
Chapter 7

Capacitors and Inductors

7.1 The Capacitors

7.2 The Inductors

7.3 Inductance and Capacitance Combination

7.4 Consequences of Linearity

7.5 Simple Op Amp Circuits with Capacitors

7.6 Duality

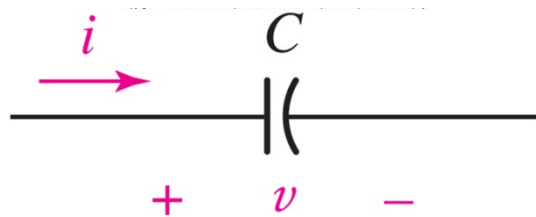
- Ideal Capacitor Model

active element: an element that is **capable of furnishing an average power** greater than zero to some external device

ideal source, op amp

passive: an element that **cannot supply an average power** that is greater than zero over an infinite time interval

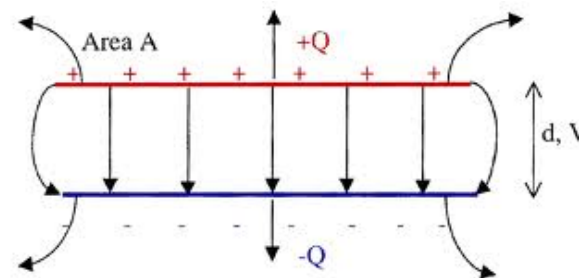
resistor, capacitor, inductor



$$i = C \frac{dv}{dt}$$

$$i = C \frac{dv}{dt} = \frac{dq}{dt} \rightarrow dq = Cdv$$

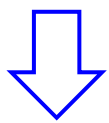
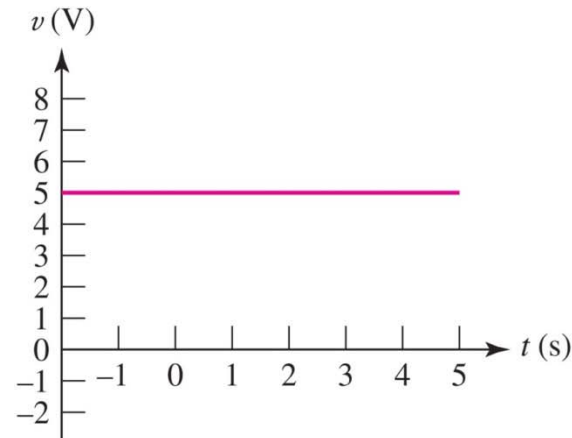
$$\Rightarrow C = \frac{dq}{dv} \text{ [unit: C/V or farad(F)]}$$



$$C = \frac{\epsilon A}{d}$$

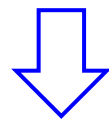
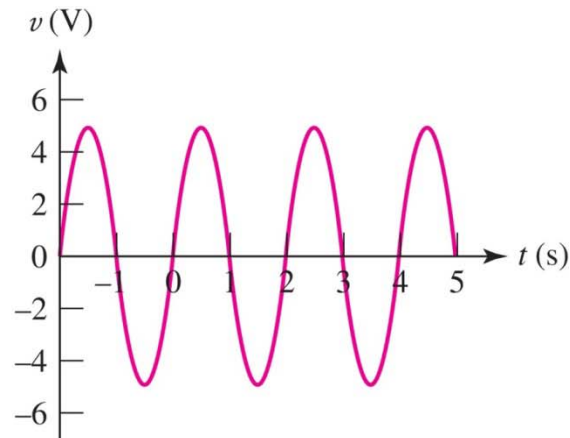
ϵ : permittivity

Example 7.1 Determine the current i flowing through the capacitor ($C=2\text{ F}$)



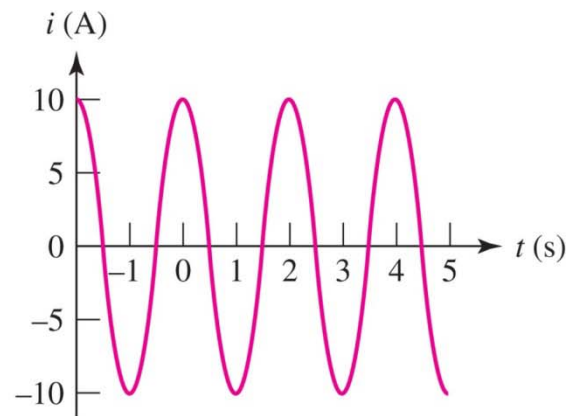
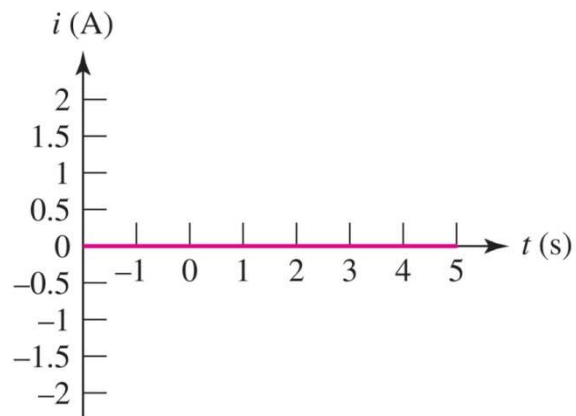
$$i = C \frac{dv}{dt} \rightarrow \frac{dv}{dt} = 0$$

$$\Rightarrow i = 0$$



$$i = C \frac{dv}{dt} = 2 \frac{dv}{dt}$$

**capacitor:
open circuit to DC**

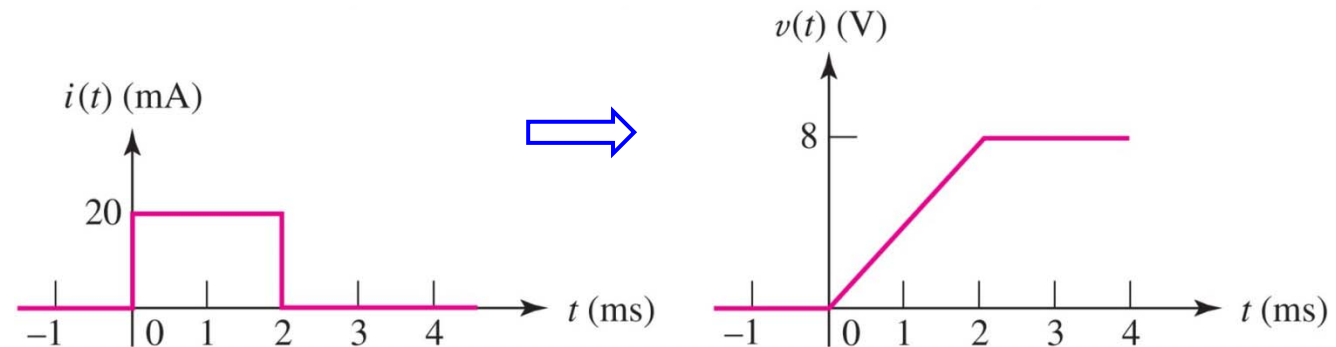


- Integral Voltage-Current Relationships

$$i = C \frac{dv}{dt} \Rightarrow dv = \frac{1}{C} i(t) dt \quad \Rightarrow v(t) = \frac{1}{C} \int_{t_0}^t i(t') dt' + v(t_0) \quad \text{or} \quad v(t) = \frac{1}{C} \int idt + k$$

$$t_0 \rightarrow -\infty, v(-\infty) = 0 \quad \Rightarrow v(t) = \frac{1}{C} \int_{-\infty}^t idt' \quad \begin{array}{l} i = C \frac{dv}{dt} = \frac{dq}{dt} \rightarrow dq = Cdv \\ \Rightarrow q(t) = Cv(t) \end{array}$$

Example 7.2 Determine the voltage across the capacitor ($C=2 \mu F$)



$$v(t) = \frac{1}{C} \int_{-\infty}^t idt' = \frac{1}{C} \left[\int_{-\infty}^0 idt' + \int_0^t idt' \right]$$

$$= \frac{1}{5 \times 10^{-6}} \int_0^t 20 \times 10^{-3} dt' + v(0)$$

$$v(t) = 4000t, 0 \leq t \leq 2 \text{ ms},$$

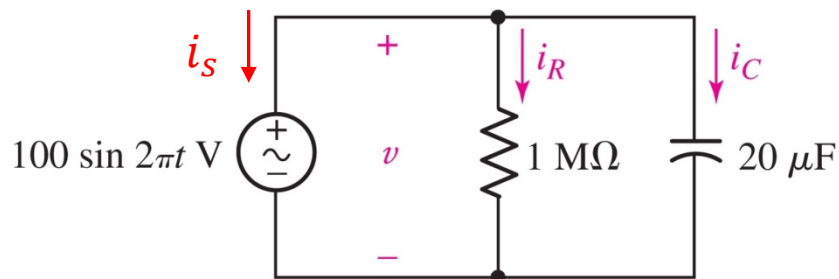
$$v(t) = 8, t > 2 \text{ ms}$$

- Energy Storage

$$\text{Power : } p = vi = vC \frac{dv}{dt} \quad E = \int_{t_0}^t p dt' = C \int_{t_0}^t v \frac{dv}{dt'} dt' = C \int_{v(t_0)}^{v(t)} v' dv' = \frac{1}{2} C [v^2(t) - v^2(t_0)]$$

$$w_C(t) - w_C(t_0) = \frac{1}{2} C [v^2(t) - v^2(t_0)] \quad w_C(t) = \frac{1}{2} C v^2(t)$$

Example 7.3 Find maximum energy stored in capacitor.



$$p_R = \frac{v^2}{R} = \frac{100^2 (\sin 2\pi t)^2}{10^6} = 10^{-2} (\sin 2\pi t)^2$$

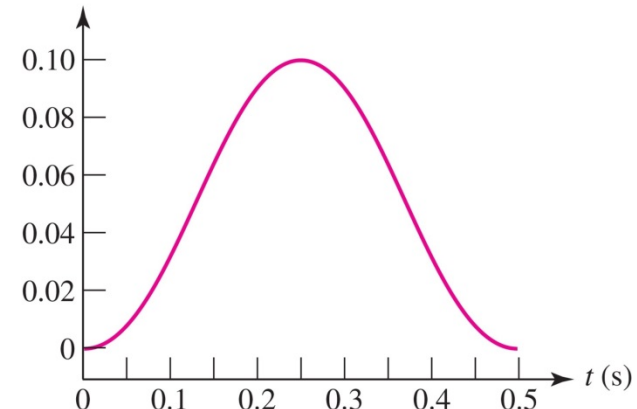
$$\Rightarrow w_R = \int_0^{0.5} p_R dt = \int_0^{0.5} 10^{-2} \sin^2 2\pi t dt = 2.5 \text{ mJ}$$

$$w_C(t) = 0.1 \sin^2 2\pi t \text{ (J)}$$

$$w_C(t) = \frac{1}{2} C v^2 = \frac{1}{2} 20 \times 10^{-6} \times 100^2 (\sin 2\pi t)^2$$

$$= 0.1 \sin^2 2\pi t \text{ J}$$

$$\rightarrow \text{maximu at } t = \frac{1}{4} \Rightarrow w_{C_max} = 100 \text{ mJ}$$



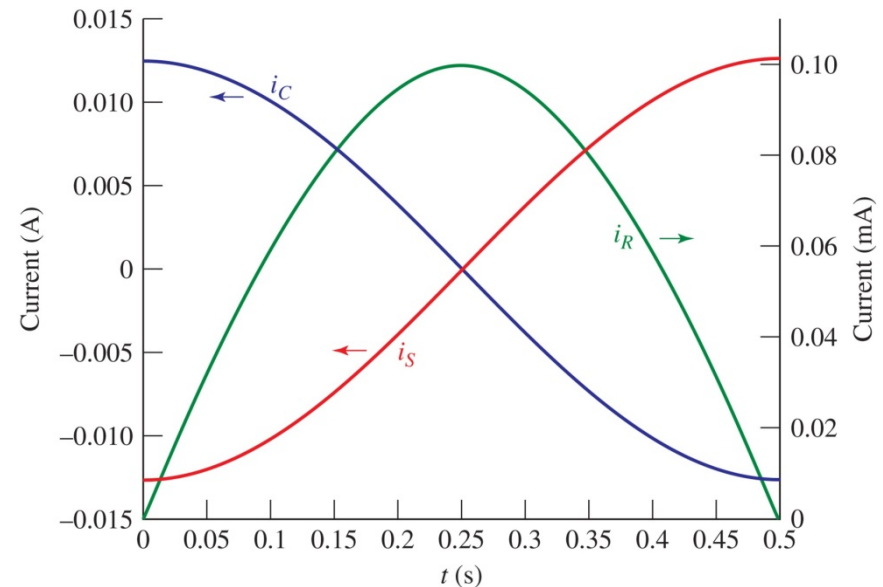
$$i_R = \frac{v}{R} = 1 \times 10^{-6} \times 100 \sin 2\pi t$$

$$= 0.0001 \sin 2\pi t$$

$$i_C = C \frac{dv}{dt} = 20 \times 10^{-6} \times 200\pi \cos 2\pi t$$

$$= 0.004\pi \cos 2\pi t = 0.01257 \cos 2\pi t$$

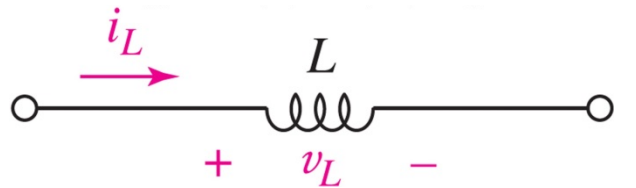
$$i_S = -(i_R + i_C)$$



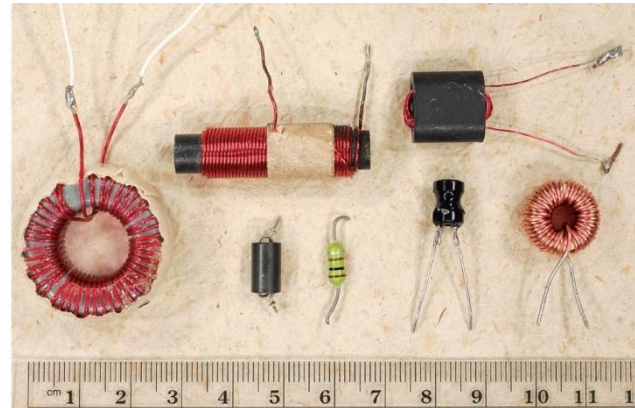
Important Characteristics of an Ideal Capacitor

1. There is no current through a capacitor if the voltage across it is not changing with time. A capacitor is therefore an ***open circuit to dc***.
2. A finite amount of energy can be stored in a capacitor even if the current through the capacitor is zero, such as when the voltage across it is constant.
3. It is impossible to change the voltage across a capacitor by a finite amount in zero time, for this requires an infinite current through the capacitor. (A capacitor resists an abrupt change in the voltage across it in a manner analogous to the way a spring resists an abrupt change in its displacement.)
4. A capacitor never dissipates energy, but only stores it. Although this is true for the ***mathematical model***, it is not true for a physical capacitor due to finite resistances associated with the dielectric as well as packaging.

- Ideal Inductor Model



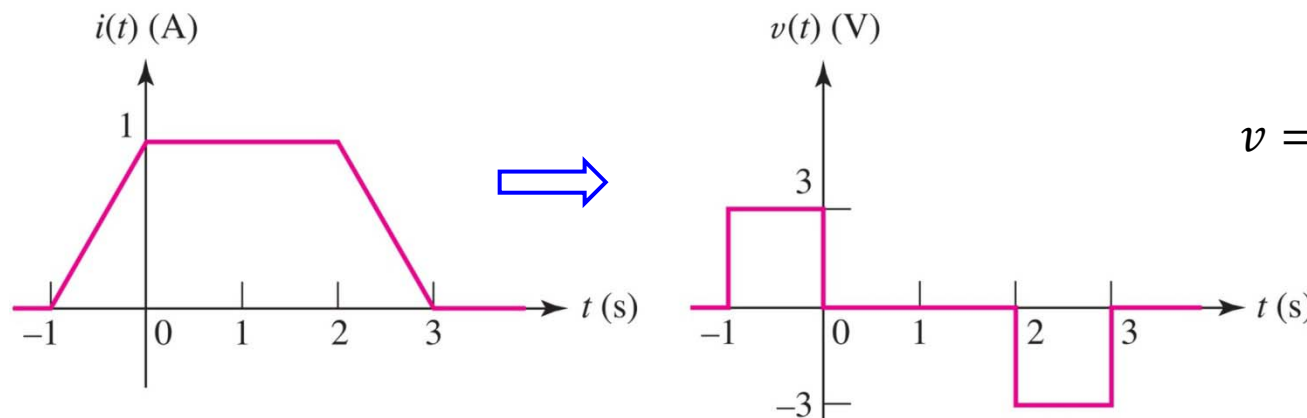
$$v = L \frac{di}{dt} \quad [\text{unit}] \text{ henry (H)}$$



Voltage is proportional to the time rate of change of the current producing the magnetic field

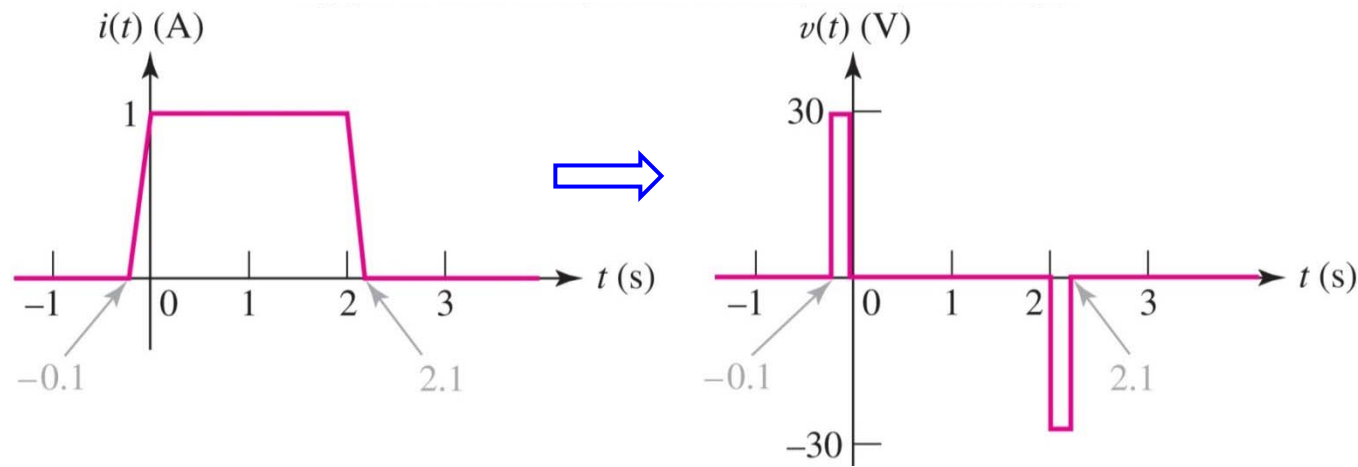
$$v = 0 \rightarrow \text{constant current} \Rightarrow \textit{short circuit to dc}$$

Example 7.4 Determine the inductor voltage. ($L = 3 \text{ H}$)

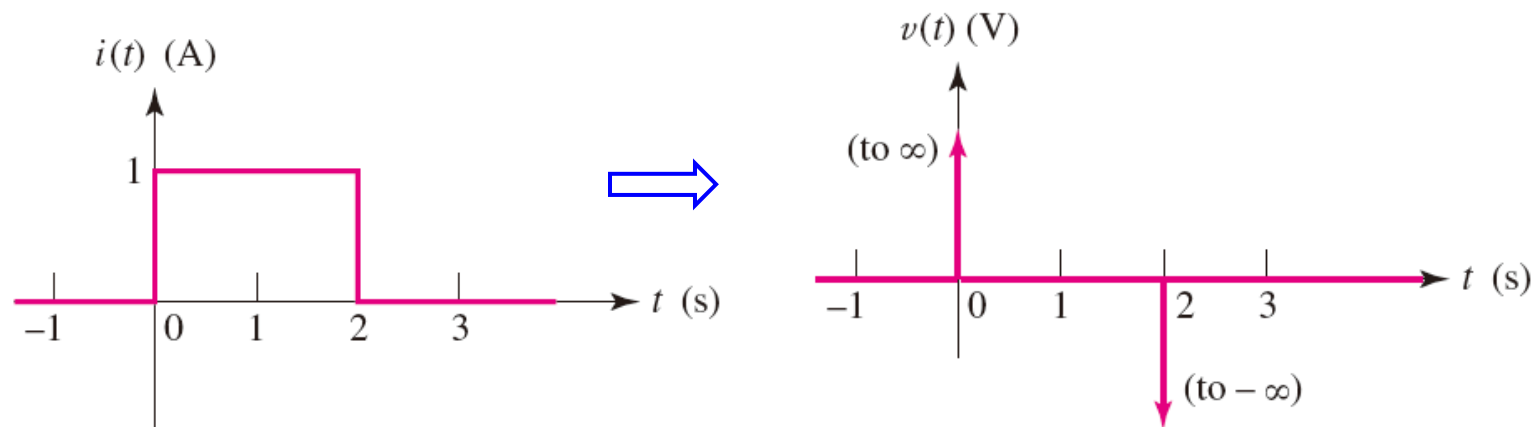


$$v = L \frac{di}{dt} = 3 \frac{di}{dt}$$

Example 7.5 Determine the inductor voltage. ($L = 3 \text{ H}$)



$$v = L \frac{di}{dt} = 3 \frac{di}{dt}$$



Infinite voltage spikes are required to produce the abrupt changes in the inductor current

- Integral Voltage-Current Relationships

$$v = L \frac{di}{dt} \rightarrow di = \frac{1}{L} v dt$$

$$\int_{i(t_0)}^{i(t)} di' = \frac{1}{L} \int_{t_0}^t v(t') dt' \Rightarrow i(t) - i(t_0) = \frac{1}{L} \int_{t_0}^t v(t') dt' \Rightarrow i(t) = \frac{1}{L} \int_{t_0}^t v(t') dt' + i(t_0)$$

$$i(t) = \frac{1}{L} \int v dt + k \quad i(t) = \frac{1}{L} \int_{-\infty}^t v dt' \quad t_0 \rightarrow -\infty, i(-\infty) = 0$$

Example 7.6 Determine the inductor current if $i(t = -\pi/2) = 1\text{A}$. ($L = 2\text{ H}$)

$$v_L = 6 \cos 5t \text{ V} \quad \Rightarrow i(t) = \frac{1}{2} \int_{t_0}^t 6 \cos 5t' dt' + i(t_0) = \frac{1}{2} \left(\frac{6}{5} \right) \sin 5t - \frac{1}{2} \left(\frac{6}{5} \right) \sin 5t_0 + i(t_0)$$

$$\Rightarrow i(t) = 0.6 \sin 5t - 0.6 \sin 5t_0 + i(t_0) = 0.6 \sin(5t) - 0.6 \sin\left(-\frac{5\pi}{2}\right) + 1 = 0.6 \sin 5t + 1.6$$

$$i(t) = \frac{1}{2} \int 6 \cos 5t dt + k = 0.6 \sin 5t + k \quad \text{Let } t_0 = -\frac{\pi}{2}$$

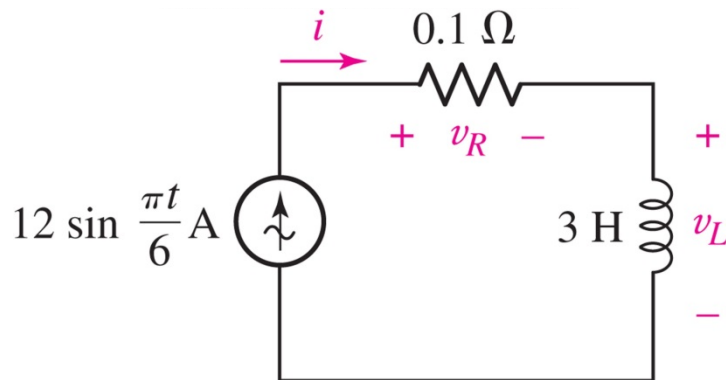
$$\Rightarrow i\left(-\frac{\pi}{2}\right) = 0.6 \sin\left(-\frac{5\pi}{2}\right) + k = 1 \Rightarrow k = 1 + 0.6 = 1.6 \quad \therefore i(t) = 0.6 \sin 5t + 1.6$$

- Energy Storage

$$\text{Power: } p = vi = iL \frac{di}{dt} \qquad E = \int_{t_0}^t p dt' = L \int_{t_0}^t i \frac{di}{dt'} dt' = L \int_{i(t_0)}^{i(t)} i' dv' = \frac{1}{2} L [i^2(t) - i^2(t_0)]$$

$$w_L(t) - \cancel{w_L(t_0)} = \frac{1}{2} L [i^2(t) - i^2(t_0)] \qquad \Rightarrow \quad w_L(t) = \frac{1}{2} Li^2 \quad \text{Assume } i(t_0) = 0$$

Example 7.7 Find the maximum energy stored in the inductor



$$w_L = \frac{1}{2} Li^2 = \frac{1}{2} \times 3 \times \left(12 \sin\left(\frac{\pi t}{6}\right) \right)^2 = 216 \sin^2 \frac{\pi t}{6}$$

$$\text{at } t = 0: w_L(t = 0) = 0 \text{ J}$$

$$\text{at } t = 3: w_L(t = 3) = 216 \sin^2 \frac{\pi}{2} = 216 \text{ J} \quad \text{maximum}$$

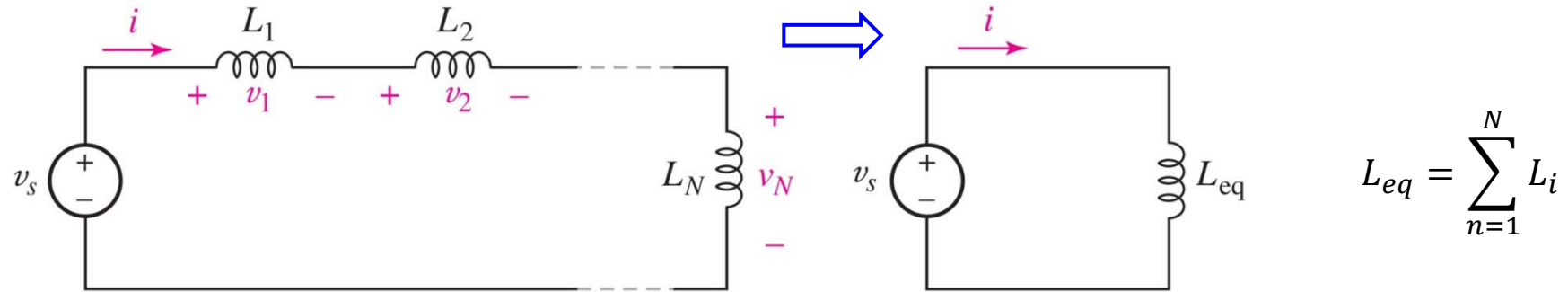
$$\text{at } t = 6: w_L(t = 6) = 0 \text{ J}$$

$$p_R = i^2 R = 0.1 \times 12^2 \sin^2 \frac{\pi t}{6} \text{ W} \qquad \Rightarrow \quad w_R = \int_0^6 p_R dt = \int_0^6 14.4 \sin^2 \frac{\pi}{6} t dt = 43.2 \text{ J}$$

Important Characteristics of an Ideal Inductor

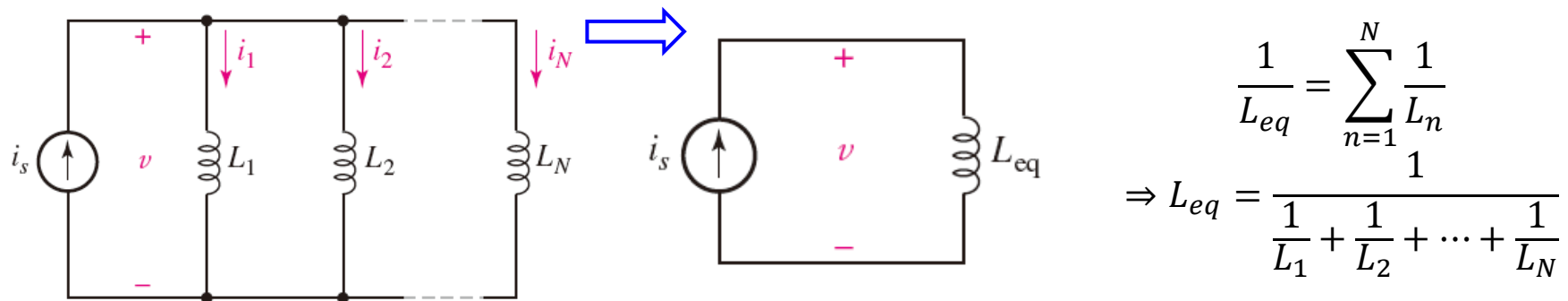
1. There is no voltage across an inductor if the current through it is not changing with time. ***An inductor is therefore a short circuit to dc.***
2. A finite amount of energy can be stored in an inductor even if the voltage across the inductor is zero, such as when the current through it is constant.
3. It is impossible to change the current through in inductor by a finite amount in zero time, for this requires an infinite voltage across the inductor. (An inductor resists an abrupt change in the current across it in a manner analogous to the way a mass resists an abrupt change in its velocity.)
4. The inductor never dissipates energy, but only stores it. Although this is true for the ***mathematical*** model, it is not true for a ***physical*** inductor due to series resistances.

- Inductors in Series



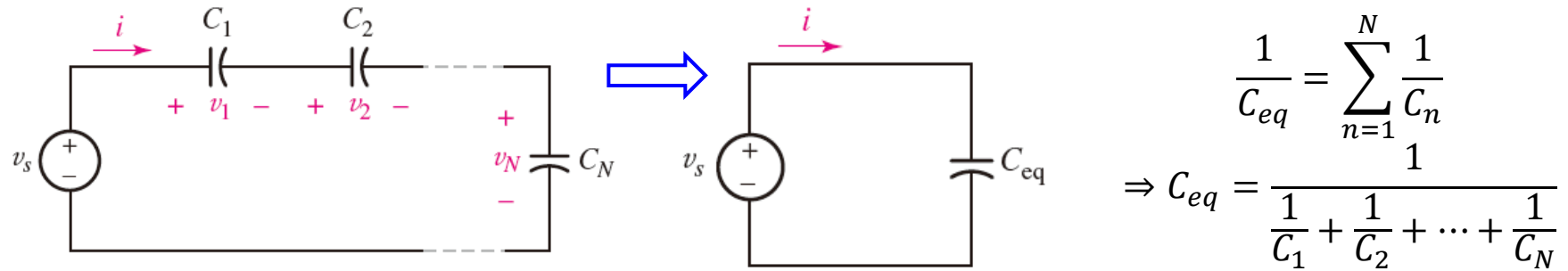
$$v_s = v_1 + v_2 + \dots + v_N = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + \dots + L_N \frac{di}{dt} = (L_1 + L_2 + \dots + L_N) \frac{di}{dt} = L_{eq} \frac{di}{dt}$$

- Inductors in Parallel



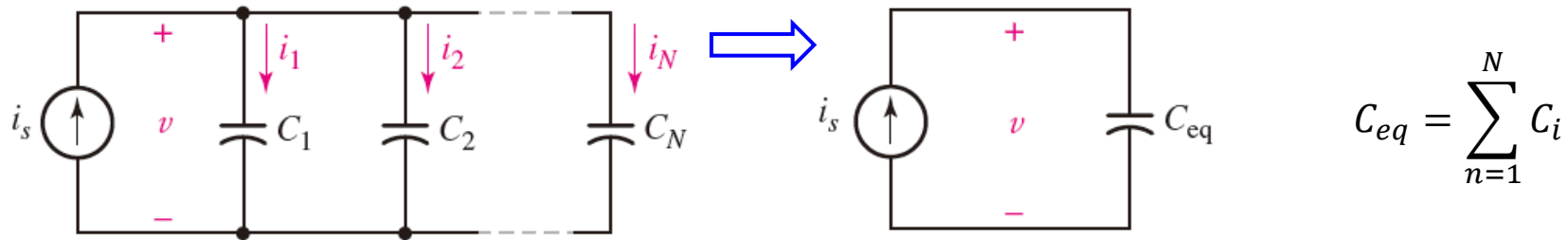
$$i_s = \sum_{n=1}^N i_n = \sum_{n=1}^N \left[\frac{1}{L_n} \int_{t_0}^t v dt' + i_n(t_0) \right] = \left(\sum_{n=1}^N \frac{1}{L_n} \right) \int_{t_0}^t v dt' + \sum_{n=1}^N i_n(t_0) = \frac{1}{L_{eq}} \int_{t_0}^t v dt' + i_s(t_0)$$

- Capacitors in Series



$$v_s = \sum_{n=1}^N v_n = \sum_{n=1}^N \left[\frac{1}{C_n} \int_{t_0}^t i dt' + v_n(t_0) \right] = \left(\sum_{n=1}^N \frac{1}{C_n} \right) \int_{t_0}^t i dt' + \sum_{n=1}^N v_n(t_0) = \frac{1}{C_{eq}} \int_{t_0}^t i dt' + v_s(t_0)$$

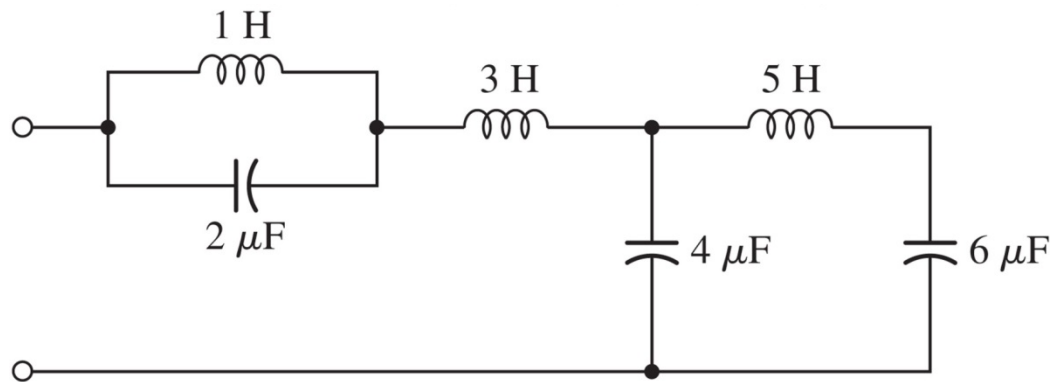
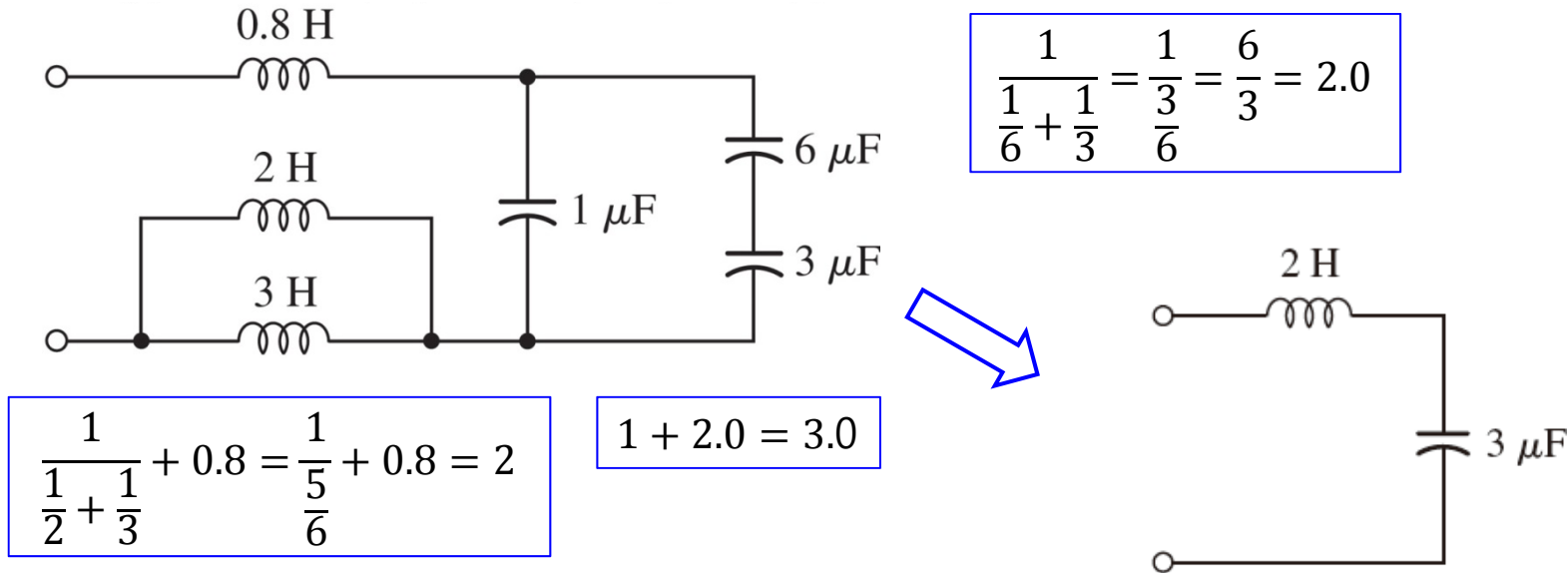
- Capacitors in Parallel



$$\begin{aligned} i_s &= i_1 + i_2 + \dots + i_N \\ &= C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + \dots + C_N \frac{dv}{dt} \\ &= (C_1 + C_2 + \dots + C_N) \frac{dv}{dt} \end{aligned}$$

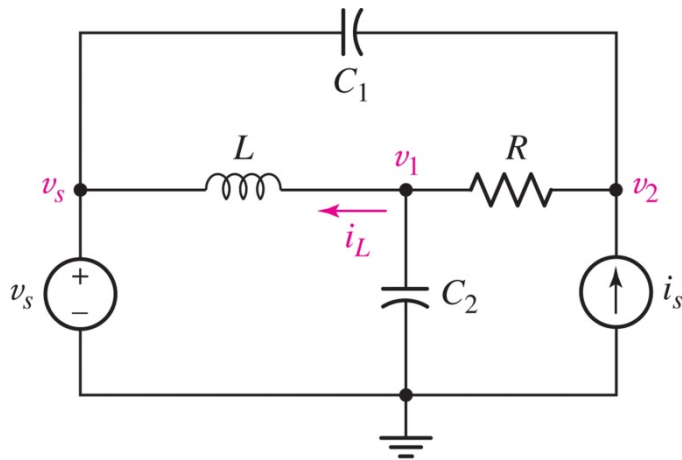
$$i_s = \sum_{n=1}^N i_n = \sum_{n=1}^N C_n \frac{dv}{dt} = \frac{dv}{dt} \sum_{n=1}^N C_n = C_{eq} \frac{dv}{dt}$$

Example 7.8 Simplify the network using series-parallel combinations.



No series or parallel combinations of either the inductors or the capacitors.
 → can't be simplified.

Example 7.9 Write appropriate nodal equations for the circuit.



$$i_s = \frac{v_2 - v_1}{R} + C_1 \frac{d(v_2 - v_s)}{dt}$$

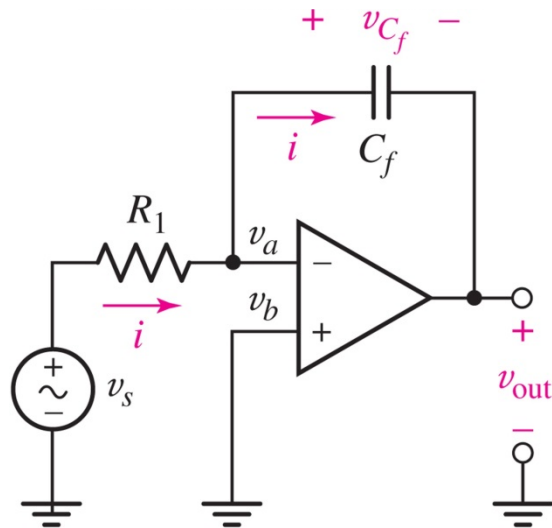
$$\Rightarrow -\frac{v_1}{R} + \frac{v_2}{R} + C_1 \frac{dv_2}{dt} = i_s + C_1 \frac{dv_s}{dt}$$

$$\frac{1}{L} \int_{t_0}^t (v_1 - v_s) dt' + i_L(t_0) + C_2 \frac{dv_1}{dt} + \frac{v_1 - v_2}{R} = 0$$

$$\Rightarrow \frac{v_1}{R} + C_2 \frac{dv_1}{dt} + \frac{1}{L} \int_{t_0}^t v_1 dt' - \frac{v_2}{R} = \frac{1}{L} \int_{t_0}^t v_s dt' - i_L(t_0)$$

1. Inductors and capacitors are linear elements.
2. The principle of proportionality between source and response can be extended to the general RLC circuit, and it follows that the principle of superposition also applies.
3. Initial inductor current and capacitor voltages must be treated as independent sources in applying the superposition principle.
4. Thévenin and Norton's theorem can be applied to linear RLC circuits, if we wish.

- Integrator



$$v_a = v_b = 0,$$

$$v_{out} = -v_{C_f}$$

$$\frac{v_s - v_a}{R_1} = C_f \frac{d(v_a - v_{out})}{dt}$$

$$\rightarrow \frac{v_s}{R_1} = -C_f \frac{dv_{out}}{dt}$$

$$\Rightarrow v_{out} = -v_{C_f} = -\frac{1}{C_f R_1} \int_0^t v_s dt' - v_{C_f}(0)$$

- Differentiator

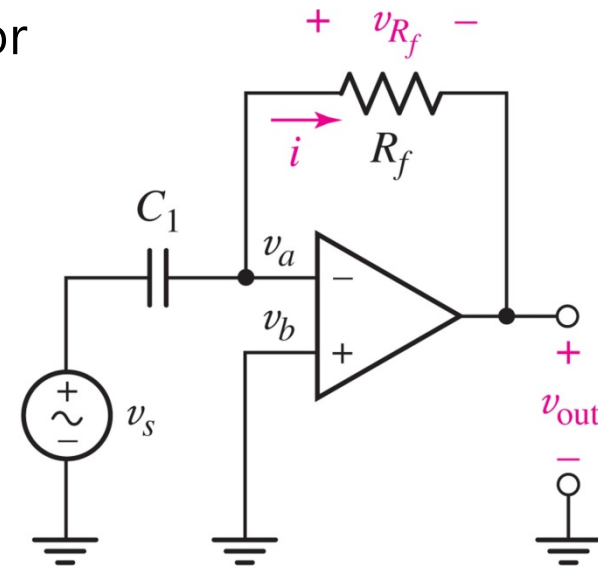
$$v_a = v_b = 0,$$

$$v_{out} = -v_{R_f}$$

$$C_1 \frac{d(v_s - v_a)}{dt} = \frac{v_a - v_{out}}{R_f}$$

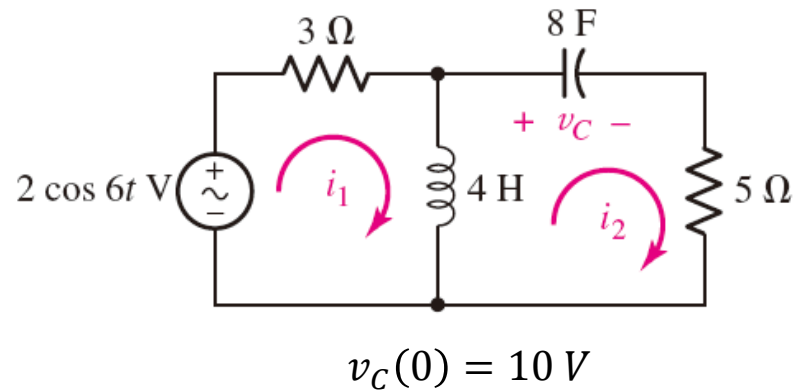
$$\rightarrow -\frac{v_{out}}{R_f} = C_1 \frac{dv_s}{dt}$$

$$\Rightarrow v_{out} = -v_{R_f} = -R_f C_1 \frac{dv_s}{dt}$$



- Duality

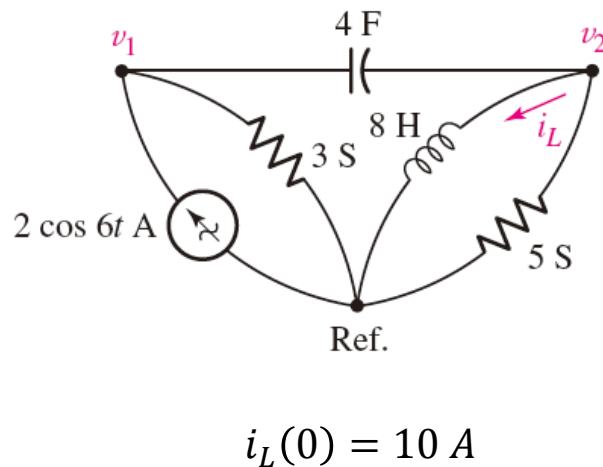
Two circuits are "dual" if the mesh equations that characterize one of them have the *same mathematical form* as the nodal equations that characterize the other.



$$3i_1 + 4 \frac{di_1}{dt} - 4 \frac{di_2}{dt} = 2 \cos 6t,$$

$$-4 \frac{di_1}{dt} + 4 \frac{di_2}{dt} + \frac{1}{8} \int_0^t i_2 dt' + v_C(0) + 5i_2 = 0$$

$$-4 \frac{di_1}{dt} + 4 \frac{di_2}{dt} + \frac{1}{8} \int_0^t i_2 dt' + 5i_2 = -10$$

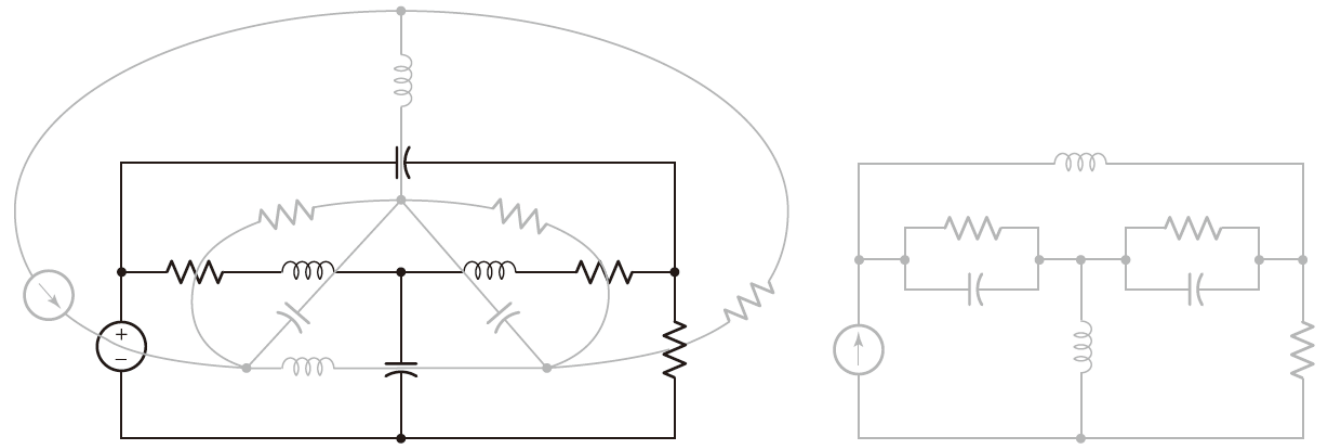
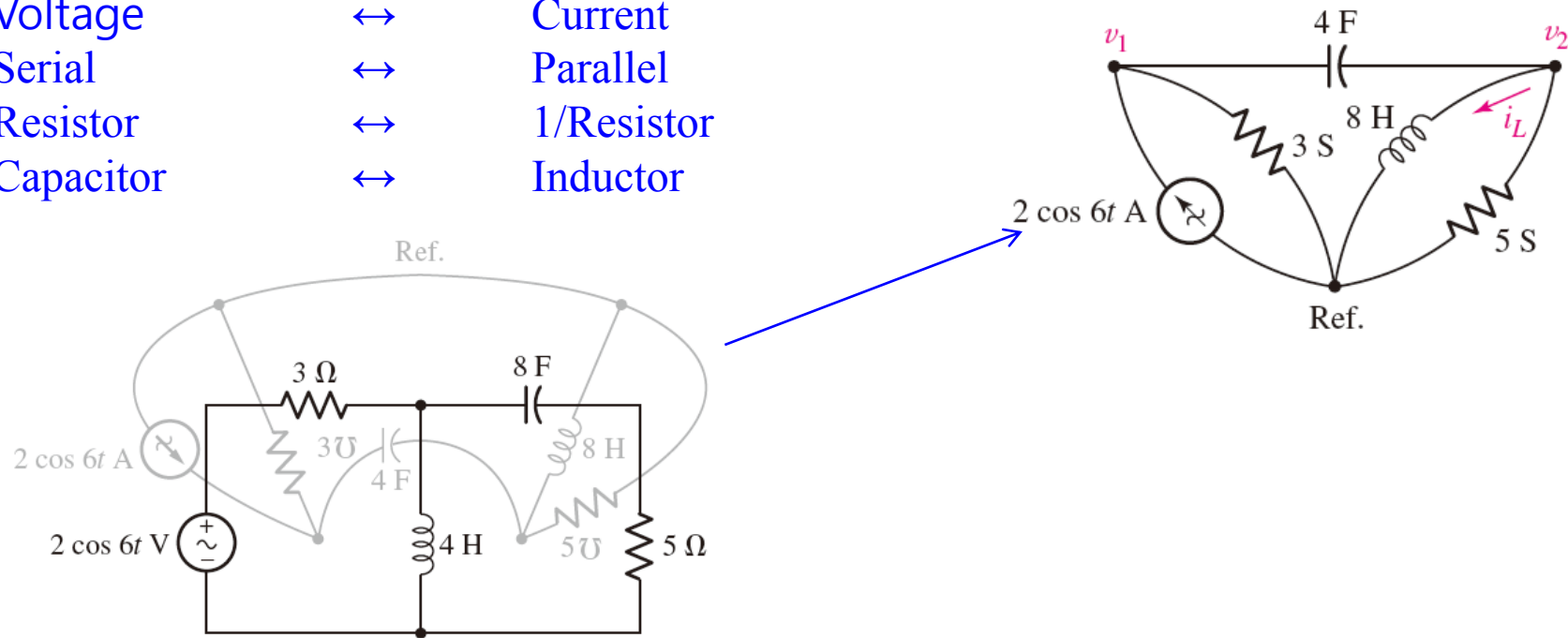


$$3v_1 + 4 \frac{dv_1}{dt} - 4 \frac{dv_2}{dt} = 2 \cos 6t,$$

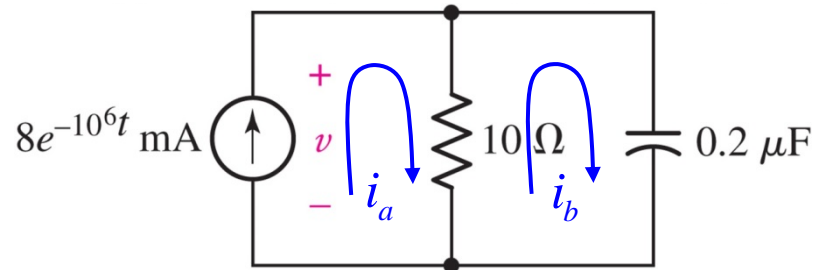
$$-4 \frac{dv_1}{dt} + 4 \frac{dv_2}{dt} + \frac{1}{8} \int_0^t v_2 dt' + i_L(0) + 5v_2 = 0$$

$$-4 \frac{dv_1}{dt} + 4 \frac{dv_2}{dt} + \frac{1}{8} \int_0^t v_2 dt' + 5v_2 = -10$$

Voltage	\leftrightarrow	Current
Serial	\leftrightarrow	Parallel
Resistor	\leftrightarrow	1/Resistor
Capacitor	\leftrightarrow	Inductor



Practice 7.11



(a)

$$8e^{-10^6 t} = \frac{v}{10} + 0.2\mu \frac{dv}{dt}$$

$$-v + 10i_a - 10i_b = 0$$

$$-10i_a + 10i_b + \frac{1}{C} \int i_b dt = 0$$

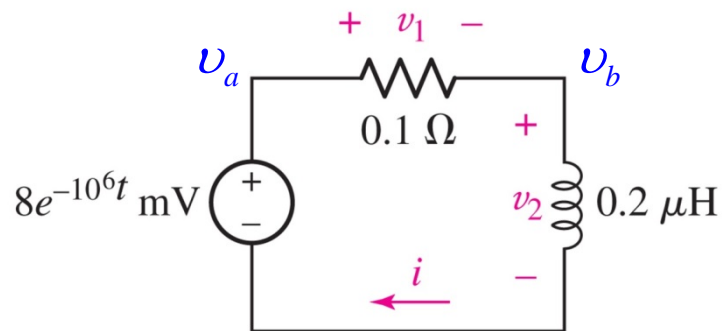
$$v = i = -80e^{-10^6 t}$$

$$i_a = v_a = 8e^{-10^6 t}$$

$$i_b = v_b = v_a - 0.1 \times i$$

$$= 8e^{-10^6 t} + 0.1 \times 80e^{-10^6 t}$$

$$= 16e^{-10^6 t}$$



(b)

$$8e^{-10^6 t} = 0.1i + 0.2\mu \frac{di}{dt}$$

$$-i + 10v_a - 10v_b = 0$$

$$-10v_a + 10v_b + \frac{1}{L} \int v_b dt = 0$$

Homework : 7장 Exercises 5의 배수 문제 (64번 문제까지)

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