
Chapter 4

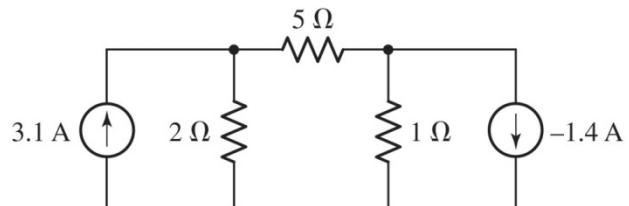
Basic Nodal and Mesh

Analysis

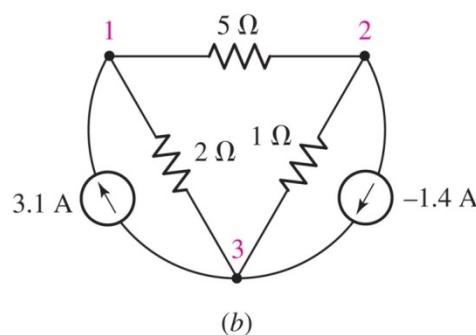
- 4.1 Nodal Analysis
- 4.2 The Supernode
- 4.3 Mesh Analysis
- 4.4 The Supermesh
- 4.5 Nodal vs. Mesh Analysis: A Comparison
- 4.6 Computer-aided Circuit Analysis

- Method

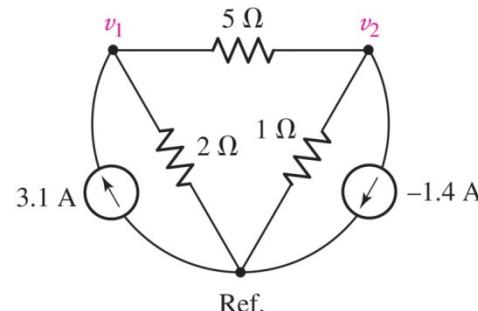
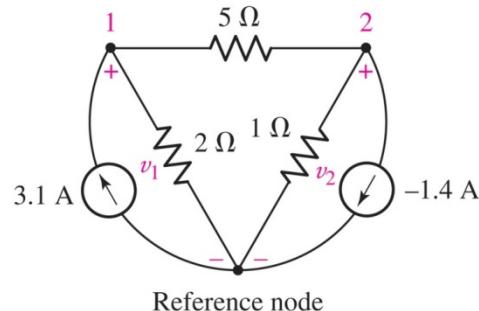
- We select one node in the circuit to be a reference node.
- Node voltage = The voltage of each of the nonreference nodes with respect to the reference node.
- Use Node voltages as unknown variables to set up Independent KCL equations (node equations). Find node voltages.
- We can find all branch voltages and currents if we know node voltages



(a)



(b)



$$3.1 = \frac{v_1}{2} + \frac{v_1 - v_2}{5}$$

$$\rightarrow 3.1 - \frac{v_1}{2} - \frac{v_1 - v_2}{5} = 0$$

$$-(-1.4) = \frac{v_2}{1} + \frac{v_2 - v_1}{5}$$

$$\rightarrow 1.4 - \frac{v_2}{1} - \frac{v_2 - v_1}{5} = 0$$

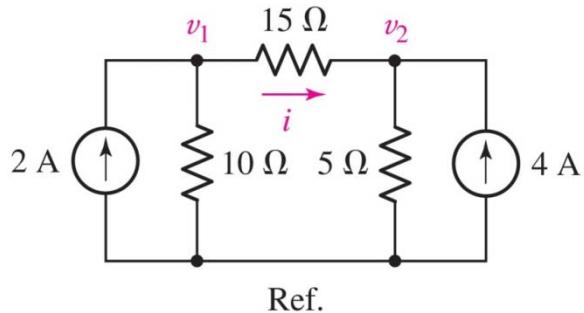


$$v_1 = 5 V, v_2 = 2 V$$

$$v_{5\Omega} = v_1 - v_2 = 3 V$$

Nodal analysis is based on KCL

Example 4.1 Determine the current through 15Ω resistor

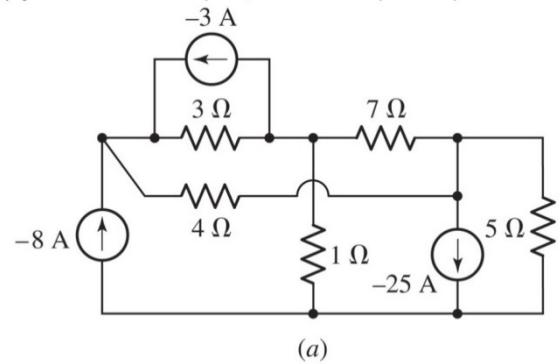


$$\text{Node 1: } 2 = \frac{v_1}{7+3} + \frac{v_1-v_2}{15} \rightarrow 5v_1 - 2v_2 = 60$$

$$\text{Node 2: } 4 = \frac{v_2}{5} + \frac{v_2-v_1}{15} \rightarrow -v_1 + 4v_2 = 60$$

$$\Rightarrow v_1 = 20 V, v_2 = 20 V \Rightarrow i = 0$$

Example 4.2 Determine the nodal voltages as referenced to the bottom node

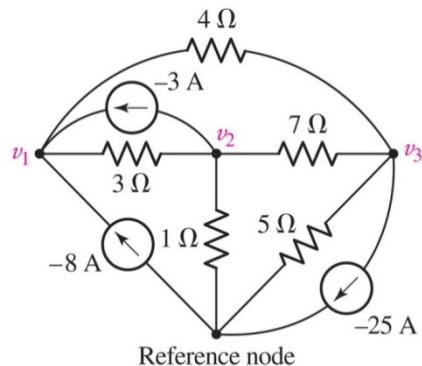


$$\begin{aligned} \text{Node 1: } -8 - 3 &= \frac{v_1-v_2}{3} + \frac{v_1-v_3}{4} \\ &\rightarrow -0.5833v_1 - 0.3333v_2 = -11 \end{aligned}$$

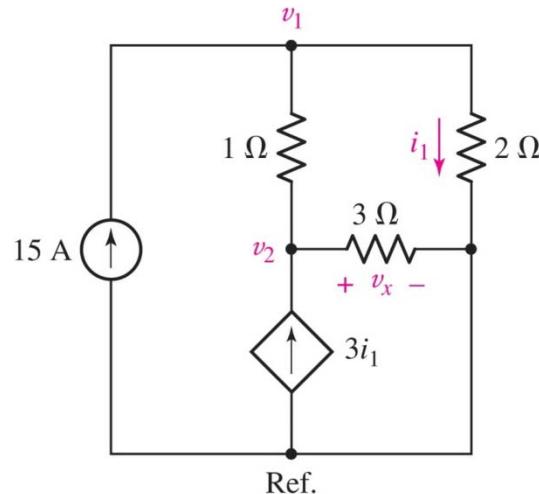
$$\begin{aligned} \text{Node 2: } -(-3) &= \frac{v_2}{1} + \frac{v_2-v_1}{3} + \frac{v_2-v_3}{7} \\ &\rightarrow -0.3333v_1 + 1.4762v_2 - 0.1429v_3 = 3 \end{aligned}$$

$$\begin{aligned} \text{Node 3: } -(-25) &= \frac{v_3}{5} + \frac{v_3-v_1}{4} + \frac{v_3-v_2}{7} \\ &\rightarrow -0.25v_1 - 0.1429v_2 + 0.5929v_3 = 25 \end{aligned}$$

$$\Rightarrow v_1 = 5.412 V, \quad v_2 = 7.736 V, \quad v_3 = 46.32 V$$



Example 4.3 Determine the power supplied by the dependent source



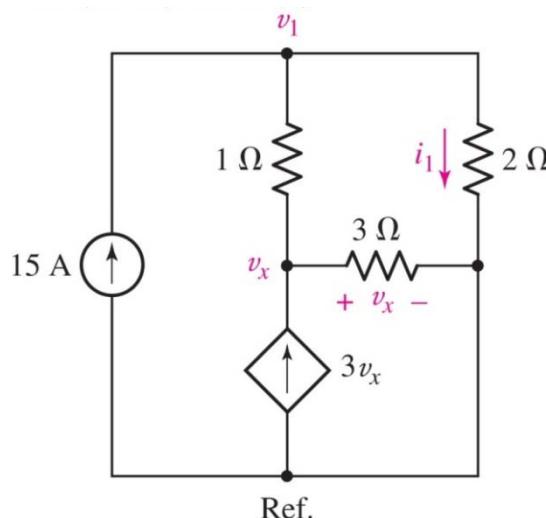
$$\text{Node 1: } 15 = \frac{v_1 - v_2}{1} + \frac{v_1}{2} \rightarrow 3v_1 - 2v_2 = 30$$

$$i_1 = \frac{v_1}{2}$$

$$\text{Node 2: } 3i_1 = \frac{v_2 - v_1}{1} + \frac{v_2}{3} \rightarrow 3 \frac{v_1}{2} = \frac{v_2 - v_1}{1} + \frac{v_2}{3} \rightarrow -15v_1 + 8v_2 = 0$$

$$\begin{aligned} \Rightarrow v_1 &= -40 \text{ V} & p_{3i_1} &= (3i_1)(v_2) \\ v_2 &= -75 \text{ V} & &= (3 \times (-20))(-75) \\ i_1 &= -20 \text{ A} & &= 4500 \text{ W} \end{aligned}$$

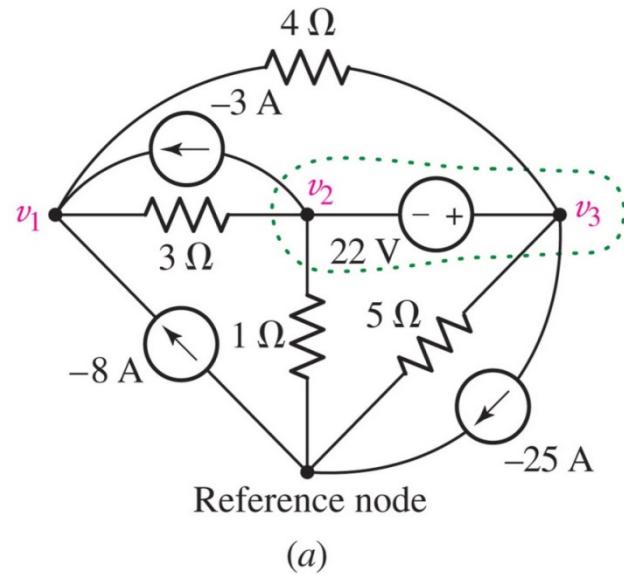
Example 4.4 Determine the power supplied by the dependent source



$$\text{Node 1: } 15 = \frac{v_1 - v_2}{1} + \frac{v_1}{2} \rightarrow 3v_1 - 2v_2 = 30$$

$$\text{Node } x: 3v_x = \frac{v_x - v_1}{1} + \frac{v_x}{3} \rightarrow 5v_x + 3v_1 = 0$$

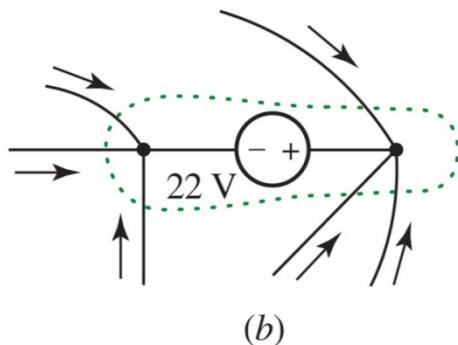
$$\begin{aligned} \Rightarrow v_1 &= \frac{50}{7} \text{ V} & p_{3v_x} &= (3v_x)(v_x) \\ v_x &= -\frac{30}{7} \text{ V} & &= 55.1 \text{ W} \end{aligned}$$



To solve the circuit (a) on the left side, a branch current between v_2 and v_3 should be assigned.
→ Four equations and four unknowns

- Supernode

- A closed region which encloses each voltage source
- KCL is available to the supernode
- All current flowing into the region sum to zero



Example 4.5 Determine the node voltage v_1

$$\text{Node 1 : } -8 - 3 = \frac{v_1 - v_2}{3} + \frac{v_1 - v_3}{4}$$

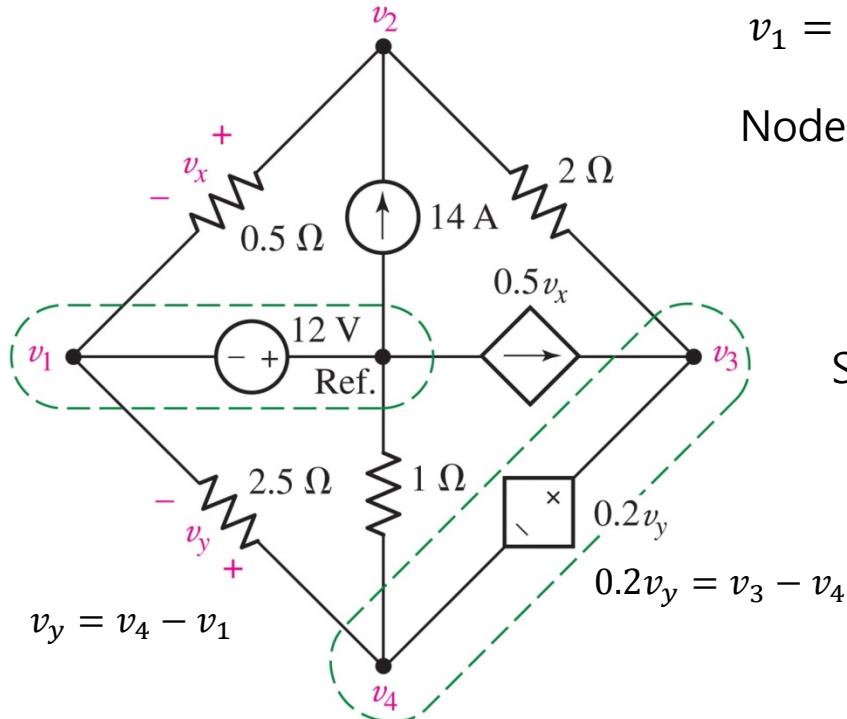
$$\rightarrow -0.5833v_1 - 0.3333v_2 = -11$$

$$\text{Supernode 2-3 : } 3 + 25 = \frac{v_2 - v_1}{3} + \frac{v_3 - v_1}{4} + \frac{v_3}{5} + \frac{v_2}{1}$$

$$\rightarrow -0.5833v_1 + 1.3333v_2 + 0.45v_3 = 28$$

$$v_3 - v_2 = 22 \quad \rightarrow \quad v_1 = 1.071 V$$

Example 4.6 Determine the node-to-reference voltages



$$v_1 = -12$$

$$\begin{aligned} \text{Node 2 : } 14 &= \frac{v_2 - v_1}{0.5} + \frac{v_2 - v_3}{2} \\ \rightarrow -2v_1 + 2.5v_2 - 0.5v_3 &= 14 \\ \rightarrow +2.5v_2 - 0.5v_3 &= -10 \end{aligned}$$

$$\begin{aligned} \text{Supernode 3-4 : } 0.5v_x &= \frac{v_3 - v_2}{2} + \frac{v_4}{1} + \frac{v_4 - v_1}{2.5} \\ \rightarrow 0.1v_1 - v_2 + 0.5v_3 + 1.4v_4 &= 0 \\ \rightarrow -v_2 + 0.5v_3 + 1.4v_4 &= 1.2 \end{aligned}$$

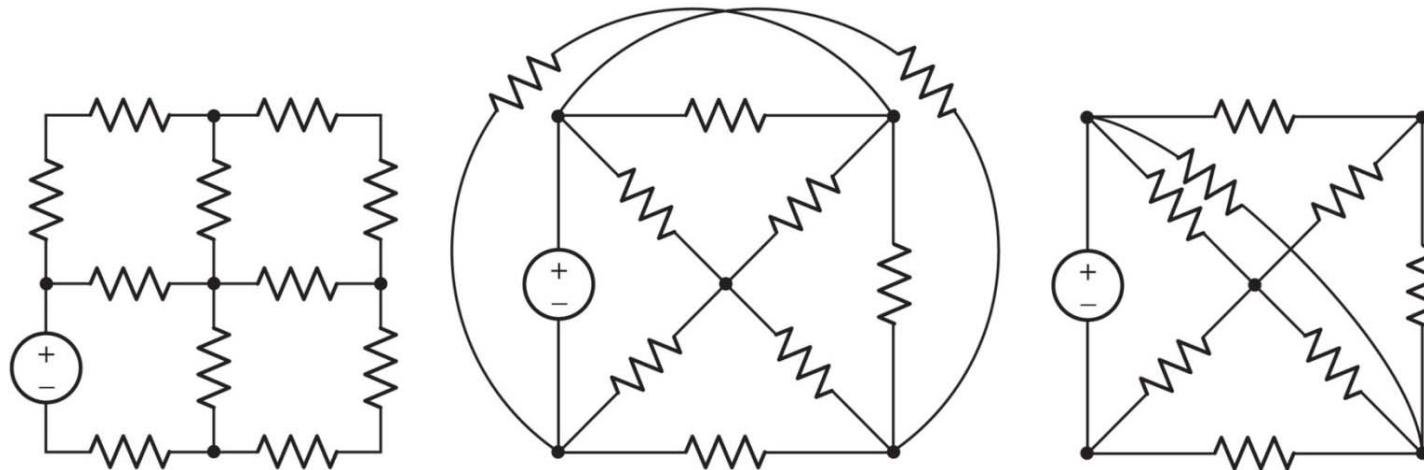
$$v_x = v_2 - v_1$$

$$\begin{aligned} 0.2v_y &= v_3 - v_4 = 0.2(v_4 - v_1) \\ \rightarrow 0.2v_1 + v_3 - 1.2v_4 &= 0 \rightarrow v_3 - 1.2v_4 = 2.4 \end{aligned}$$

$$v_y = v_4 - v_1$$

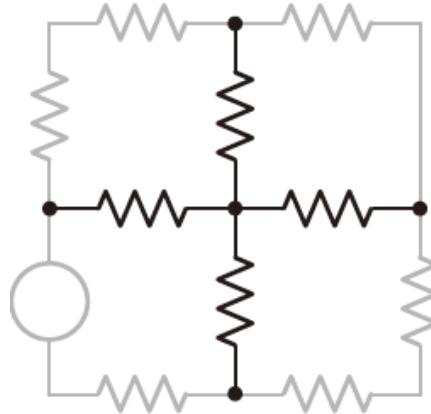
$$\therefore v_1 = -12 V, \quad v_2 = -4 V, \quad v_3 = 0 V, \quad v_4 = -2 V$$

- Loop and mesh
 - Loop : Any closed path in the circuit
 - Mesh : A loop that does not contain any other loops within it
- Mesh Current = The current that flows only around the perimeter of the mesh. (Independent loop current)
- Use Mesh Current as unknown variables to set up Independent KVL equations (mesh equations)
- We can find all branch voltages and currents if we know mesh currents.
- We restrict attention only to **planar circuits** (No branch passes over or under any other branch)

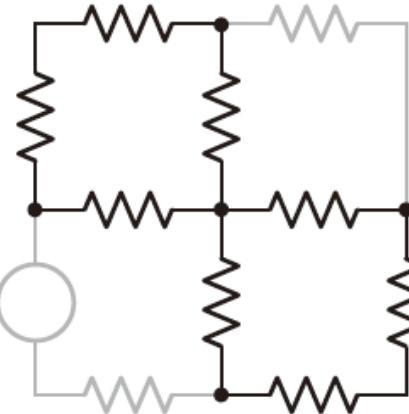


Non-planar circuit

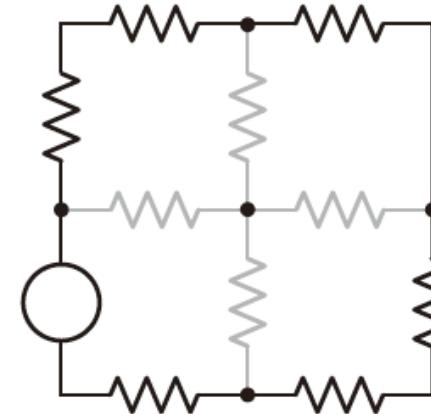
Loop and Mesh



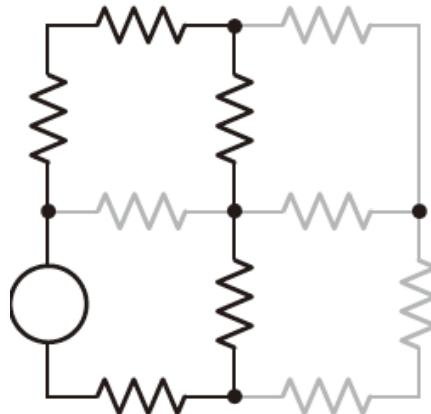
No path, no loop



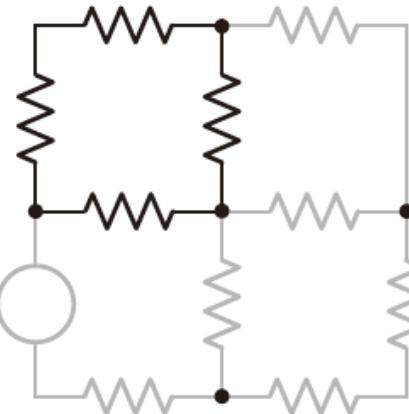
No path, no loop



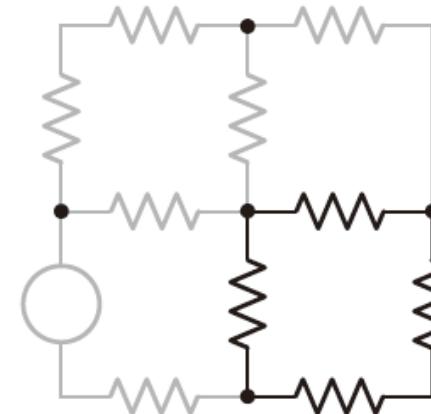
Loop, No mesh



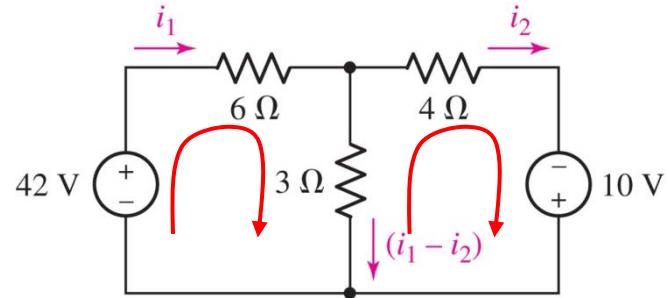
Loop, No mesh



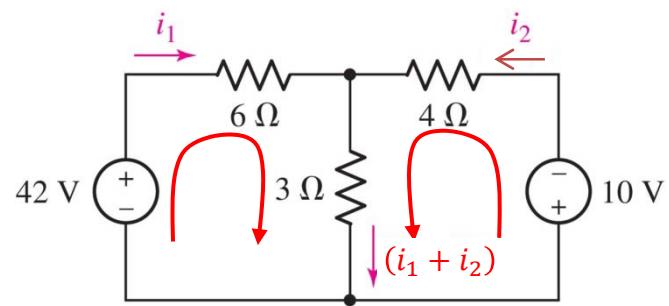
Loop and Mesh



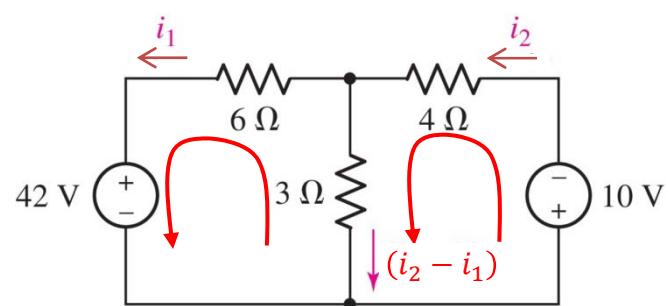
Loop and Mesh



$$\begin{aligned} \text{Loop 1: } & -42 + 6i_1 + 3(i_1 - i_2) = 0 \rightarrow 9i_1 - 3i_2 = 42 \\ \text{Loop 2: } & 3(i_2 - i_1) + 4i_2 - 10 = 0 \rightarrow -3i_1 + 7i_2 = 10 \\ \rightarrow & \quad i_1 = 6 \text{ A}, \quad i_2 = 4 \text{ A}, \quad (i_1 - i_2) = 2 \text{ A} \end{aligned}$$

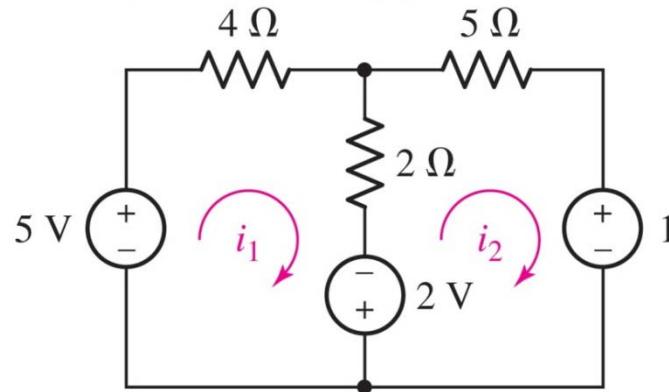


$$\begin{aligned} \text{Loop 1: } & -42 + 6i_1 + 3(i_1 + i_2) = 0 \rightarrow 9i_1 + 3i_2 = 42 \\ \text{Loop 2: } & 10 + 4i_2 + 3(i_1 + i_2) = 0 \rightarrow 3i_1 + 7i_2 = -10 \\ \rightarrow & \quad i_1 = 6 \text{ A}, \quad i_2 = -4 \text{ A}, \quad (i_1 + i_2) = 2 \text{ A} \end{aligned}$$



$$\begin{aligned} \text{Loop 1: } & 3(i_1 - i_2) + 6i_1 + 42 = 0 \rightarrow 9i_1 - 3i_2 = -42 \\ \text{Loop 2: } & 10 + 4i_2 + 3(i_2 - i_1) = 0 \rightarrow -3i_1 + 7i_2 = -10 \\ \rightarrow & \quad i_1 = -6 \text{ A}, \quad i_2 = -4 \text{ A}, \quad (i_2 - i_1) = 2 \text{ A} \end{aligned}$$

Example 4.7 Determine the power supplied by the 2 V source

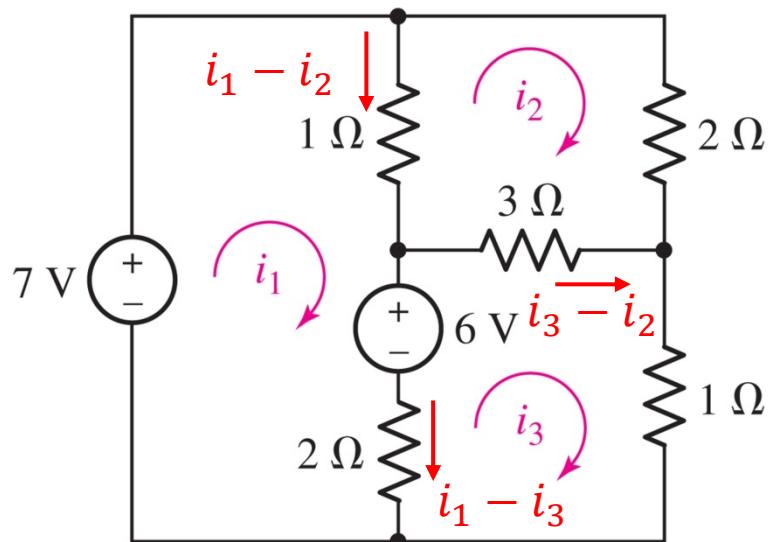


$$\text{Loop 1: } -5 + 4i_1 + 2(i_1 - i_2) - 2 = 0 \rightarrow 6i_1 - 2i_2 = 7$$

$$\text{Loop 2: } 2 + 2(i_2 - i_1) + 5i_2 + 1 = 0 \rightarrow -2i_1 + 7i_2 = -3$$

$$\begin{aligned} \Rightarrow i_1 &= \frac{43}{38} A, i_2 = -\frac{2}{19} A, & p_{2V} &= (2)(i_1 - i_2) \\ &&&= (2)\left(\frac{47}{38}\right) = 2.474 W \end{aligned}$$

Example 4.8 Determine the three mesh currents



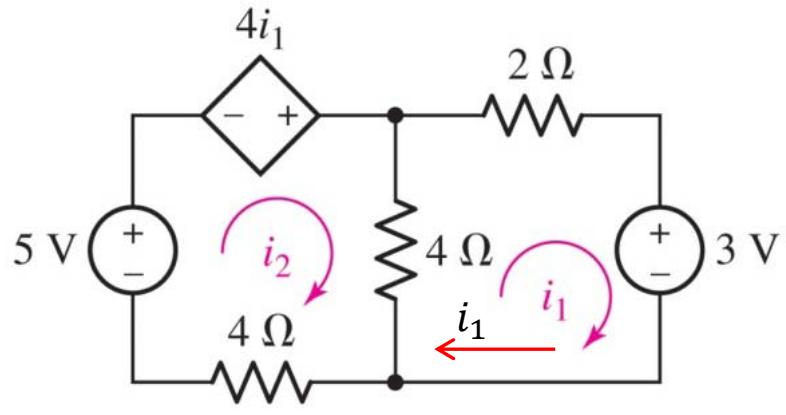
$$\begin{aligned} \text{Loop 1: } -7 + 1(i_1 - i_2) + 6 + 2(i_1 - i_3) &= 0 \\ \rightarrow 3i_1 - i_2 - 2i_3 &= 1 \end{aligned}$$

$$\begin{aligned} \text{Loop 2: } 1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) &= 0 \\ \rightarrow -i_1 + 6i_2 - 3i_3 &= 0 \end{aligned}$$

$$\begin{aligned} \text{Loop 3: } 2(i_3 - i_1) - 6 + 3(i_3 - i_2) + i_3 &= 0 \\ \rightarrow -2i_1 - 3i_2 + 6i_3 &= 6 \end{aligned}$$

$$\Rightarrow i_1 = 3A, i_2 = 2A, i_3 = 3A$$

Example 4.9 Determine the current i_1

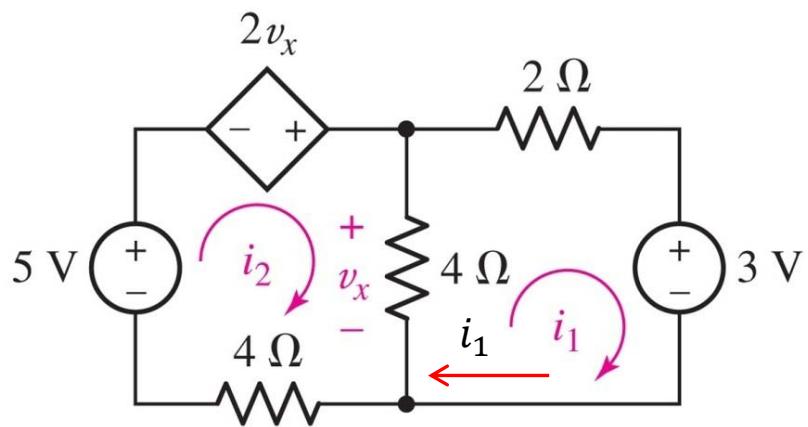


$$\begin{aligned}\text{Loop 1 : } & -5 - 4i_1 + 4(i_2 - i_1) + 4i_2 = 0 \\ \rightarrow & -8i_1 + 8i_2 = 5\end{aligned}$$

$$\begin{aligned}\text{Loop 2 : } & 2i_1 + 3 + 4(i_1 - i_2) = 0 \\ \rightarrow & 6i_1 - 4i_2 = -3\end{aligned}$$

$$\Rightarrow i_1 = -\frac{1}{4} A, i_2 = \frac{3}{8} A$$

Example 4.10 Determine the current i_1



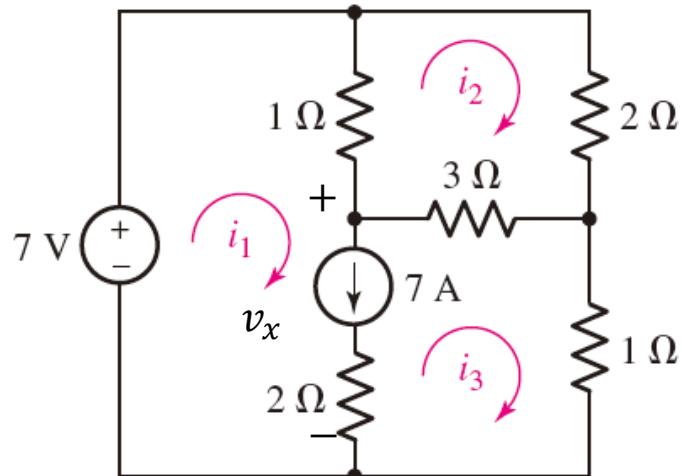
$$\begin{aligned}\text{Loop 1: } & -5 - 2v_x + 4(i_2 - i_1) + 4i_2 = 0 \\ & -5 - 2(4(i_2 - i_1)) + 4(i_2 - i_1) + 4i_2 = 0 \\ \rightarrow & 4i_1 = 5\end{aligned}$$

$$\begin{aligned}\text{Loop 2 : } & 2i_1 + 3 + 4(i_1 - i_2) = 0 \\ \rightarrow & 6i_1 - 4i_2 = -3\end{aligned}$$

$$\Rightarrow i_1 = \frac{5}{4} A$$

- Circuits with current sources
- The current sources reduce the # of simultaneous mesh equations

Example 4.11



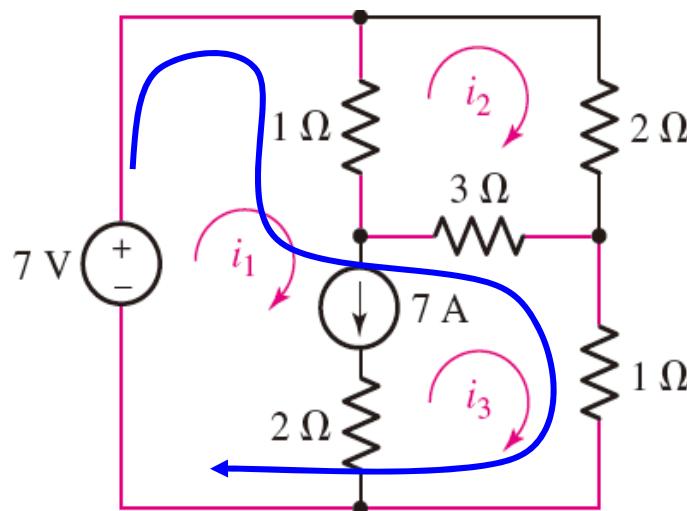
Using conventional method

$$\text{Loop 1: } -7 + 1(i_1 - i_2) + v_x = 0 \rightarrow i_1 - i_2 + v_x = 7$$

$$\begin{aligned} \text{Loop 2: } & 2i_2 + 3(i_2 - i_3) + 1(i_2 - i_1) = 0 \\ & \rightarrow -i_1 + 6i_2 - 3i_3 = 0 \end{aligned}$$

$$\text{Loop 3: } i_3 - v_x + 3(i_3 - i_2) = 0 \rightarrow -3i_2 + 4i_3 - v_x = 0$$

$$i_1 - i_3 = 7 \quad \Rightarrow \quad i_3 = 2 \text{ A}, i_1 = 9 \text{ A}, i_2 = \frac{15}{6} \text{ A}, v_x = 0.5 \text{ V}$$



Using supermesh

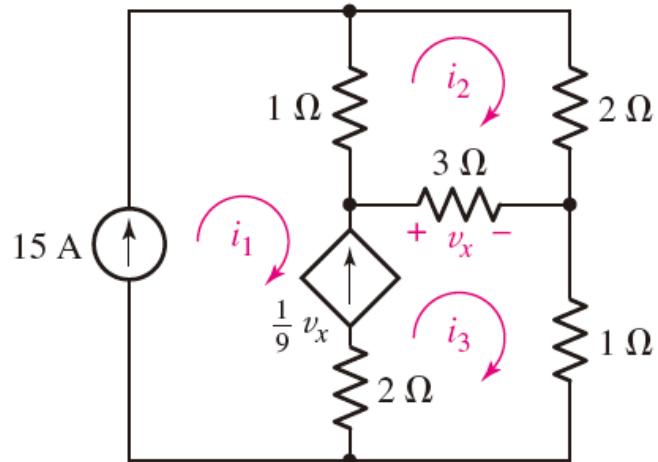
$$\begin{aligned} \text{Loop 1: } & -7 + 1(i_1 - i_2) + 3(i_3 - i_2) + 1i_3 = 0 \\ & \rightarrow i_1 - 4i_2 + 4i_3 = 7 \end{aligned}$$

$$\begin{aligned} \text{Loop 2: } & 1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0 \\ & \rightarrow -i_1 + 6i_2 - 3i_3 = 0 \end{aligned}$$

$$i_1 - i_3 = 7 \quad \Rightarrow \quad i_3 = 2 \text{ A}, i_1 = 9 \text{ A}, i_2 = \frac{15}{6} \text{ A}$$

3 equations and 3 unknowns

Example 4.12 Evaluate the three unknown currents

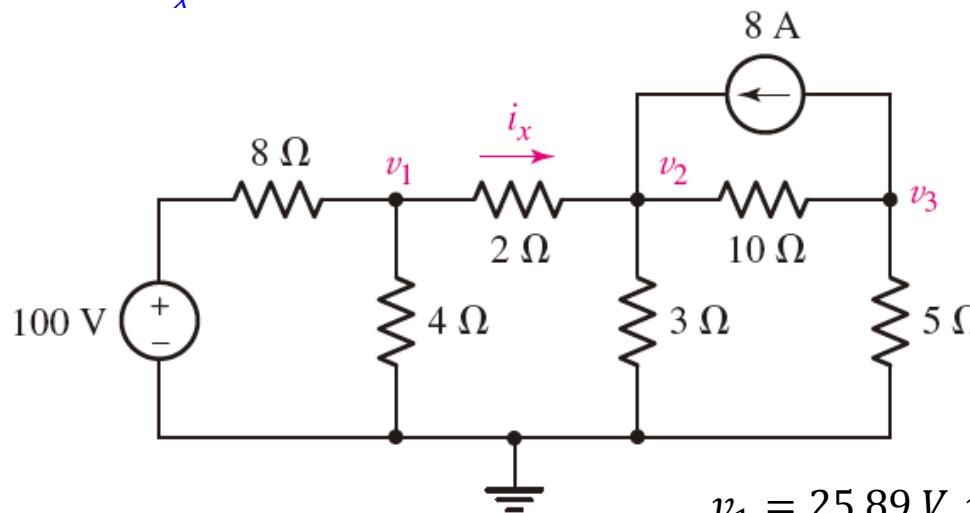


$$\begin{aligned}
 i_1 &= 15 & i_3 - i_1 &= \frac{v_x}{9} = \frac{3(i_3 - i_2)}{9} \quad \leftarrow v_x = 3(i_3 - i_2) \\
 i_3 - 15 &= \frac{1}{3}i_3 - \frac{1}{3}i_2 \quad \rightarrow \frac{1}{3}i_2 + \frac{2}{3}i_3 &= 15 \\
 &\Rightarrow 1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) &= 0 \\
 &\rightarrow -i_1 + 6i_2 - 3i_3 &= 0 \rightarrow 6i_2 - 3i_3 &= 15 \\
 &\Rightarrow i_2 = 11 A, \quad i_3 = 17 A
 \end{aligned}$$

- In choosing nodal or mesh analysis,

- Non-planar circuit → Nodal analysis
- N nodes → N-1 KCL equations (Each supernode reduce the # of equations by 1)
- M meshes → M KVL equations (Each supermesh reduce this number by 1)
- Controlling quantity in dependent sources : nodal voltage → nodal analysis
mesh current → mesh analysis
- Location of source : current source in mesh line → mesh analysis
voltage source connected to ref. → nodal analysis
- Asking current → mesh analysis, asking voltage → nodal analysis

Find i_x

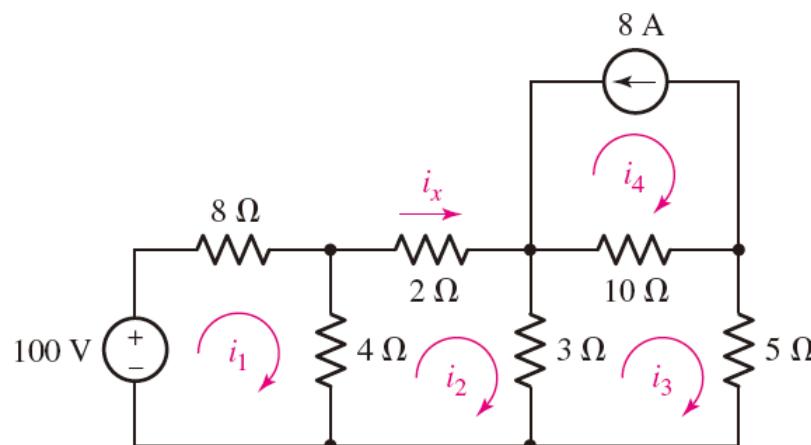


$$\text{Node 1: } \frac{100 - v_1}{8} = \frac{v_1}{4} + \frac{v_1 - v_2}{2} (= -i_x) \\ \rightarrow 0.875v_1 - 0.5v_2 = 12.5$$

$$\text{Node 2: } \frac{v_1 - v_2}{2} = \frac{v_2}{3} + \frac{v_2 - v_3}{10} - 8 \\ \rightarrow -0.5v_1 + 0.933v_2 - 0.1v_3 = 8$$

$$\text{Node 3: } -8 = \frac{v_3}{5} + \frac{v_3 - v_2}{10} \\ \rightarrow -0.1v_2 + 0.3v_3 = -8$$

$$v_1 = 25.89 \text{ V}, v_2 = 20.31 \text{ V} \quad \Rightarrow \quad i_x = \frac{v_1 - v_2}{2} = 2.79 \text{ A}$$



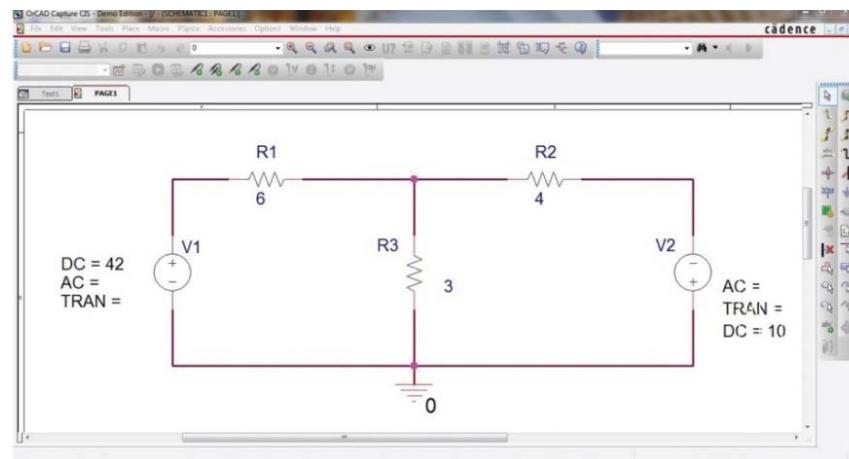
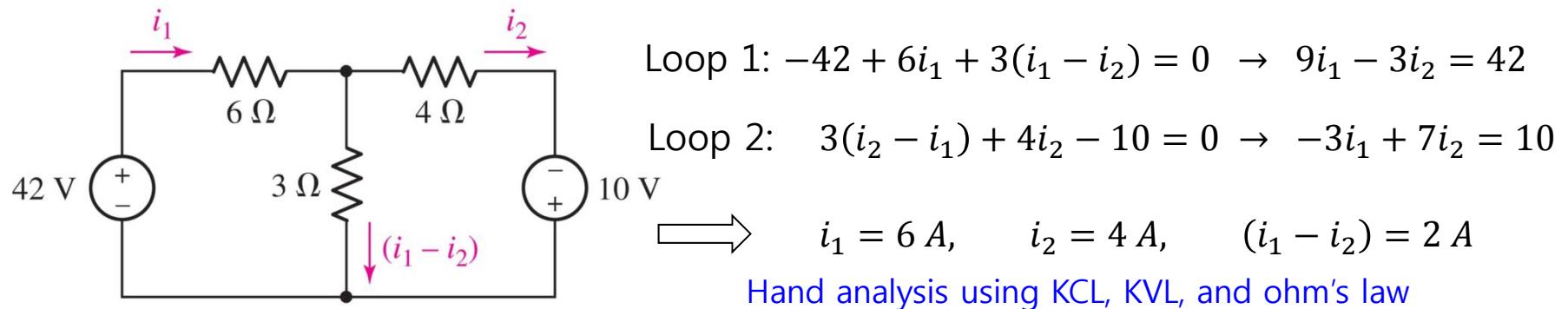
$$\text{Loop 1: } -100 + 8i_1 + 4(i_1 - i_2) = 0 \\ \rightarrow 12i_1 - 4i_2 = 100$$

$$\text{Loop 2: } 2i_2 + 3(i_2 - i_3) + 4(i_2 - i_1) = 0 \\ \rightarrow -4i_1 + 9i_2 - 3i_3 = 0$$

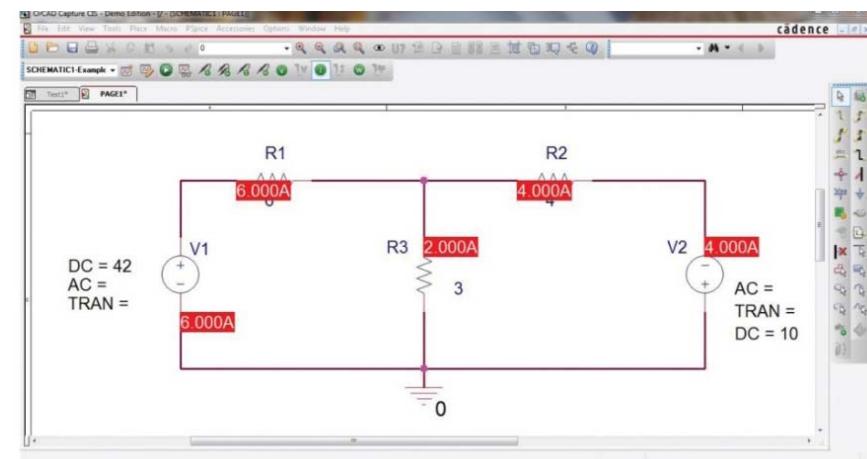
$$\text{Loop 3: } 3(i_3 - i_2) + 10(i_3 - (-8)) + 5i_3 = 0 \\ \rightarrow -3i_2 + 18i_3 = -80$$

$$\Rightarrow i_x = i_2 = 2.79 \text{ A}$$

- To analyze a complex circuit, a computer software package is useful
- SPICE (Simulation Program with Integrated Circuit Emphasis)
- Pspice : SPICE with intuitive graphical interface by MicroSim Corp.

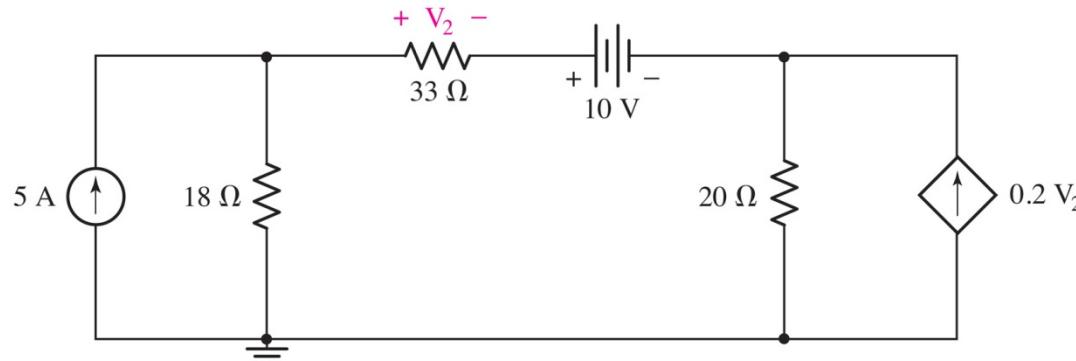


Orcad schematic capture software

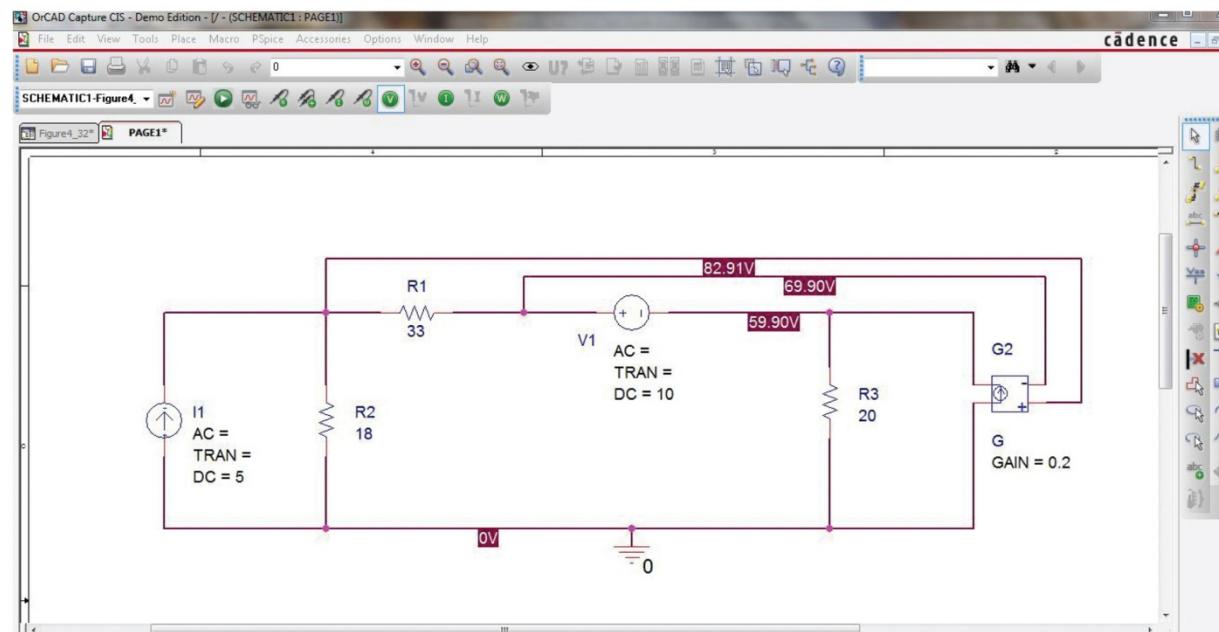


Circuit after simulation run

- Circuit simulation example with dependent source



(a)



- Node-based Pspice schematic creation (Text-based input format)

```

00 * Example SPICE input deck for simple voltage divider circuit.
01
02 .OP
03
04 R1 1 2 1k
05 R2 2 0 1k
06 V1 1 0 DC 5
07
08 * End of input deck
09

```

The screenshot shows the Cadence PSpice A/D Demo interface. The main window displays the SPICE input deck for a simple voltage divider circuit. The input deck consists of lines 00 through 09, defining components R1, R2, and V1, and ending with a comment line. Below the input deck, there are two smaller windows: one for analysis results and another for cursor data.

Input Deck

```

21 **** 07/27/10 14:51:28 ***** PSpice Lite (June 2009) ***** ID# 10813 ****
22
23 * Example SPICE input deck for simple voltage divider circuit.
24
25
26
27
28
29
30 **** SMALL SIGNAL BIAS SOLUTION TEMPERATURE = 27.000 DEG C
31
32
33
34
35 NODE VOLTAGE NODE VOLTAGE NODE VOLTAGE NODE VOLTAGE
36 ( 1) 5.0000 ( 2) 2.5000
37
38
39
40
41 VOLTAGE SOURCE CURRENTS
42 NAME CURRENT
43 V1 -2.500E-03
44
45 TOTAL POWER DISSIPATION 1.25E-02 WATTS
46
47
48
49 **** 07/27/10 14:51:28 ***** PSpice Lite (June 2009) ***** ID# 10813 ****
50

```

The screenshot shows the Cadence PSpice A/D Demo interface with the output deck displayed. The output deck includes header information, component values, bias solution, voltage source currents, and total power dissipation. Below the output deck, there are two smaller windows: one for analysis results and another for cursor data.

Output Deck

Homework : 4장 Exercises 4의 배수 문제 (57번 문제까지)

- Due day : 4장 수업 끝나고 일주일 후 수업시작 전까지 제출.

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