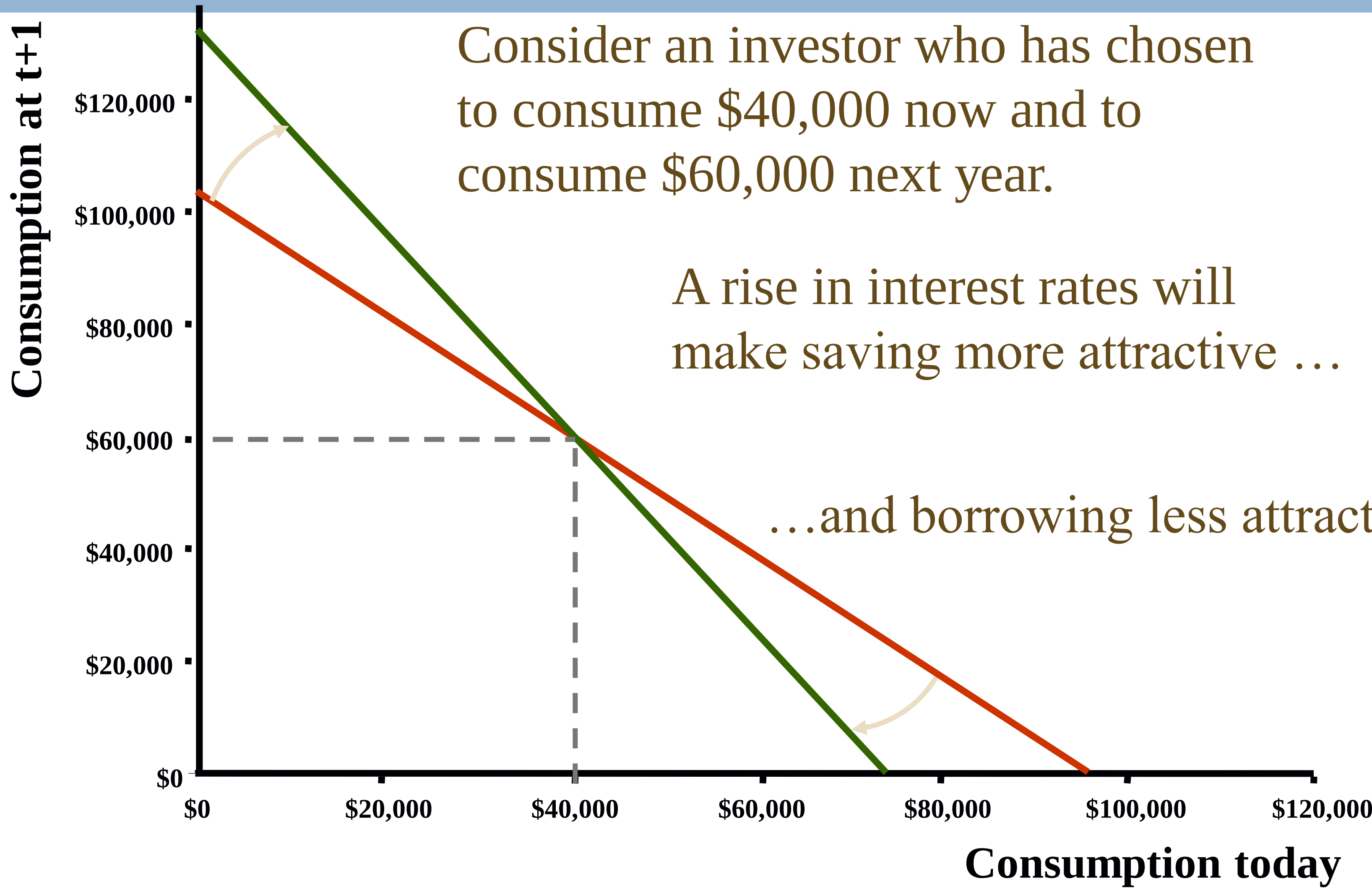


Taking Advantage of Our Opportunities



A person's preferences will determine what point on the opportunity set she will choose.

Changing Our Opportunities



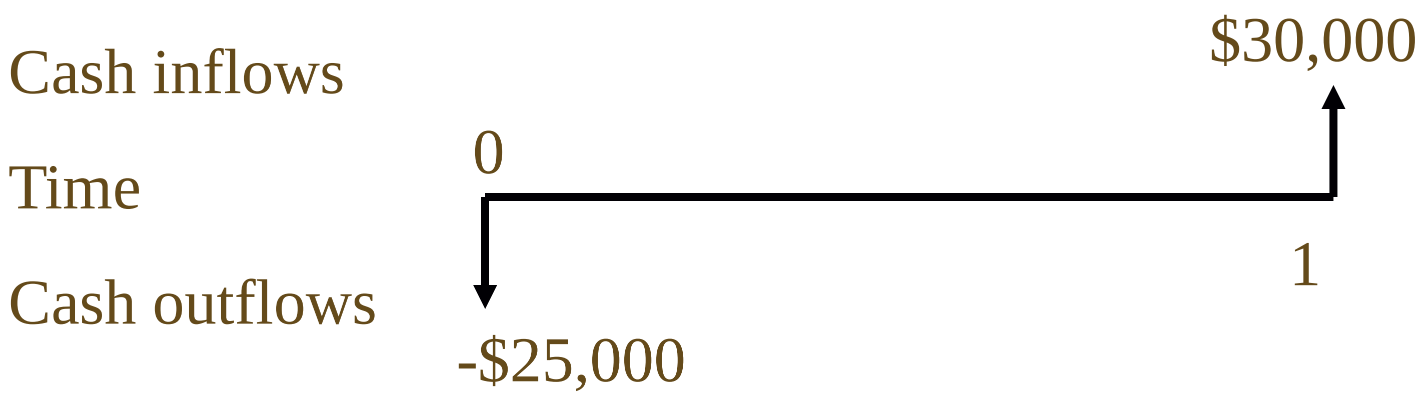
Consider an investor who has chosen to consume $40,000 now and to consume $60,000 next year.

A rise in interest rates will make saving more attractive …

…and borrowing less attractive.

Illustrating the Investment Decision

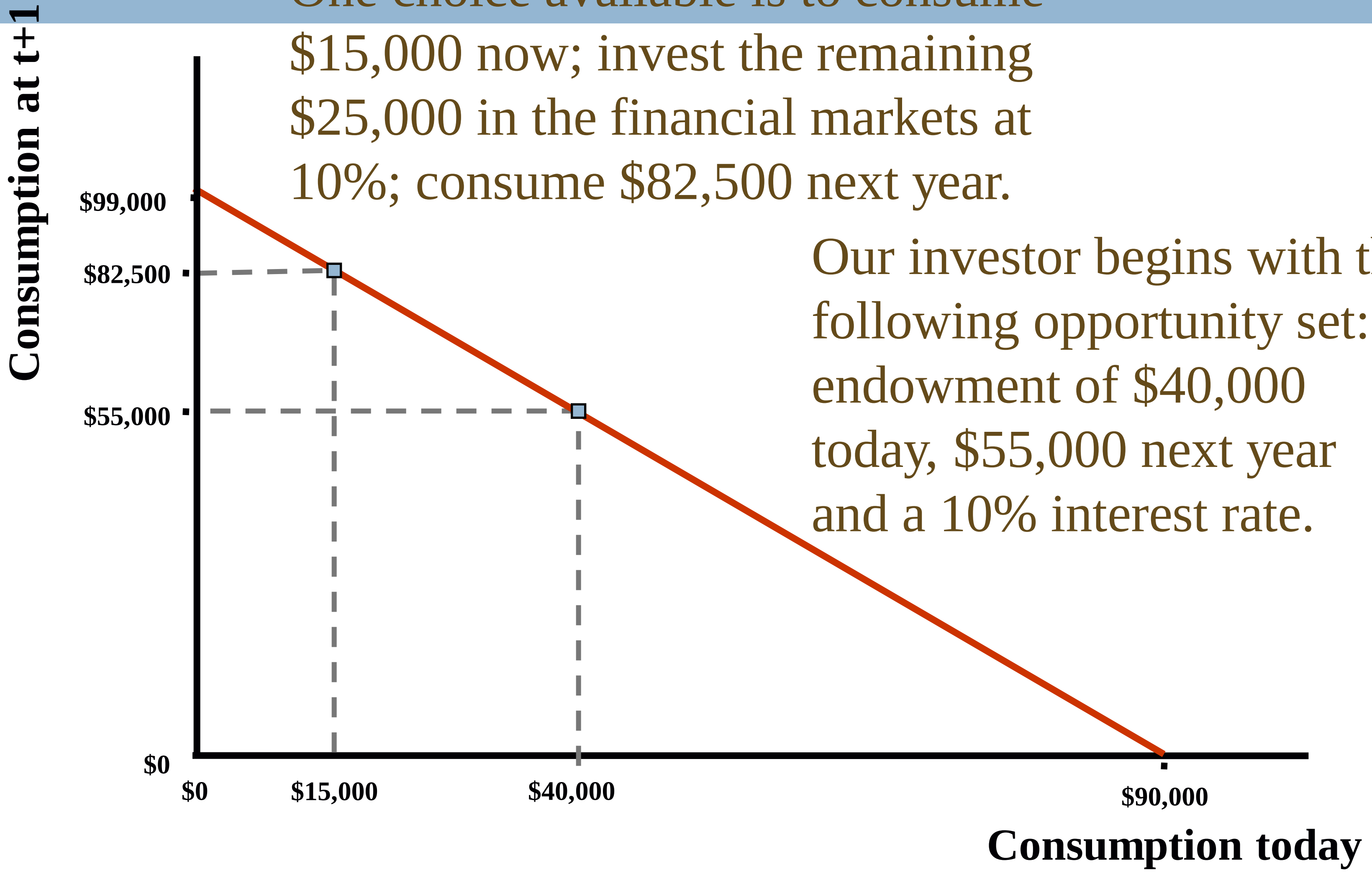
- Consider an investor who has an initial endowment of income of \$40,000 this year and \$55,000 next year.
- Suppose that she faces a 10-percent interest rate and is offered the following investment.



Illustrating the Investment Decision

One choice available is to consume $15,000 now; invest the remaining $25,000 in the financial markets at 10%; consume $82,500 next year.

Our investor begins with the following opportunity set: endowment of $40,000 today, $55,000 next year and a 10% interest rate.

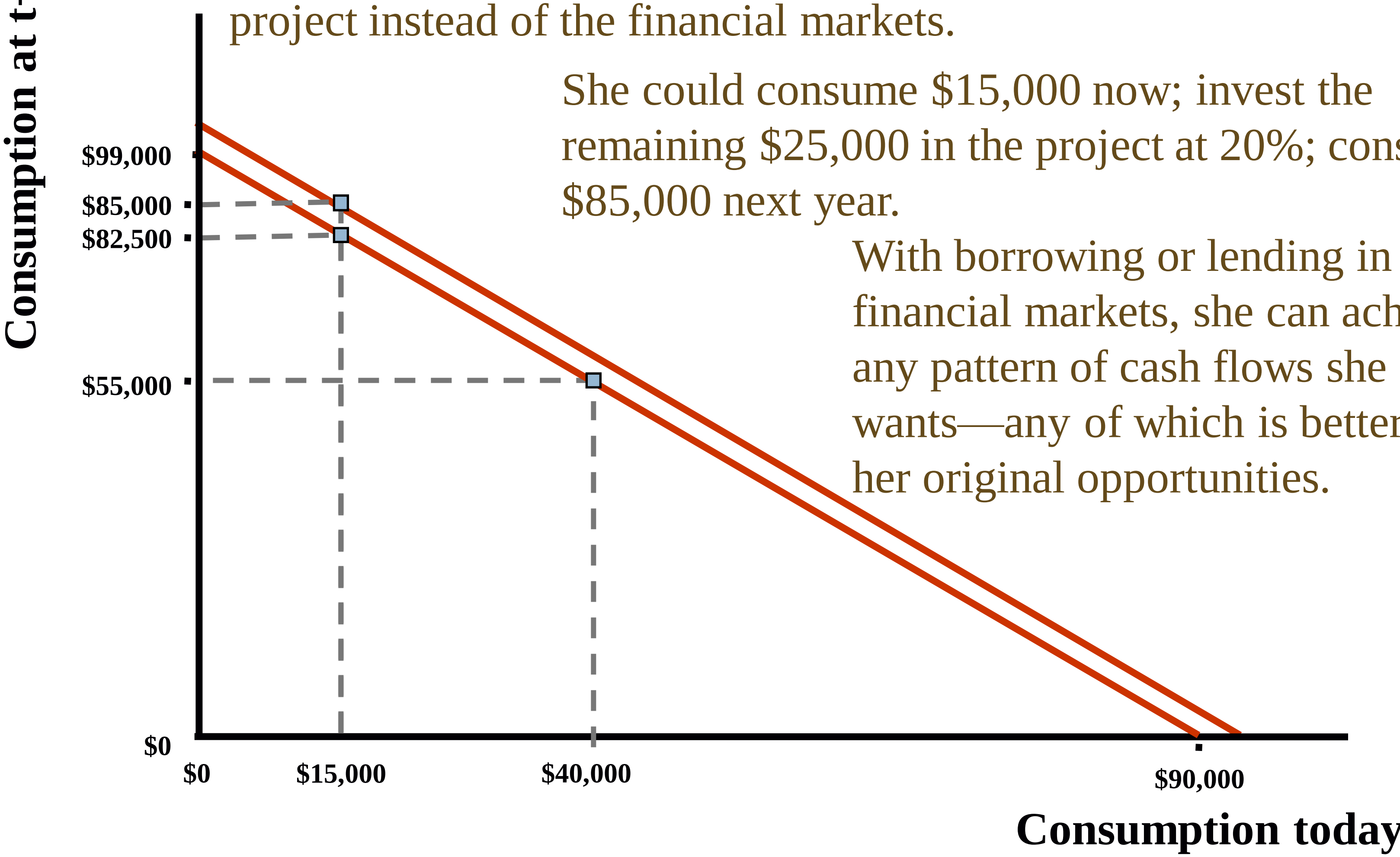


Illustrating the Investment Decision

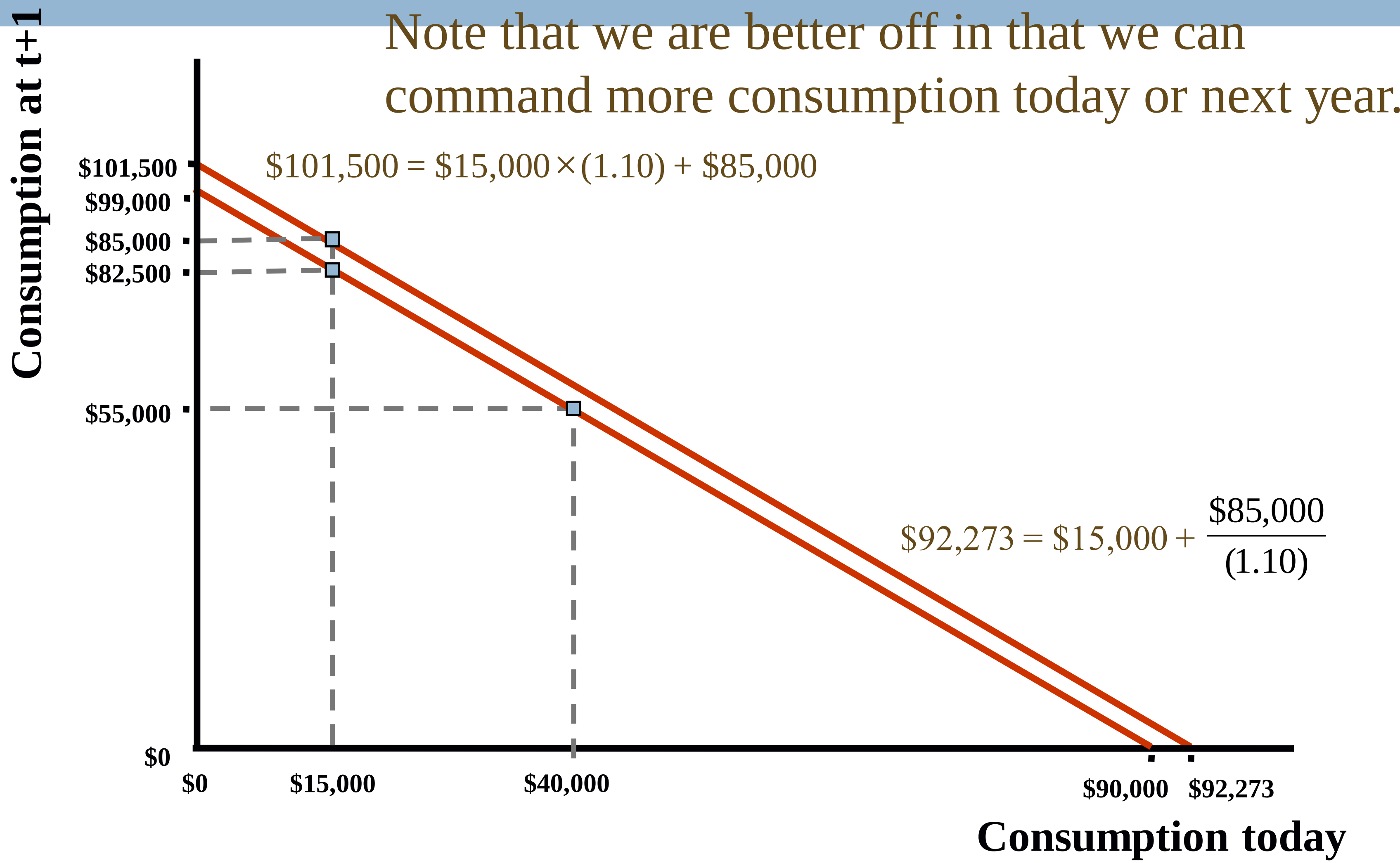
A better alternative would be to invest in the project instead of the financial markets.

She could consume \$15,000 now; invest the remaining \$25,000 in the project at 20%; consume \$85,000 next year.

With borrowing or lending in the financial markets, she can achieve any pattern of cash flows she wants—any of which is better than her original opportunities.



Illustrating the Investment Decision



Net Present Value

- The value created by the investment opportunity increased our possible consumption.
- This opportunity, therefore, created value.
- The current value of the opportunity is the investment's NPV.