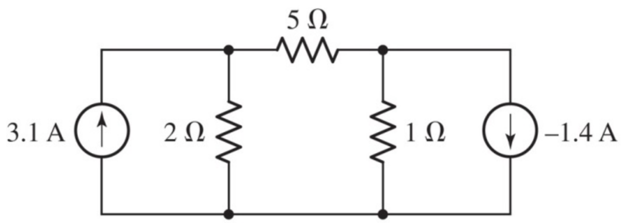
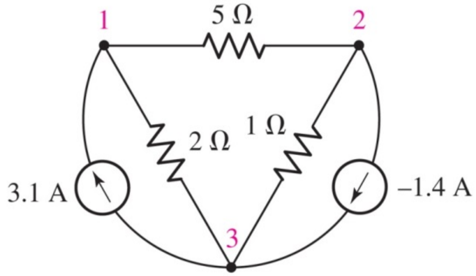
Chapter 4
Basic Nodal and Mesh Analysis

- Method

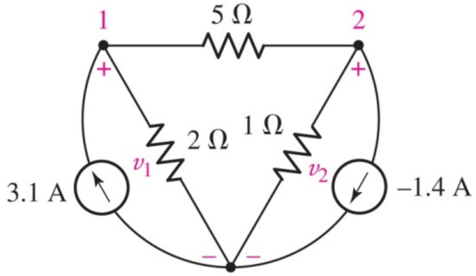
- We select one node in the circuit to be a reference node.
- Node voltage = The voltage of each of the nonreference nodes with respect to the reference node.
- Use Node voltages as unknown variables to set up Independent KCL equations (node equations). Find node voltages.
- We can find all branch voltages and currents if we know node voltages



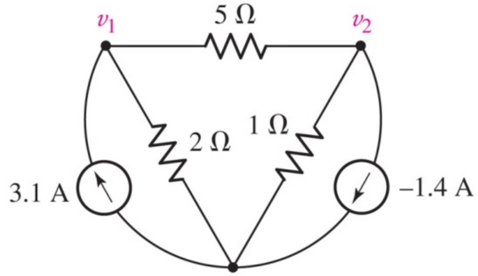
(a)



(b)



Reference node



Ref.

$$3.1 = \frac{v_1}{2} + \frac{v_1 - v_2}{5}$$

$$\rightarrow 3.1 - \frac{v_1}{2} - \frac{v_1 - v_2}{5} = 0$$

$$-(-1.4) = \frac{v_2}{1} + \frac{v_2 - v_1}{5}$$

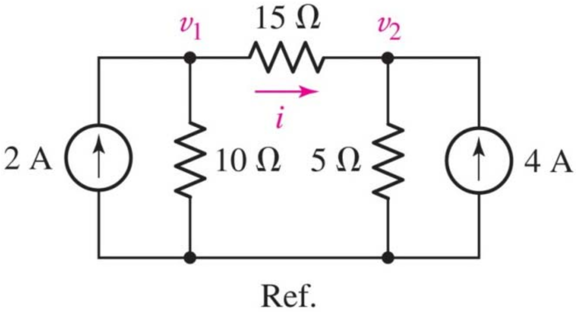
$$\rightarrow 1.4 - \frac{v_2}{1} - \frac{v_2 - v_1}{5} = 0$$

$$v_1 = 5\,V, v_2 = 2\,V$$

$$v_{5\Omega} = v_1 - v_2 = 3\,V$$

Nodal analysis is based on KCL

Example 4.1 Determine the current through 15Ω resistor

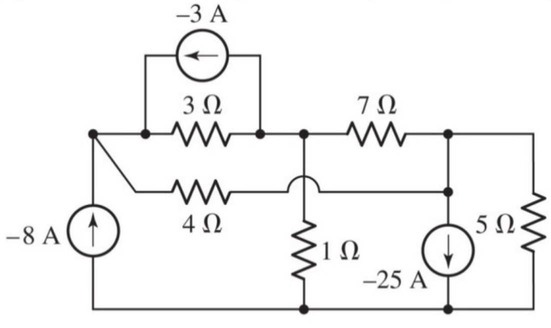


Node 1: $2 = \frac{v_1}{7+3} + \frac{v_1 - v_2}{15} \rightarrow 5v_1 - 2v_2 = 60$

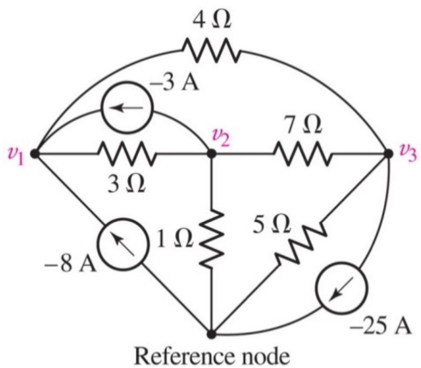
Node 2: $4 = \frac{v_2}{5} + \frac{v_2 - v_1}{15} \rightarrow -v_1 + 4_{v2} = 60$

⇒ $v_1 = 20\,V, v_2 = 20\,V$ ⇒ $i = 0$

Example 4.2 Determine the nodal voltages as referenced to the bottom node



(a)



Node 1: $-8 - 3 = \frac{v_1 - v_2}{3} + \frac{v_1 - v_3}{4}$

$\rightarrow -0.5833v_1 - 0.3333v_2 = -11$

Node 2: $-(-3) = \frac{v_2}{1} + \frac{v_2 - v_1}{3} + \frac{v_2 - v_3}{7}$

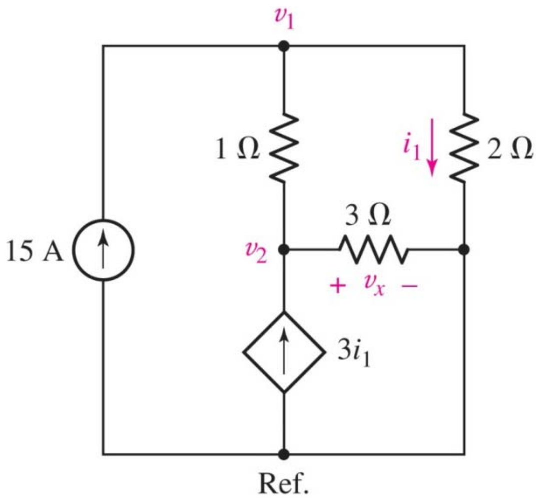
$\rightarrow -0.3333v_1 + 1.4762v_2 - 0.1429v_3 = 3$

Node 3: $-(-25) = \frac{v_3}{5} + \frac{v_3 - v_1}{4} + \frac{v_3 - v_2}{7}$

$\rightarrow -0.25v_1 - 0.1429v_2 + 0.5929v_3 = 25$

⇒ $v_1 = 5{,}412\,V,\quad v_2 = 7.736\,V,\quad v_3 = 46.32\,V$

Example 4.3 Determine the power supplied by the dependent source



Node 1: $15 = \frac{v_1 - v_2}{1} + \frac{v_1}{2} \rightarrow 3v_1 - 2v_2 = 30$ $i_1 = \frac{v_1}{2}$

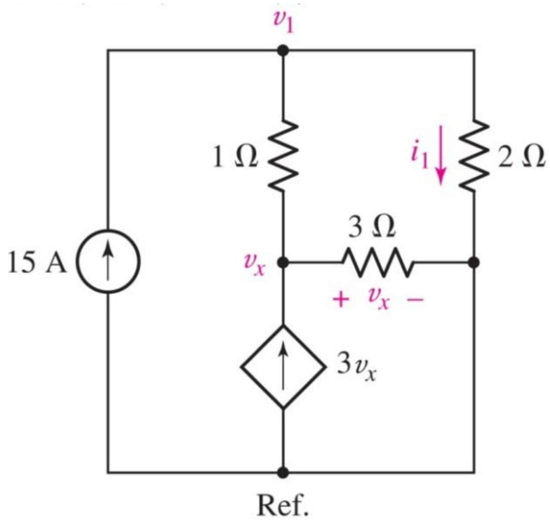
Node 2: $3i_1 = \frac{v_2 - v_1}{1} + \frac{v_2}{3} \rightarrow 3\frac{v_1}{2} = \frac{v_2 - v_1}{1} + \frac{v_2}{3} \rightarrow -15v_1 + 8v_2 = 0$

⇨

$$v_1 = -40\,V$$
$$v_2 = -75\,V$$
$$i_1 = -20\,A$$

$$p_{3i_1} = (3i_1)(v_2) = \big(3 \times (-20)\big)(-75) = 4500\,W$$

Example 4.4 Determine the power supplied by the dependent source



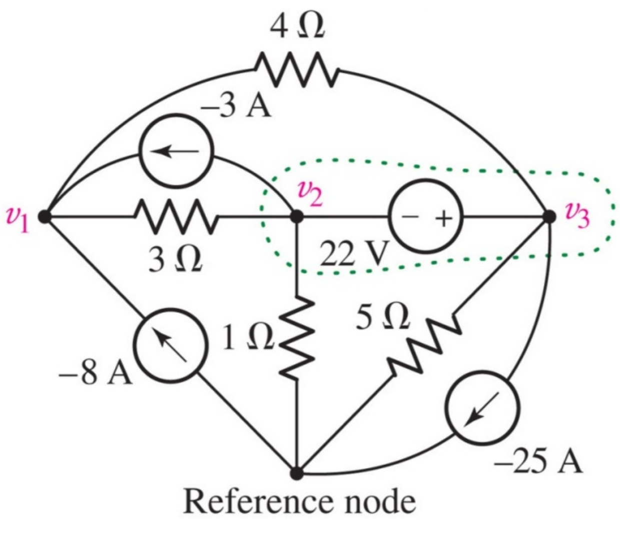
Node 1: $15 = \frac{v_1 - v_2}{1} + \frac{v_1}{2} \rightarrow 3v_1 - 2v_2 = 30$

Node x: $3v_x = \frac{v_x - v_1}{1} + \frac{v_x}{3} \rightarrow 5v_x + 3v_1 = 0$

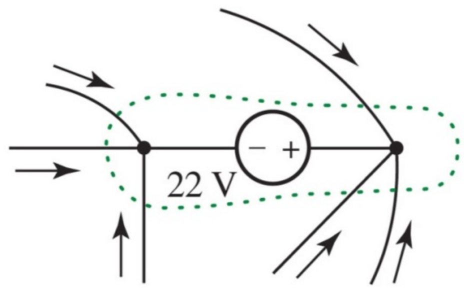
⇨

$$v_1 = \frac{50}{7}V$$
$$v_x = -\frac{30}{7}V$$

$$p_{3v_x} = (3v_x)(v_x) = 55.1\,W$$



(a)



(b)

To solve the circuit (a) on the left side, a branch current between v_2 and v_3 should be assigned.
→ Four equations and four unknowns

- Supernode
  - A closed region which encloses each voltage source
  - KCL is available to the supernode
  - All current flowing into the region sum to zero

Example 4.5 Determine the node voltage v_1

Node 1 : $-8-3=\frac{v_1-v_2}{3}+\frac{v_1-v_3}{4}$

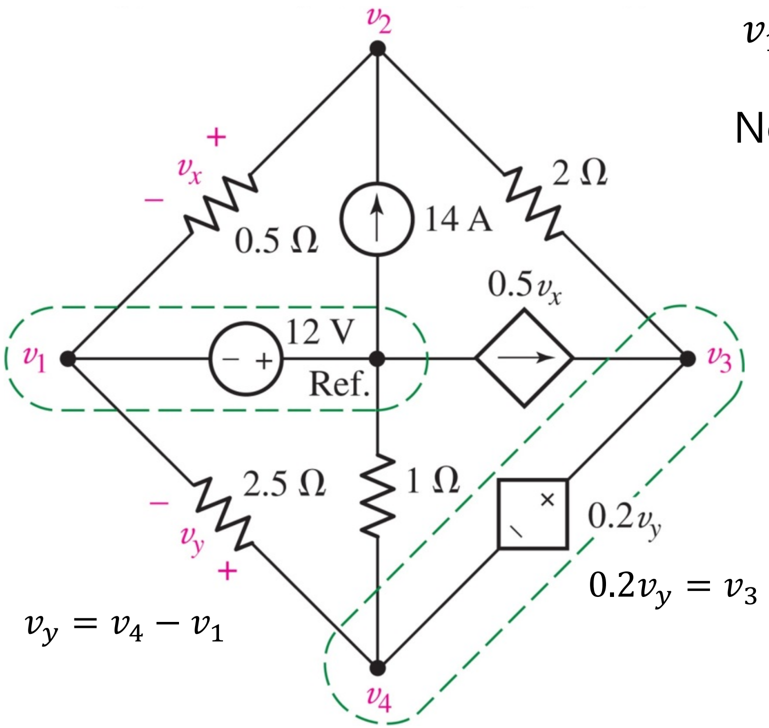
$\rightarrow -0.5833v_1-0.3333v_2=-11$

Supernode 2-3 : $3+25=\frac{v_2-v_1}{3}+\frac{v_3-v_1}{4}+\frac{v_3}{5}+\frac{v_2}{1}$

$\rightarrow -0.5833v_1+1.3333v_2+0.45v_3=28$

$v_3-v_2=22$ ⇨ $v_1=1.071\ V$

Example 4.6 Determine the node-to-reference voltages



$v_y = v_4 - v_1$

$0.2v_y = v_3 - v_4$

$$v_1 = -12$$

Node 2 : $14 = \frac{v_2 - v_1}{0.5} + \frac{v_2 - v_3}{2}$

$\rightarrow -2v_1 + 2.5v_2 - 0.5v_3 = 14$

$\rightarrow +2.5v_2 - 0.5v_3 = -10$

Supernode 3-4 : $0.5v_x = \frac{v_3 - v_2}{2} + \frac{v_4}{1} + \frac{v_4 - v_1}{2.5}$

$\rightarrow 0.1v_1 - v_2 + 0.5v_3 + 1.4v_4 = 0$

$\rightarrow -v_2 + 0.5v_3 + 1.4v_4 = 1.2$

$v_x = v_2 - v_1$

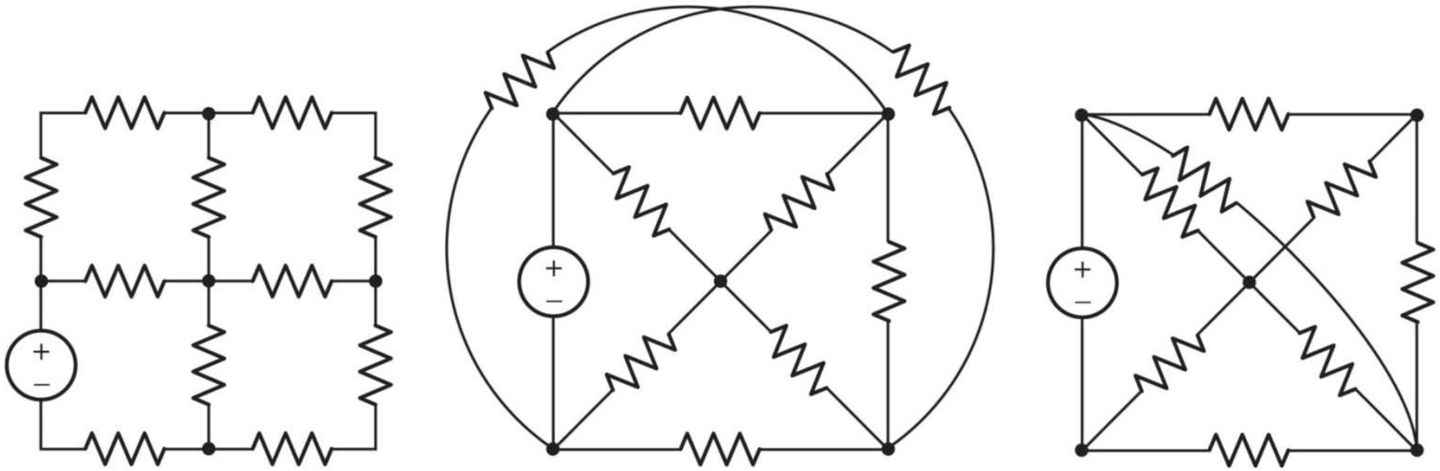
$$0.2v_y = v_3 - v_4 = 0.2(v_4 - v_1)$$

$\rightarrow 0.2v_1 + v_3 - 1.2v_4 = 0 \rightarrow v_3 - 1.2v_4 = 2.4$

$v_y = v_4 - v_1$

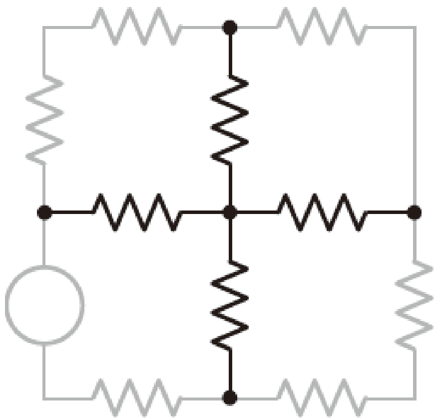
$\therefore v_1 = -12\,V, \quad v_2 = -4\,V, \quad v_3 = 0\,V, \quad v_4 = -2\,V$

- Loop and mesh
  - Loop : Any closed path in the circuit
  - Mesh : A loop that does not contain any other loops within it
- Mesh Current = The current that flows only around the perimeter of the mesh. (Independent loop current)
- Use Mesh Current as unknown variables to set up Independent KVL equations (mesh equations)
- We can find all branch voltages and currents if we know mesh currents.
- We restrict attention only to **planar circuits** (No branch passes over or under any other branch)

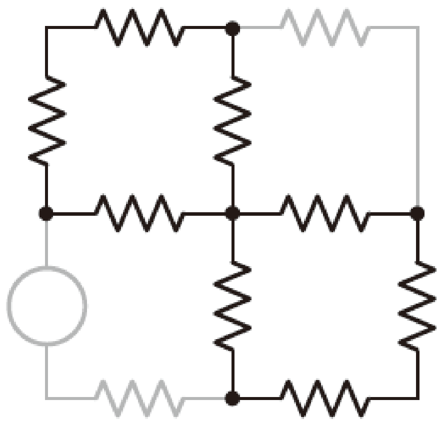


Non-planar circuit

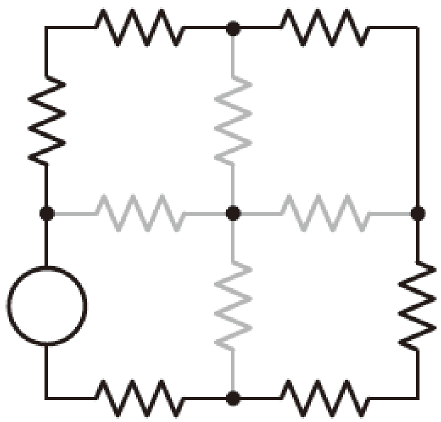
Loop and Mesh



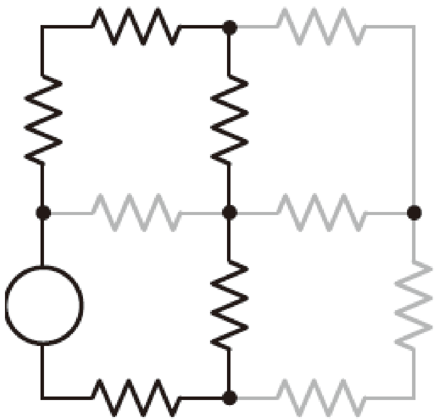
No path, no loop



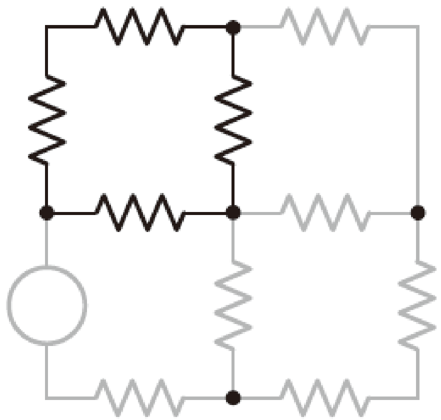
No path, no loop



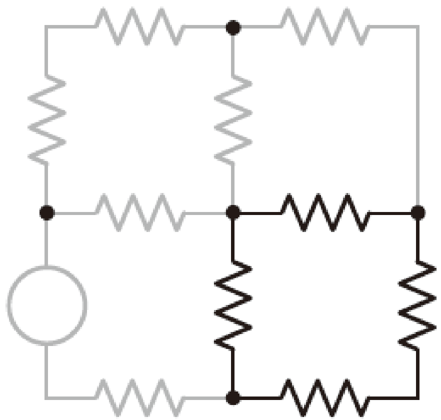
Loop, No mesh



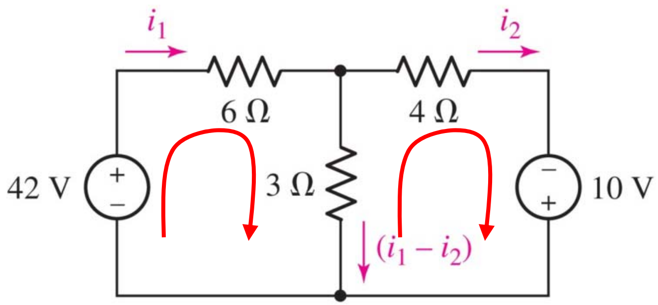
Loop, No mesh



Loop and Mesh



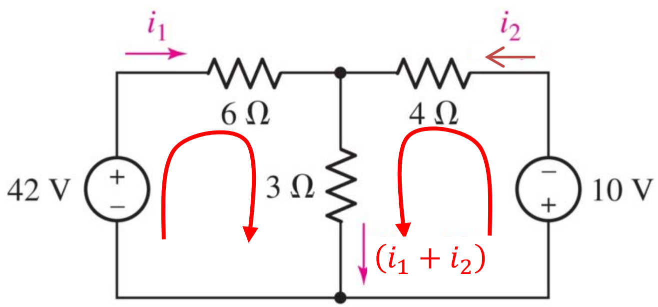
Loop and Mesh



Loop 1: $-42 + 6i_1 + 3(i_1 - i_2) = 0 \rightarrow 9i_1 - 3i_2 = 42$

Loop 2: $3(i_2 - i_1) + 4i_2 - 10 = 0 \rightarrow -3i_1 + 7i_2 = 10$

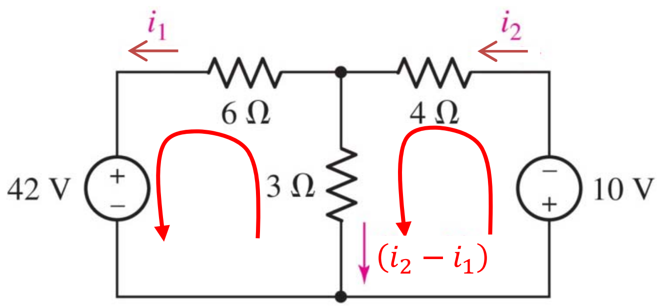
$\Longrightarrow \quad i_1 = 6\,A, \quad i_2 = 4\,A, \quad (i_1 - i_2) = 2\,A$



Loop 1: $-42 + 6i_1 + 3(i_1 + i_2) = 0 \rightarrow 9i_1 + 3i_2 = 42$

Loop 2: $10 + 4i_2 + 3(i_1 + i_2) = 0 \rightarrow 3i_1 + 7i_2 = -10$

$\Longrightarrow \quad i_1 = 6\,A, \quad i_2 = -4\,A, \quad (i_1 + i_2) = 2\,A$

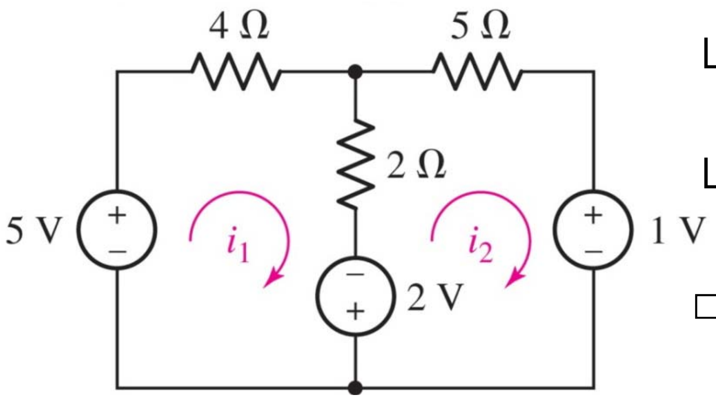


Loop 1: $3(i_1 - i_2) + 6i_1 + 42 = 0 \rightarrow 9i_1 - 3i_2 = -42$

Loop 2: $10 + 4i_2 + 3(i_2 - i_1) = 0 \rightarrow -3i_1 + 7i_2 = -10$

$\Longrightarrow \quad i_1 = -6\,A, \quad i_2 = -4\,A, \quad (i_2 - i_1) = 2\,A$

Example 4.7 Determine the power supplied by the 2 V source



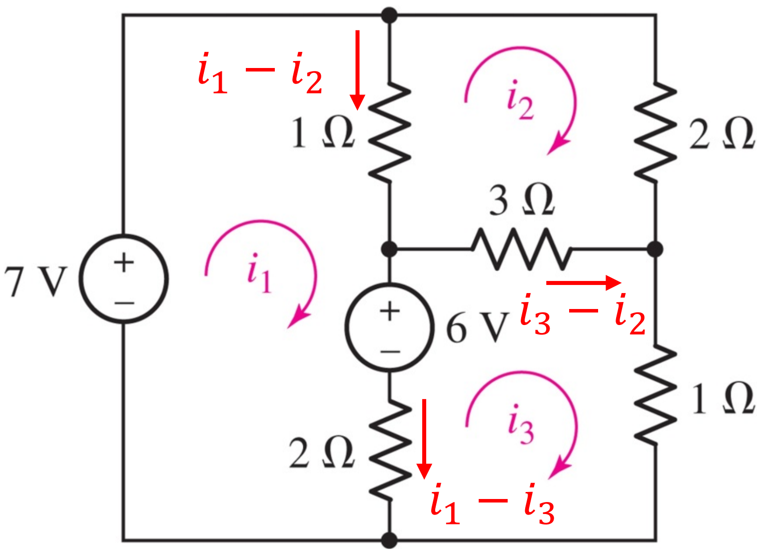
Loop 1: $-5 + 4i_1 + 2(i_1 - i_2) - 2 = 0 \rightarrow 6i_1 - 2i_2 = 7$

Loop 2: $2 + 2(i_2 - i_1) + 5i_2 + 1 = 0 \rightarrow -2i_1 + 7i_2 = -3$

$\Rightarrow$ $i_1 = \frac{43}{38} A, i_2 = -\frac{2}{19} A,$

$p_{2V} = (2)(i_1 - i_2) = (2)\left(\frac{47}{38}\right) = 2.474\, W$

Example 4.8 Determine the three mesh currents



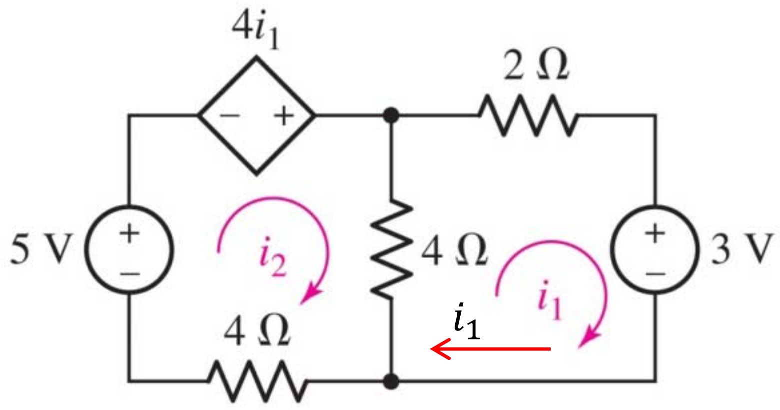
Loop 1 : $-7 + 1(i_1 - i_2) + 6 + 2(i_1 - i_3) = 0$
$\rightarrow 3i_1 - i_2 - 2i_3 = 1$

Loop 2 : $1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$
$\rightarrow -i_1 + 6i_2 - 3i_3 = 0$

Loop 3 : $2(i_3 - i_1) - 6 + 3(i_3 - i_2) + i_3 = 0$
$\rightarrow -2i_1 - 3i_2 + 6i_3 = 6$

$\Rightarrow$ $i_1 = 3A,\ i_2 = 2\ A,\ i_3 = 3\ A$

Example 4.9 Determine the current i_1

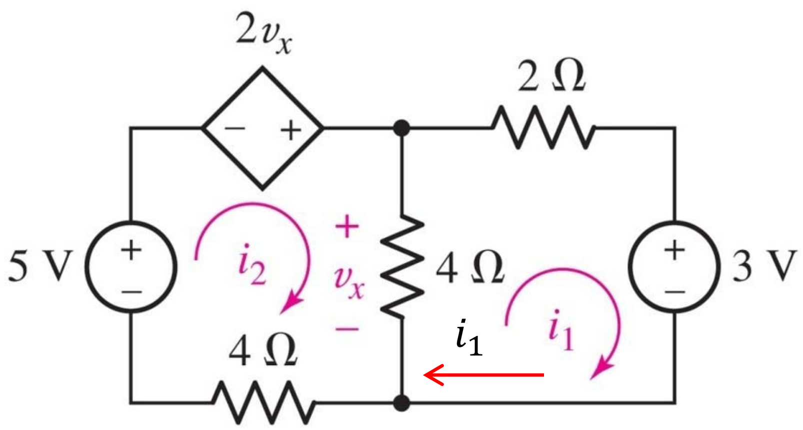


Loop 1 : $-5 - 4i_1 + 4(i_2 - i_1) + 4i_2 = 0$
$\rightarrow -8i_1 + 8i_2 = 5$

Loop 2 : $2i_i + 3 + 4(i_1 - i_2) = 0$
$\rightarrow 6i_1 - 4i_2 = -3$

$\Longrightarrow$ $i_1 = -\frac{1}{4}\,A,\ i_2 = \frac{3}{8}\,A$

Example 4.10 Determine the current i_1



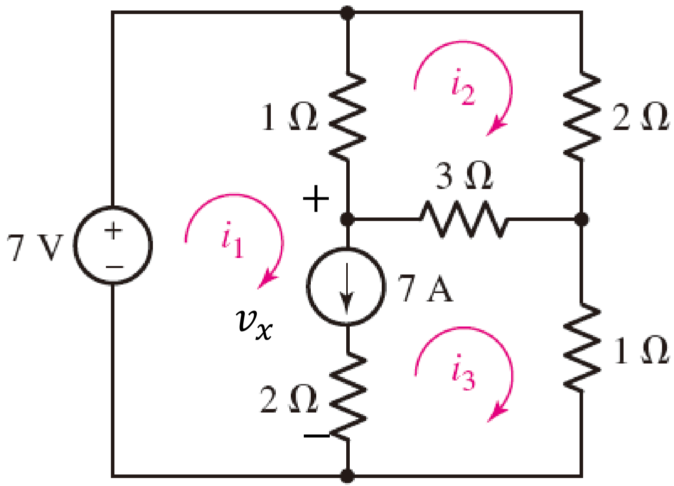
Loop 1: $-5 - 2v_x + 4(i_2 - i_1) + 4i_2 = 0$
$-5 - 2\big(4(i_2 - i_1)\big) + 4(i_2 - i_1) + 4i_2 = 0$
$\rightarrow 4i_1 = 5$

Loop 2 : $2i_i + 3 + 4(i_1 - i_2) = 0$
$\rightarrow 6i_1 - 4i_2 = -3$

$\Longrightarrow$ $i_1 = \frac{5}{4}\,A$

- Circuits with current sources
- The current sources reduce the # of simultaneous mesh equations

Example 4.11



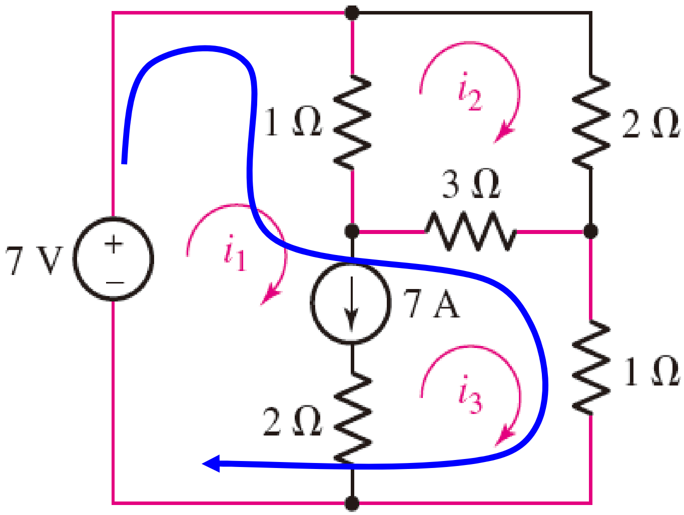
Using conventional method

Loop 1: $-7 + 1(i_1 - i_2) + v_x = 0 \;\rightarrow\; i_1 - i_2 + v_x = 7$

Loop 2: $2i_2 + 3(i_2 - i_3) + 1(i_2 - i_1) = 0$
$\rightarrow -i_1 + 6i_2 - 3i_3 = 0$

Loop 3: $i_3 - v_x + 3(i_3 - i_2) = 0 \;\rightarrow\; -3i_2 + 4i_3 - v_x = 0$

$i_1 - i_3 = 7 \;\Rightarrow\; i_3 = 2\,A, i_1 = 9\,A, i_2 = \frac{15}{6}A, v_x = 0.5\,V$



Using supermesh

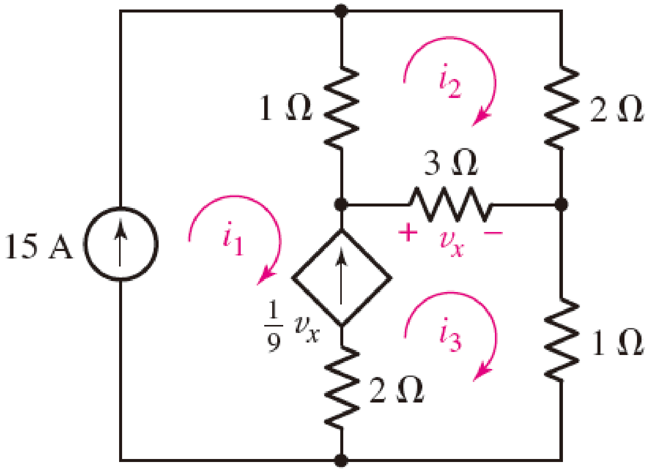
Loop 1: $-7 + 1(i_1 - i_2) + 3(i_3 - i_2) + 1i_3 = 0$
$\rightarrow i_1 - 4i_2 + 4i_3 = 7$

Loop 2: $1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$
$\rightarrow -i_1 + 6i_2 - 3i_3 = 0$

$i_1 - i_3 = 7 \;\Rightarrow\; i_3 = 2\,A, i_1 = 9\,A, i_2 = \frac{15}{6}A$

3 equations and 3 unknowns

Example 4.12 Evaluate the three unknown currents



$i_1 = 15 \qquad i_3 - i_1 = \dfrac{v_x}{9} = \dfrac{3(i_3 - i_2)}{9} \leftarrow v_x = 3(i_3 - i_2)$

$i_3 - 15 = \dfrac{1}{3} i_3 - \dfrac{1}{3} i_2 \rightarrow \dfrac{1}{3} i_2 + \dfrac{2}{3} i_3 = 15$

$\Longrightarrow \quad 1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$

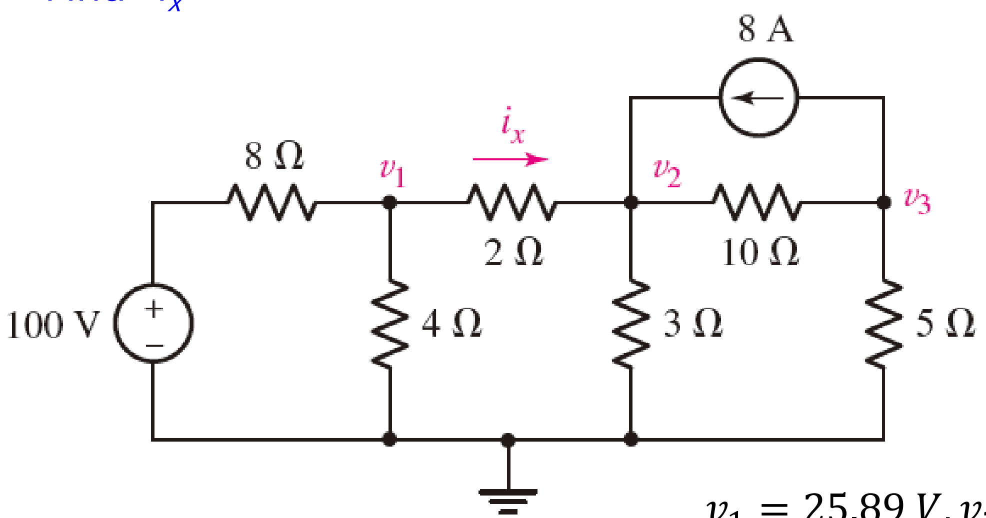
$\rightarrow -i_1 + 6i_2 - 3i_3 = 0 \rightarrow 6i_2 - 3i_3 = 15$

$\Longrightarrow \quad i_2 = 11\ A, \qquad i_3 = 17\ A$

- In choosing nodal or mesh analysis,

- Non-planar circuit → Nodal analysis
- N nodes → N-1 KCL equations (Each supernode reduce the # of equations by 1)
- M meshes → M KVL equations (Each supermesh reduce this number by 1)
- Controlling quantity in dependent sources : nodal voltage → nodal analysis
  mesh current → mesh analysis
- Location of source : current source in mesh line → mesh analysis
  voltage source connected to ref. → nodal analysis
- Asking current → mesh analysis, asking voltage → nodal analysis

Find i_x



Node 1: $\frac{100-v_1}{8} = \frac{v_1}{4} + \frac{v_1-v_2}{2} (= -i_x)$

$\rightarrow 0.875v_1 - 0.5v_2 = 12.5$

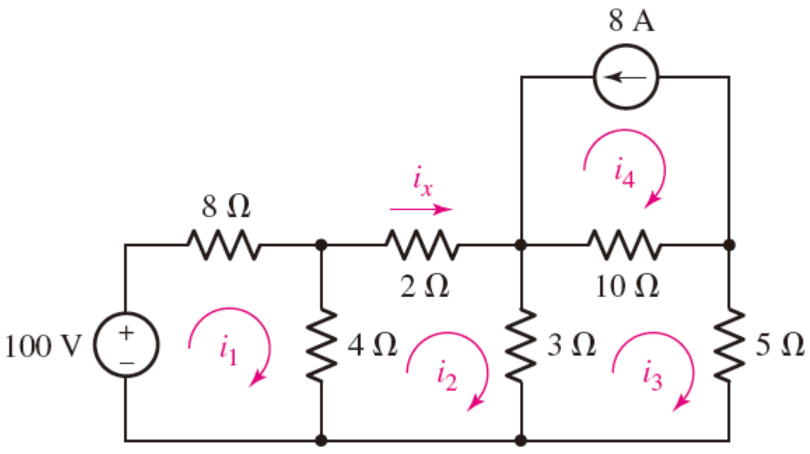
Node 2: $\frac{v_1-v_2}{2} = \frac{v_2}{3} + \frac{v_2-v_3}{10} - 8$

$\rightarrow -0.5v_1 + 0.933v_2 - 0.1v_3 = 8$

Node 3: $-8 = \frac{v_3}{5} + \frac{v_3-v_2}{10}$

$\rightarrow -0.1v_2 + 0.3v_3 = -8$

$v_1 = 25.89\,V, v_2 = 20.31\,V \implies i_x = \frac{v_1 - v_2}{2} = 2.79\,A$



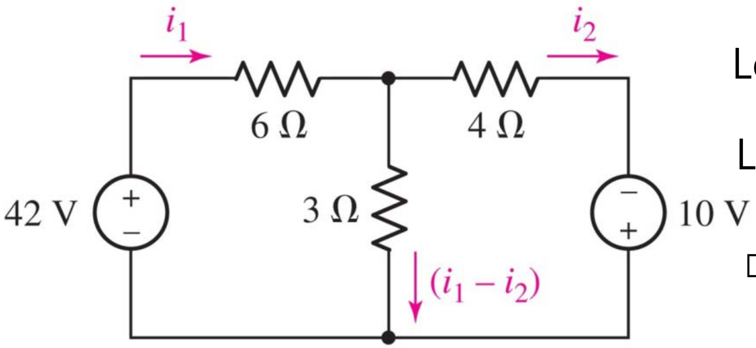
Loop 1: $-100 + 8i_1 + 4(i_1 - i_2) = 0$
$\rightarrow 12i_1 - 4i_2 = 100$

Loop 2: $2i_2 + 3(i_2 - i_3) + 4(i_2 - i_1) = 0$
$\rightarrow -4i_1 + 9i_2 - 3i_3 = 0$

Loop 3: $3(i_3 - i_2) + 10(i_3 - (-8)) + 5i_3 = 0$
$\rightarrow -3i_2 + 18i_3 = -80$

$\Rightarrow$ $i_x = i_2 = 2.79\ A$

- To analyze a complex circuit, a computer software package is useful
- SPICE (Simulation Program with Integrated Circuit Emphasis)
- Pspice : SPICE with intuitive graphical interface by MicroSim Corp.

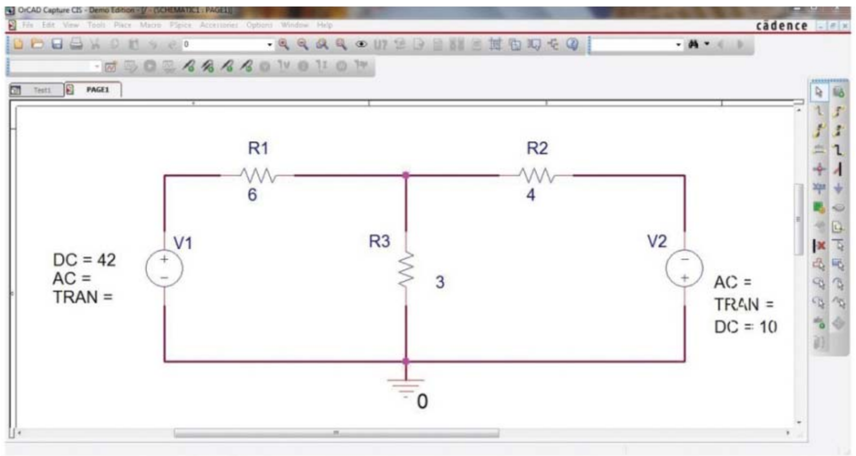


Loop 1: $-42 + 6i_1 + 3(i_1 - i_2) = 0 \;\rightarrow\; 9i_1 - 3i_2 = 42$

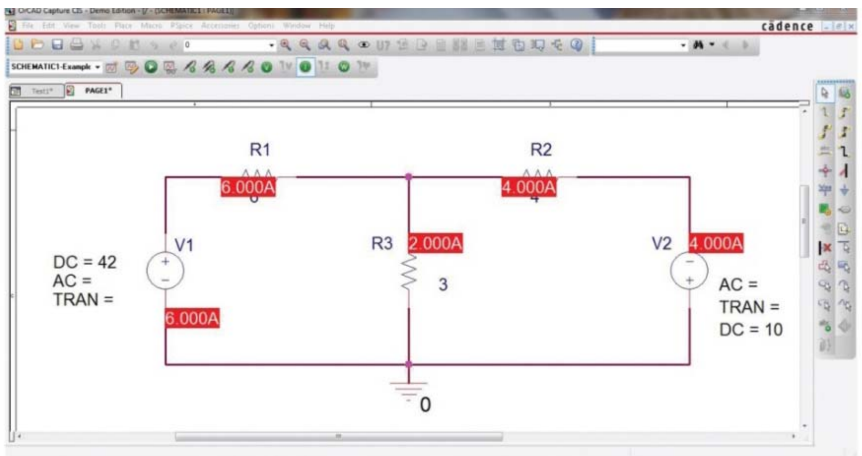
Loop 2: $3(i_2 - i_1) + 4i_2 - 10 = 0 \;\rightarrow\; -3i_1 + 7i_2 = 10$

$\Rightarrow\quad i_1 = 6\,A, \qquad i_2 = 4\,A, \qquad (i_1 - i_2) = 2\,A$

Hand analysis using KCL, KVL, and ohm's law

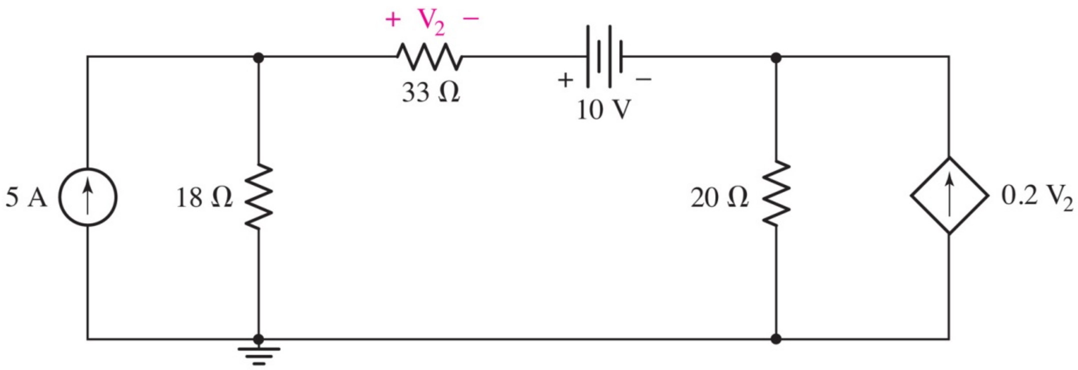


Orcad schematic capture software

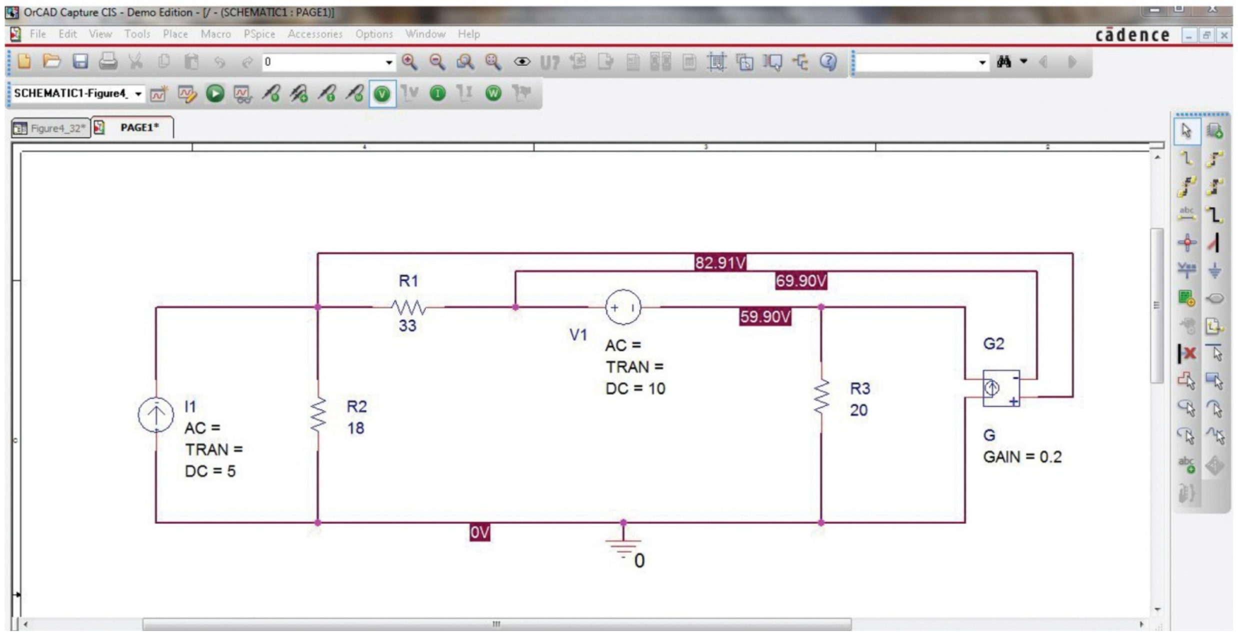


Circuit after simulation run

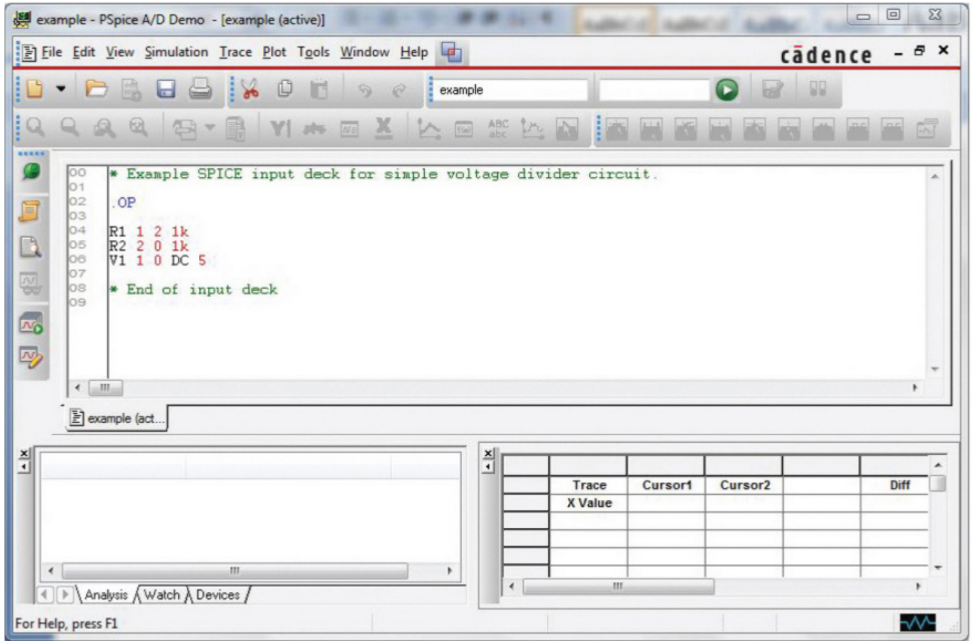
- Circuit simulation example with dependent source



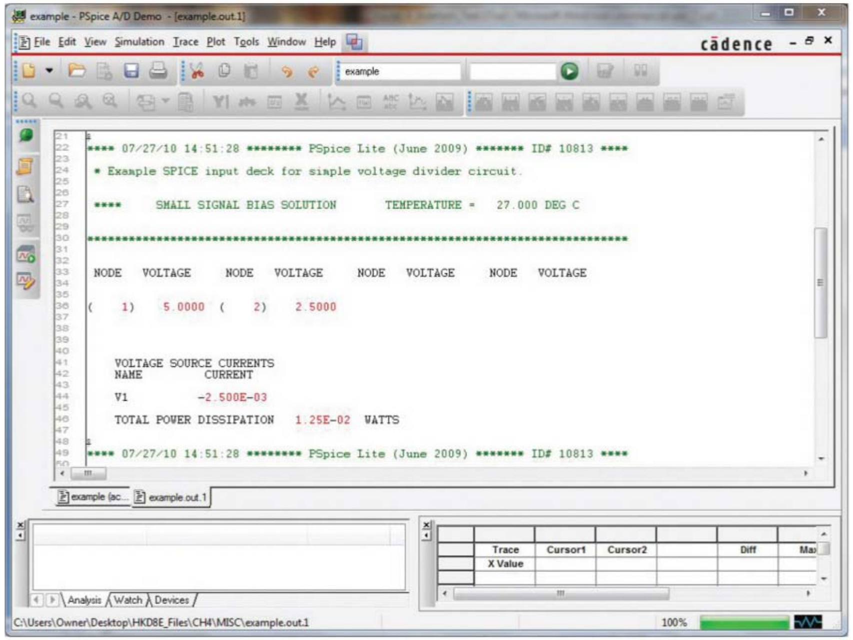
(*a*)



- Node-based Pspice schematic creation (Text-based input format)



Input Deck



Output Deck

Homework : 4장 Exercises 4의 배수 문제 (57번 문제까지)

- Due day : 4장 수업 끝나고 일주일 후 수업시작 전까지 제출.

이 자료에 나오는 모든 그림은 McGraw·hill 출판사로부터 제공받은 그림을 사용하였음.

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