경제수학 제 10 장

적분법

적분법 (Integration)

- Integration: the reverse process to differentiation
- Operating the two procedures successively does not always immediately bring you back to exactly the function you started with

Integration Notation

- If $18x^5$ is to be integrated with respect to x we show this by writing

$$\int 18x^5 \, \mathrm{d}x$$

- In general, we write the integral of f(x) with respect to x

$$\mathrm{F}(x) = \int \mathrm{f}(x) \, \mathrm{d}x$$

To Integrate a Power Function

- Add 1 to the power and divide by the new power

- Always check your integration by differentiating your answer

Constant of Integration

- Constants differentiate to zero
- As we reverse the differentiation procedure there is no way of immediately discovering whether there should be a constant in our answer, or what value it should have
- To recognize the possibility of constant term, as we integrate we add c to the expression for the integral
- c is called the constant of integration

Value of the Constant of Integration

- Sometimes there is extra information in the question from which the value of c can be deduced
- The constant does have an economic interpretation
- Example:

  When you integrate marginal cost to find total cost, the constant is fixed cost

Integral of a Power Function

$$\int \mathrm{a}x^{\mathrm{n}}dx = \frac{\mathrm{a}}{\mathrm{n}+1}x^{\mathrm{n}+1} + \mathrm{c}$$

Integral of x^{-1}

- The power-function rule does not work for x^{-1}
- Raising -1 by 1 would make it zero, which causes a problem as we try to divide by it
- Remember if $y = \log_e x$

$$\frac{dy}{dx} = \frac{1}{x} = x^{-1}$$

$$\int x^{-1}\ dx = \int \frac{1}{x}\ dx = \log_e x + c$$

Alternative Expression for Integral of x^{-1}

- If we write $c = \log_e$ m, where m is just another constant

$$\int x^{-1}\ \mathrm{d}x = \int \frac{1}{x}\,\mathrm{d}x = \log_e x + \log_e \mathrm{m}$$

and by the rules of logarithms

$$\log_e x + \log_e \mathrm{m} = \log_e \mathrm{m}x$$

so

$$\int \frac{1}{x}\,\mathrm{d}x = \log_e \mathrm{m}x$$

Integral of an Exponential Function

$$\int e^{\mathrm{m}x}\ \mathrm{d}x = \frac{1}{\mathrm{m}} e^{\mathrm{m}x} + \mathrm{c}$$

Integral of a Sum or Difference

- Integrate the terms of a sum separately and add

$$\int [f(x) + g(x)]\, dx = \int f(x)\, dx + \int g(x)\, dx$$

- Integrate the terms of a difference separately and then subtract

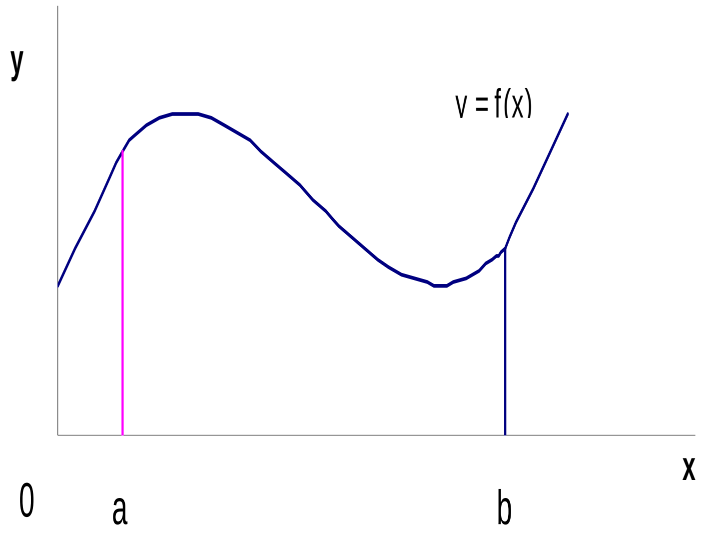
$$\int [f(x) - g(x)]\, dx = \int f(x)\, dx - \int g(x)\, dx$$

Finding Total Functions from Marginal Functions

- We integrate a marginal function to find the corresponding total function
- The constant of integration represents the constant term in the total function
- Integrating marginal revenue gives total revenue
- The total revenue function takes the value zero at zero output and so the constant of integration is zero
- Total revenue has no constant term

Area Under a Curve

- Economic analysis based on diagrams often involves a comparison of areas
- Definite integration allows us to find the size of the area lying under the curve between two values of x, points a and b



정적분 (Definite Integration)

- Integrating f(x) gives F(x) = $\int$ f(x) dx
- Substitute the higher x value, b, which gives F(b)
- Substitute the lower x value, a, to find F(a)
- The difference between these values

  F(b) – F(a) is the value of the definite integral between the limits a and b
- This is the size of the area under the curve f(x) between a and b

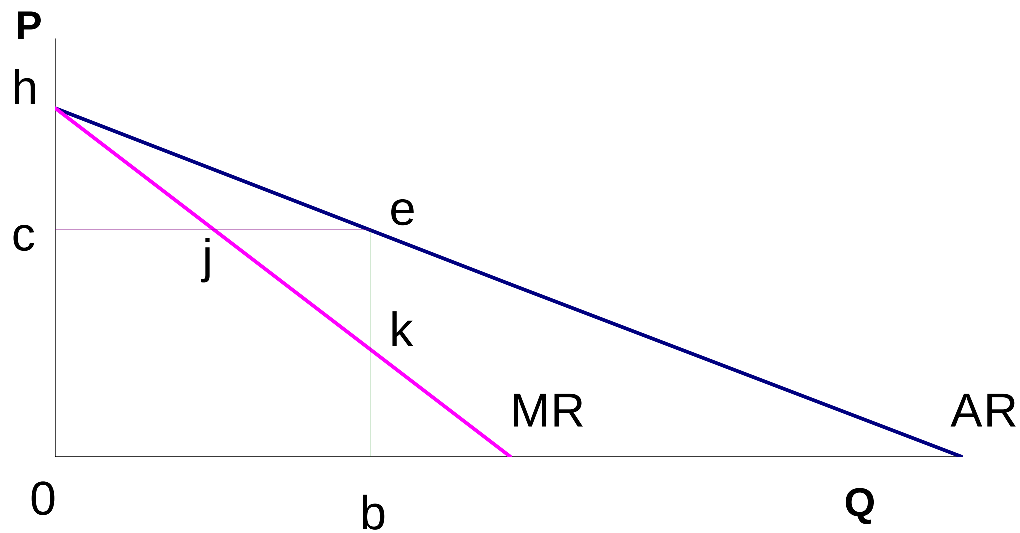
Notation for Definite Integration

- The area under the curve f(x) between a and b is given by

$$\int_a^b \mathrm{f}(x)\,\mathrm{d}x = \mathrm{F}(b) - \mathrm{F}(a)$$

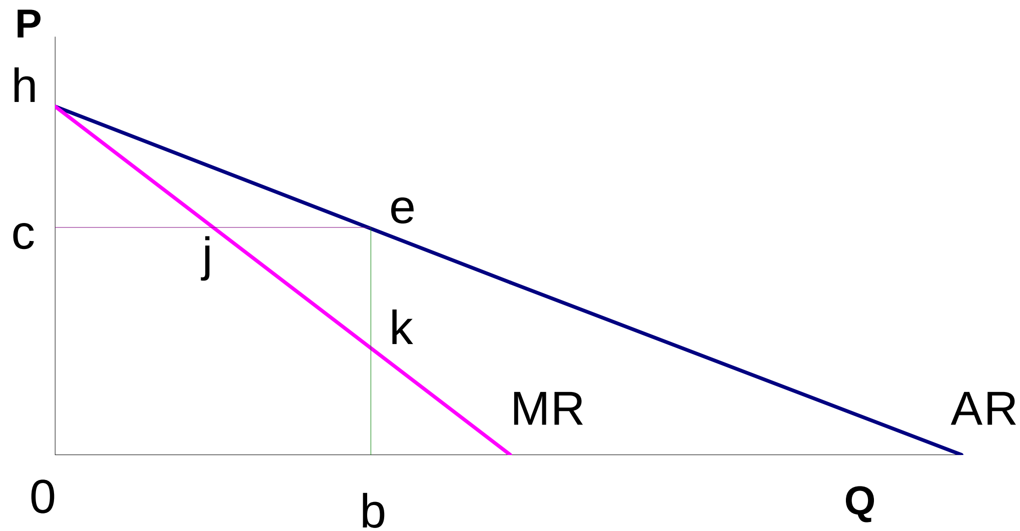
- a and b are called the limits of integration and are shown against the integral sign
- In our examples, the lower limit, a, is always smaller than the upper limit, b, and also the function f(x) lies above the x axis

Total Revenue = Price*Quantity



- For output b, price = c
- total revenue is the area of the rectangle Oceb

Total Revenue = Area under MR



- $\mathrm{TR} = \int_0^b \mathrm{MR} \;\, \mathrm{d}Q$
- TR is represented as the area under the marginal revenue curve from the origin to b
- TR = area Ohkb