Chapter 4

Discounted Cash Flow Valuation

Acknowledgement

- This work is reproduced, based on the book [Ross, Westerfield, Jaffe and Jordan “Core Principles and Applications of Corporate Finance”].
- This work can be used in the financial management course with the original text book.
- This work uses the figures and tables from the original text book.

Future Value

- In the one-period case, the formula for *FV* can be written as:

$$FV = C_0 \times (1 + r)$$

Where C_0 is cash flow today (time zero), and
r is the appropriate interest rate.

Present Value

- In the one-period case, the formula for *PV* can be written as:

$$PV = \frac{C_1}{1+r}$$

Where C_1 is cash flow at date 1, and

r is the appropriate interest rate.

Net Present Value

In the one-period case, the formula for *NPV* can be written as:

$$NPV = -Cost + PV$$

4.2 The Multiperiod Case

- The general formula for the future value of an investment over many periods can be written as:

$$FV = C_0 \times (1 + r)^T$$

Where

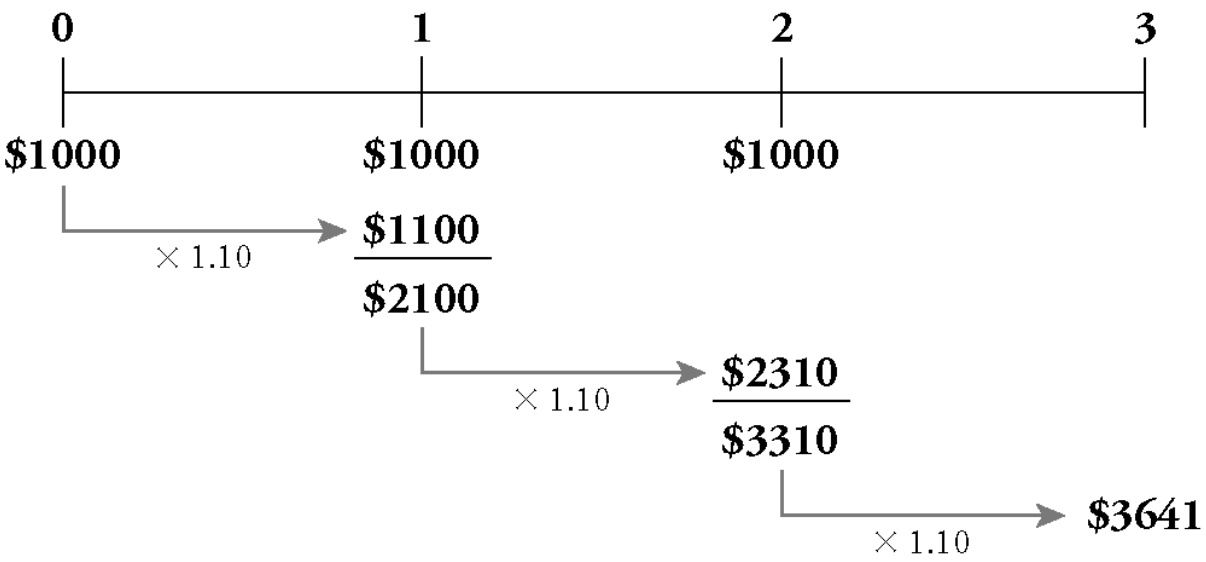
C_0 is cash flow at date 0,

r is the appropriate interest rate, and

T is the number of periods over which the cash is invested.

Valuing a Stream of Cash Flows

- Continuing in the same fashion, we can solve the problem as follows: 10% interest rate



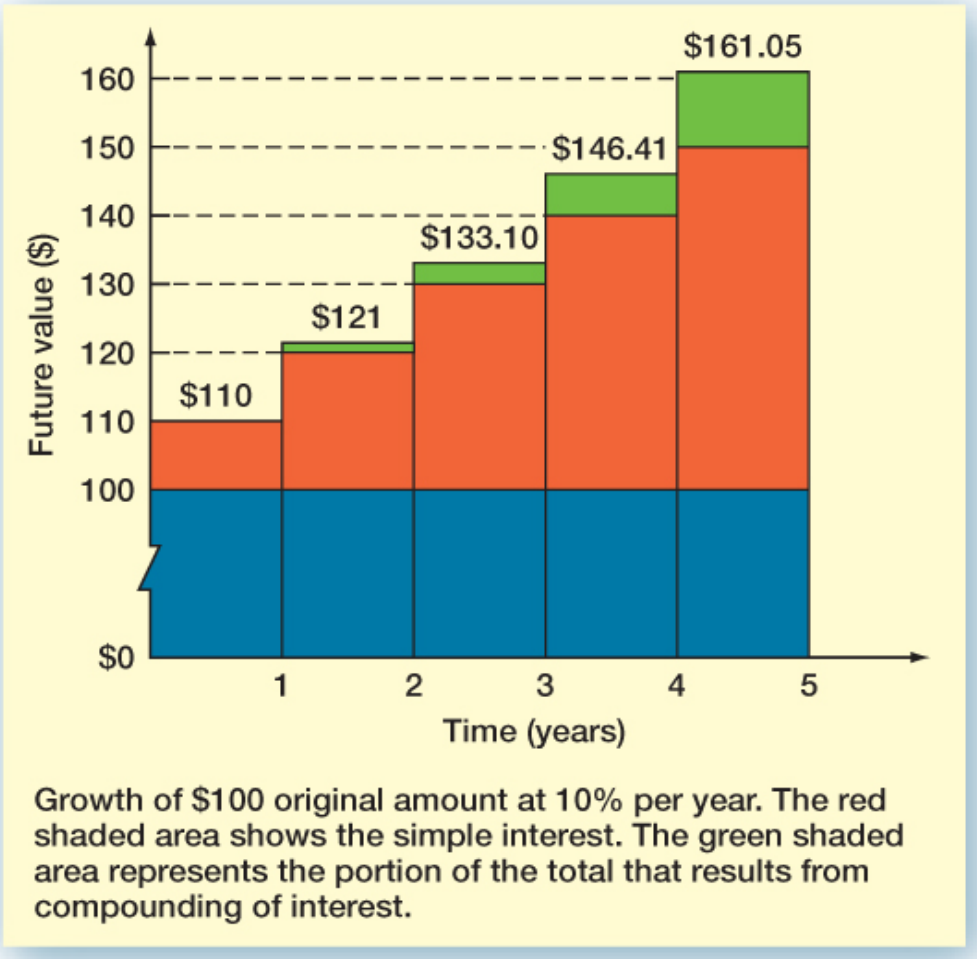
Simple and Compound Interest

- **Future Value of $100 at 10 percent**

Year	Beginning Amount	Simple Interest	Compound Interest	Total Interest Earned	Ending Amount
1	$100.00	$10	$ .00	$10.00	$110.00
2	110.00	10	1.00	11.00	121.00
3	121.00	10	2.10	12.10	133.10
4	133.10	10	3.31	13.31	146.41
5	146.41	10	4.64	14.64	161.05
		Total $50 simple interest	Total $11.05 compound interest	Total $61.05 interest	

Cited by the text book (p. 123)

Simple and Compound Interest



Growth of $100 original amount at 10% per year. The red shaded area shows the simple interest. The green shaded area represents the portion of the total that results from compounding of interest.

Cited by the text book (p. 123)

Futures Values

- Future Value Interest Factors

	Interest Rate			
Number of Periods	5%	10%	15%	20%
1	.9524	.9091	.8696	.8333
2	.9070	.8264	.7561	.6944
3	.8638	.7513	.6575	.5787
4	.8227	.6830	.5718	.4823
5	.7835	.6209	.4972	.4019

Cited by the text book (p. 124)

Present Value and Discounting

- Discounting

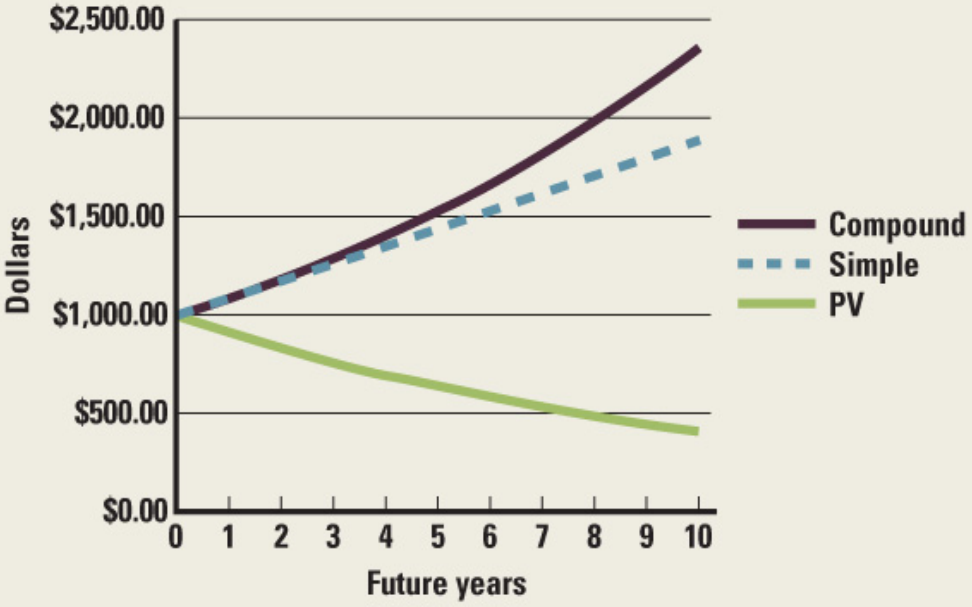


FIGURE 4.8
Compounding and Discounting

The top line shows the growth of $1,000 at compound interest with the funds invested at 9%: $\$1{,}000 \times (1.09)^{10} = \$2{,}367.36$. Simple interest is shown on the next line. It is $\$1{,}000 + [10 \times (\$1{,}000 \times .09)] = \$1{,}900$. The bottom line shows the discounted value of $1,000 if the interest rate is 9%.

Cited by the text book (p. 127)

Present Value – Important Relationship

- For a given interest rate – the longer the time period, the lower the present value
- For a given time period – the higher the interest rate, the smaller the present value

	Interest Rate			
Number of Periods	5%	10%	15%	20%
1	.9524	.9091	.8696	.8333
2	.9070	.8264	.7561	.6944
3	.8638	.7513	.6575	.5787
4	.8227	.6830	.5718	.4823
5	.7835	.6209	.4972	.4019

Cited by the text book (p. 128)

4.3 Compounding Periods

- Your investment compounding periods may not be annual, but any of a variety of time periods.
- You assumed annual interest rates; however, many projects / investments have different periods.
- For example, bonds typically pay interest semi-annually, and house loans are on a monthly payment schedule.
- Compounding an investment *m* times a year for *T* years provides for the future value of wealth:

$$FV = C_0 \times \left(1 + \frac{r}{m}\right)^{mT}$$

Effective Annual Rates of Interest

- Stated or quoted interest rate – rate before considering any compounding effects, such as 10% compounded quarterly
- Effective annual rate of interest – rate on an annual basis that reflects compounding effects (e.g., 10% compounded quarterly has an effective rate of 10.38%).

$$EAR = [1 + (\text{quoted rate})/m]^m - 1,$$

where m is the number of periods per year

Effective Annual Rates of Interest

- A reasonable question to ask in the above example is “what is the effective *annual* rate of interest on that investment?”

$$FV = \$50 \times (1 + \frac{.12}{2})^{2\times 3} = \$50 \times (1.06)^{6} = \$70.93$$

- The Effective Annual Rate (EAR) of interest is the annual rate that would give us the same end-of-investment wealth after 3 years:

$$\$50 \times (1 + EAR)^{3} = \$70.93$$

Effective Annual Rates of Interest

$$FV = \$50 \times (1 + EAR)^3 = \$70.93$$

$$(1 + EAR)^3 = \frac{\$70.93}{\$50}$$

$$EAR = \left(\frac{\$70.93}{\$50} \right)^{1/3} - 1 = .1236$$

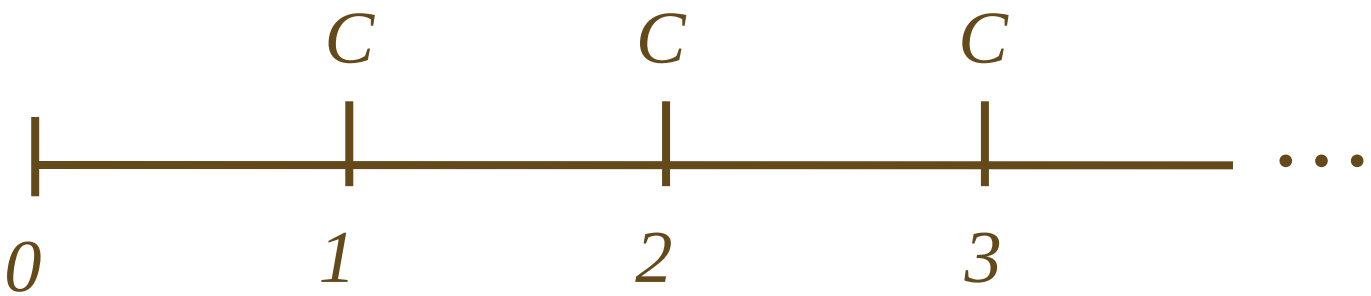
So, investing at 12.36% compounded annually is the same as investing at 12% compounded semi-annually.

4.4 Simplifications

- Perpetuity
  - A constant stream of cash flows that lasts forever
- Growing perpetuity
  - A stream of cash flows that grows at a constant rate forever
- Annuity
  - A stream of constant cash flows that lasts for a fixed number of periods
- Growing annuity
  - A stream of cash flows that grows at a constant rate for a fixed number of periods

Perpetuity

A constant stream of cash flows that lasts forever



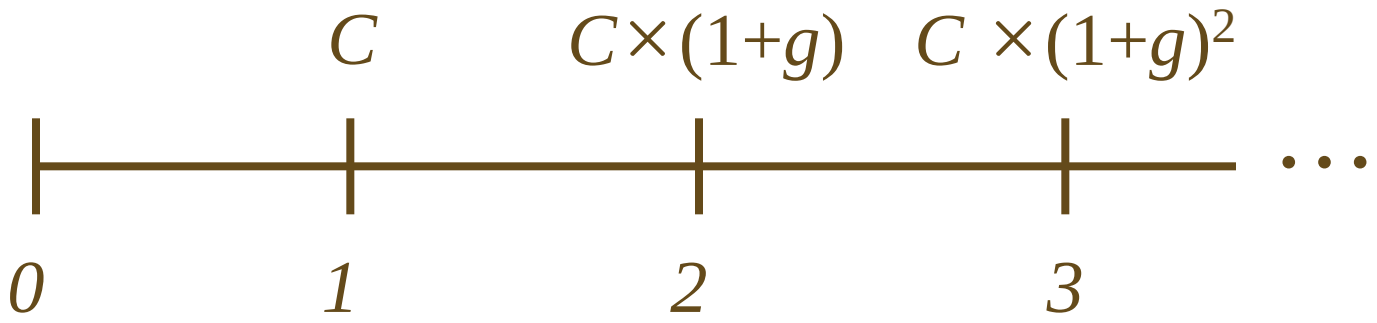
$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots$$

$$PV = \frac{C}{r}$$

Cited by the text book (p. 137)

Growing Perpetuity

A growing stream of cash flows that lasts forever



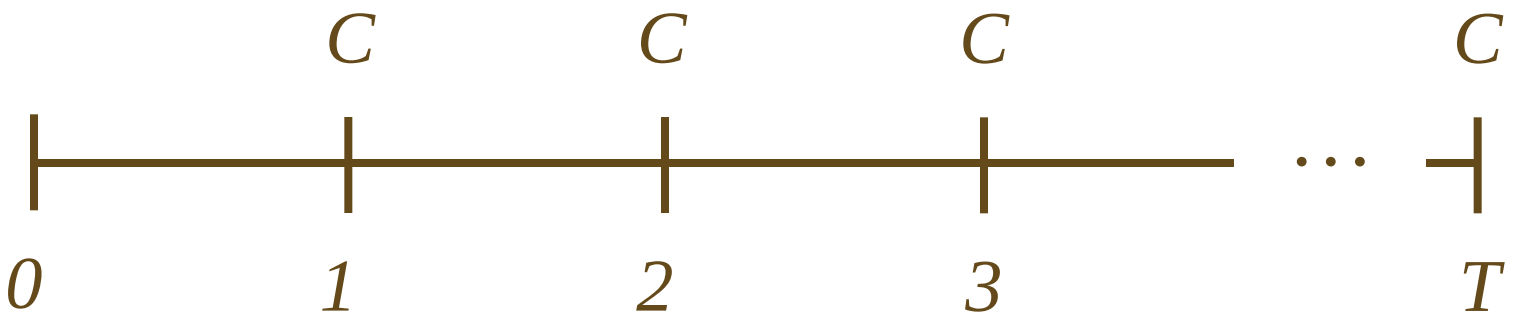
$$PV = \frac{C}{(1+r)} + \frac{C\times(1+g)}{(1+r)^2} + \frac{C\times(1+g)^2}{(1+r)^3} + \cdots$$

$$PV = \frac{C}{r-g}$$

Cited by the text book (p. 138)

Annuity

A constant stream of cash flows with a fixed maturity



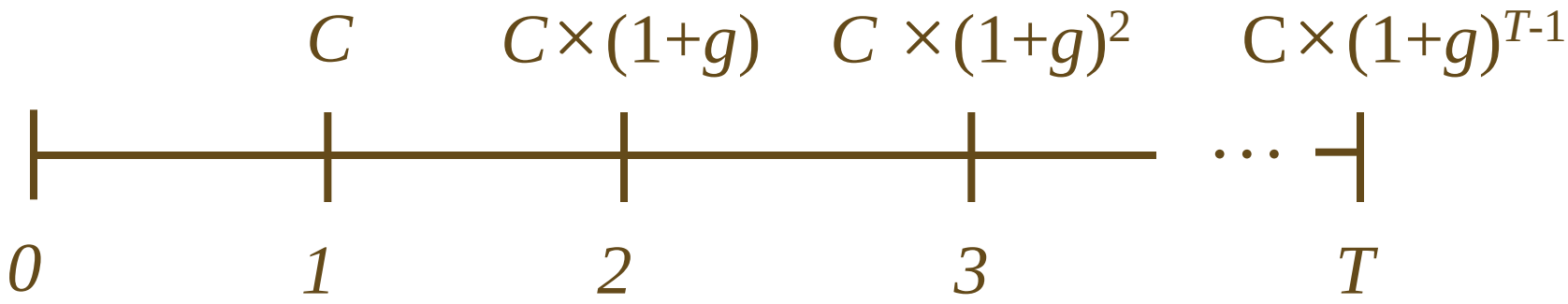
$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots \frac{C}{(1+r)^T}$$

$$PV = \frac{C}{r}\left[1 - \frac{1}{(1+r)^T}\right]$$

Cited by the text book (p. 140)

Growing Annuity

A growing stream of cash flows with a fixed maturity



$$PV = \frac{C}{(1+r)} + \frac{C\times(1+g)}{(1+r)^2} + \cdots + \frac{C\times(1+g)^{T-1}}{(1+r)^T}$$

$$PV = \frac{C}{r-g}\left[1-\left(\frac{(1+g)}{(1+r)}\right)^T\right]$$

Cited by the text book (p. 141)

4.5 Loan Types and Loan Amortization

- Pure Discount Loans are the simplest form of loan. The borrower receives money today and repays a single lump sum (principal and interest) at a future time.
- Interest-Only Loans require an interest payment each period, with full principal due at maturity.
- Amortized Loans require repayment of principal over time, in addition to required interest.

References

- Ross, Westerfield, Jaffe and Jordan, Core Principles and Application of Corporate Finance, 3ed, McGraw Hill.
- Jordan, Miller, and Dolvin, Fundamentals of Investments, 6ed, MacGraw Hill.
- Berk, DeMarzo and Harford, Fundamentals of Corporate Fiance, 2nd ed, Pearson.