경제수학 제 7 장

고급 미분법

Chain Rule

- If $y = \mathrm{f}(u)$ where $u = \mathrm{g}(x)$
- $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} . \frac{\mathrm{d}u}{\mathrm{d}x}$
- *Chain rule*: multiply the derivative of the outer function by the derivative of the inner function

Ex) Find dy/dx when $y = u^5$ and $u = 1 - x^3$.

Ex) Find dy/dx when $y = \dfrac{10}{(x^2 + 4x + 5)^7}$. (Hint: If $u = x^2 + 4x + 5$, then $y = 10u^{-7}$)

Ex) Find dy/dx when $y = \left(\dfrac{x-1}{x+3}\right)^{1/3}$

Ex) Find dy/dx when $y = \sqrt{x^2 + 1}$

Product Rule

- If $y = \mathrm{f}(x)\mathrm{g}(x)$
- $u = \mathrm{f}(x)$, $v = \mathrm{g}(x)$

- $\frac{dy}{dx} = v.\ \frac{du}{dx} + u.\ \frac{dv}{dx}$

- *Product rule*: the derivative of the first term times the second plus the derivative of the second term times the first

Ex) Compute $\dfrac{d}{dx}\left(3x^8 + \dfrac{x^{100}}{100}\right)$

Ex) Find $h'(x)$ when $h(x) = (x^3 - x)(5x^4 + x^2)$

Quotient rule

- If $y = \mathrm{f}(x)/\mathrm{g}(x)$
- $u = \mathrm{f}(x)$, $v = \mathrm{g}(x)$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v.\frac{\mathrm{d}u}{\mathrm{d}x} - u.\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$$

- *Quotient rule*: the derivative of the first term times the second minus the derivative of the second term times the first, all divided by the square of the second term

Ex) Compute $F'(x)$ and $F'(4)$ when $F(x) = \dfrac{3x-5}{x-2}$

Ex) Let $C(x)$ be the total cost of producing x units of a commodity. Then $C(x)/x$ s the average cost of producing x units. Find an expression for $\dfrac{d}{dx}\left[C(x)/x\right]$.

Problems)

1. Find the derivative of the following:

(a) $y = \frac{3}{5}x^2 - 2x^7 + \frac{1}{8} - \sqrt{x}$ (b) $y = \left(x^5 + \frac{1}{x}\right)(x^5 + 1)$ (c) $y = x^{-1}(x^2 + 1)\sqrt{x}$ (d) $y = \frac{(x+1)(x-1)}{(x^2+2)(x+3)}$

(e) $y = \frac{1}{2} + \frac{1}{3}\left(\frac{x-1}{x+1}\right)(1 + x^{-2})$

2. If $D(P)$ denotes the demand for a product when the price per unit is P , then the revenue function $R(P)$ is given by $R(P) = PD(P)$. Find an expression for $R'(P)$.

Marginal Revenue, Price Elasticity and Maximum Total Revenue

- For any demand curve, given that E is point price elasticity of demand and is negative

$$\text{MR} = P\left(1 + \frac{1}{E}\right)$$

and maximum total revenue occurs when $E = -1$

Optimal Production and Cost Relationships

- Maximum output occurs where $\mathrm{d}Q/\mathrm{d}L = 0$
- A firm operating in perfectly competitive product and labor markets:
  - has short-run marginal cost curve MC = W/MPL where MPL is the marginal product of labor and W is the wage rate
  - to maximize profits, it employs labor until

    $$\mathrm{MVP} = W$$

    where P is the price of its product and

    MVP = P.MPL is the marginal value product of labor

Marginal and average cost

- MC is below AC before a minimum turning point of AC
- At the turning point of AC, MC intersects AC from below

Exponential Functions

- For the exponential function $y = \mathrm{e}^{x}$
- $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{x}$
- More generally we can write the rule as shown below:
- For the exponential function $y = \mathrm{ae}^{\mathrm{m}x}$
- $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{mae}^{\mathrm{m}x}$

Ex) Find the derivative of the following:

(a) $y = e^{-x}$ (b) $y = x^p e^{ax}$ (c) $y = \sqrt{e^{2x} + x}$ (d) $y = \dfrac{e^x}{x}$ (e) $y = x^4 e^{-2x}$ (f) $y = xe^{-\sqrt{x}}$

Natural Logarithmic Functions 1

- If $y = \log_e x$

$$\frac{dy}{dx} = x^{-1}$$

Ex) Compute y' of the following:

(a) $y = x^3 + \ln x$ (b) $y = x^2 \ln x$ (c) $y = \ln x / x$

Natural Logarithmic Functions 2

- More generally: if $y = \log_e mx$

$$\frac{dy}{dx} = \frac{1}{x} = x^{-1}$$

- and if $y = \log_e ax^m$

$$\frac{dy}{dx} = \frac{m}{x}$$

Ex) Compute y' of the following:

(a) $y = \ln(1 - x)$ (b) $y = \ln(4 - x^2)$ (c) $y = \ln\left(\dfrac{x-1}{x+1}\right) - \dfrac{1}{4}x$