

제 5 장
생산공정과 비용
The Production Process and Costs



생산분석

Production Analysis

- Production Function 생산함수
 - $Q = F(K,L)$
 - The *maximum* amount of output that can be produced with K units of capital and L units of labor (technical efficiency).
- Short-Run vs. Long-Run Decisions
- Fixed vs. Variable Inputs

총생산

Total Product

- Cobb-Douglas Production Function

- Example: $Q = F(K,L) = K^{.5} L^{.5}$

- K is fixed at 16 units.

- Short run production function:

$$Q = (16)^{.5} L^{.5} = 4 L^{.5}$$

- Production when 100 units of labor are used?

$$Q = 4 (100)^{.5} = 4(10) = 40 \text{ units}$$

한계생산성의 측정

Marginal Productivity Measures

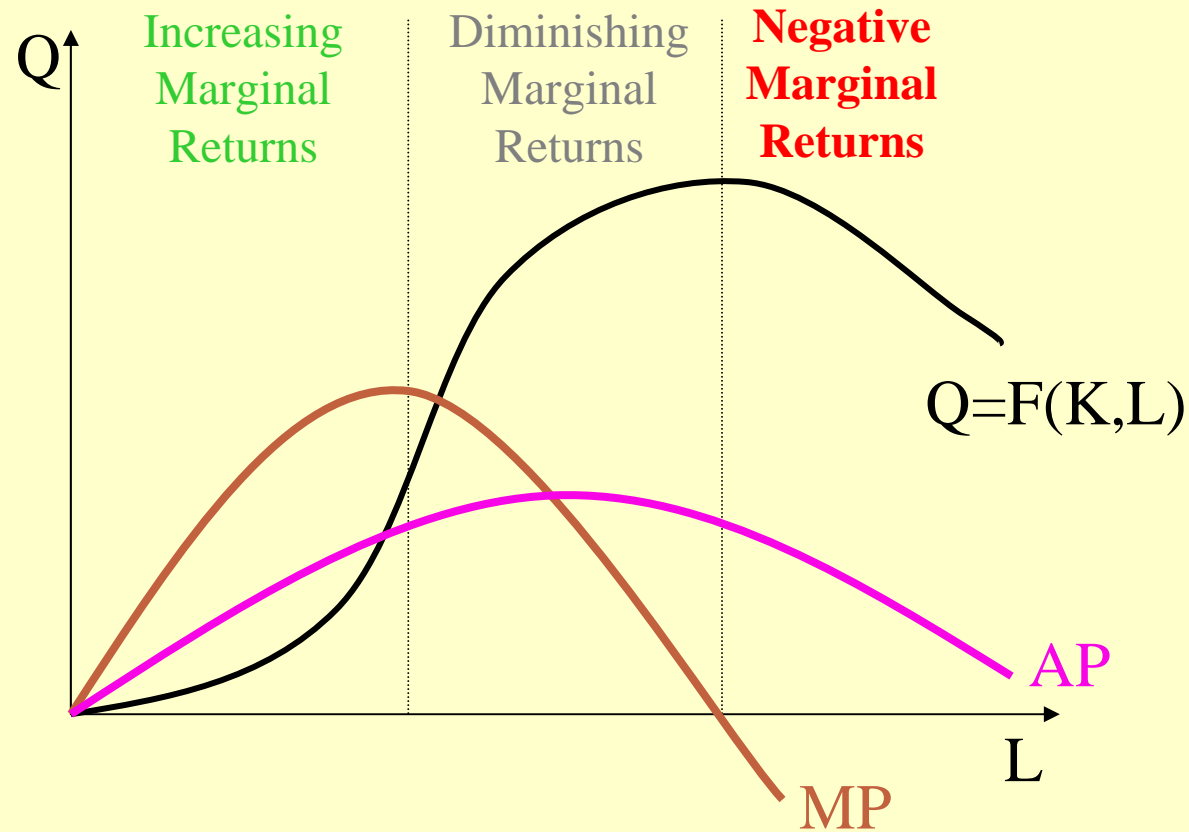
- Marginal Product of Labor: $MP_L = \Delta Q / \Delta L$
 - Measures the output produced by the last worker.
 - Slope of the short-run production function (with respect to labor).
- Marginal Product of Capital: $MP_K = \Delta Q / \Delta K$
 - Measures the output produced by the last unit of capital.
 - When capital is allowed to vary in the short run, MP_K is the slope of the production function (with respect to capital).

평균생산성의 측정

Average Productivity Measures

- Average Product of Labor
 - ☐ $AP_L = Q/L$.
 - ☐ Measures the output of an “average” worker.
 - ☐ Example: $Q = F(K,L) = K^{.5} L^{.5}$
 - If the inputs are $K = 16$ and $L = 16$, then the average product of labor is $AP_L = [(16)^{0.5}(16)^{0.5}]/16 = 1$.
- Average Product of Capital
 - ☐ $AP_K = Q/K$.
 - ☐ Measures the output of an “average” unit of capital.
 - ☐ Example: $Q = F(K,L) = K^{.5} L^{.5}$
 - If the inputs are $K = 16$ and $L = 16$, then the average product of labor is $AP_L = [(16)^{0.5}(16)^{0.5}]/16 = 1$.

Increasing, Diminishing and Negative Marginal Returns



생산공정에의 적용

Guiding the Production Process

- Producing on the production function
 - Aligning incentives to induce maximum worker effort.
- Employing the right level of inputs
 - When labor or capital vary in the short run, to maximize profit a manager will hire
 - labor until the value of marginal product of labor equals the wage: $VMP_L = w$, where $VMP_L = P \times MP_L$.
 - capital until the value of marginal product of capital equals the rental rate: $VMP_K = r$, where $VMP_K = P \times MP_K$.

等量線

Isoquant Curve

- The combinations of inputs (K, L) that yield the producer the same level of output.
- The shape of an isoquant reflects the ease with which a producer can substitute among inputs while maintaining the same level of output.

한계기술대체율

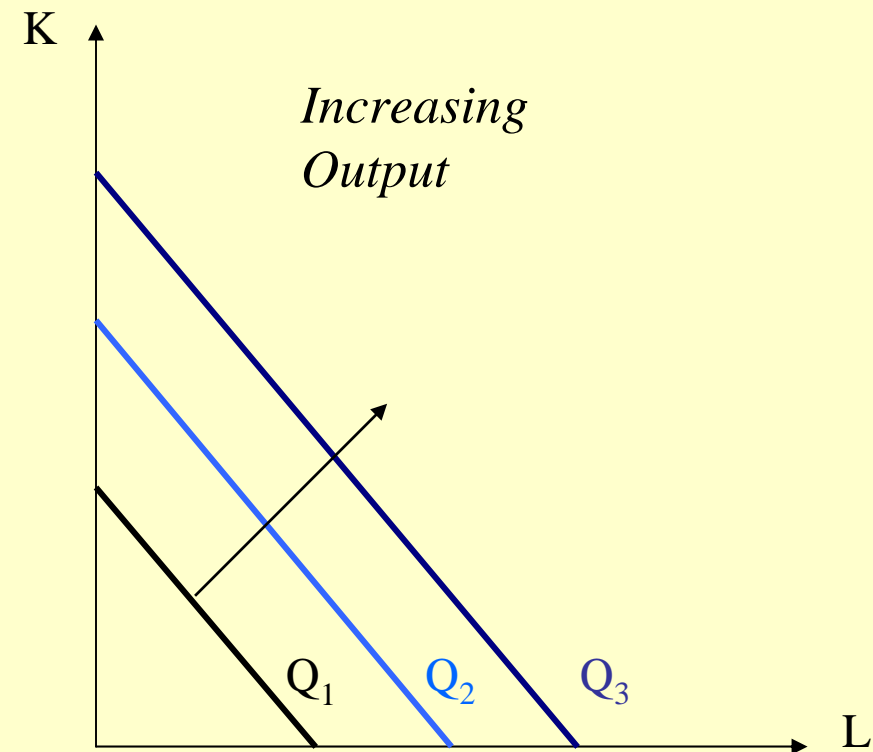
Marginal Rate of Technical Substitution (MRTS)

- The rate at which two inputs are substituted while maintaining the same output level.

$$MRTS_{KL} = \frac{MP_L}{MP_K}$$

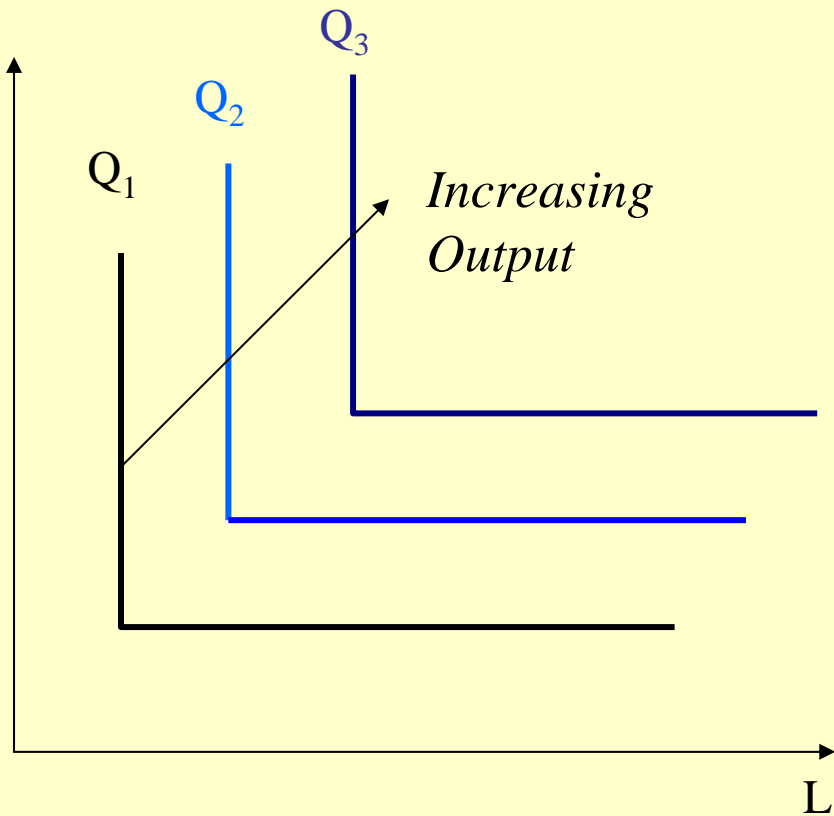
Linear Isoquants

- Capital and labor are perfect substitutes
 - $Q = aK + bL$
 - $MRTS_{KL} = b/a$
 - Linear isoquants imply that inputs are substituted at a constant rate, independent of the input levels employed.



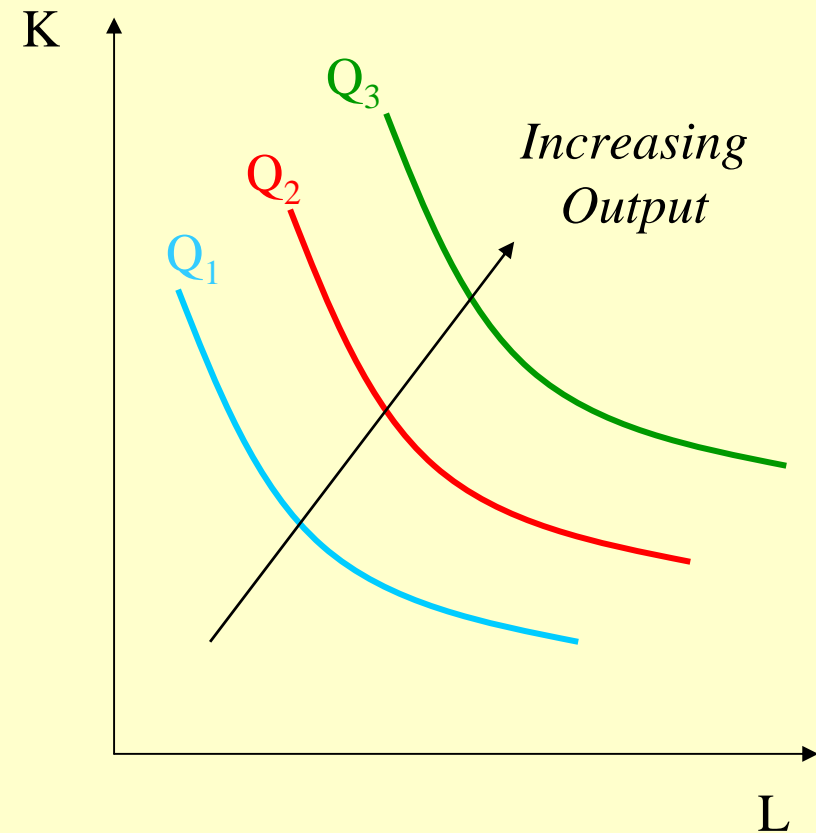
Leontief Isoquants

- Capital and labor are perfect complements.
- Capital and labor are used in fixed-proportions.
- $Q = \min \{bK, cL\}$
- Since capital and labor are consumed in fixed proportions there is no input substitution along isoquants (hence, no $MRTS_{KL}$).



Cobb-Douglas Isoquants

- Inputs are not perfectly substitutable.
- Diminishing marginal rate of technical substitution.
 - As less of one input is used in the production process, increasingly more of the other input must be employed to produce the same output level.
- $Q = K^a L^b$
- $MRTS_{KL} = MP_L / MP_K$



등비용선

Isocost Curve

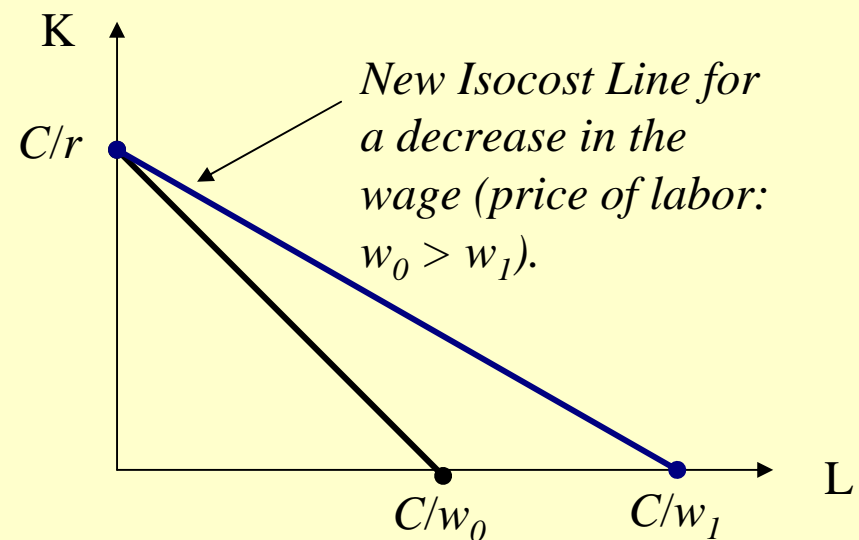
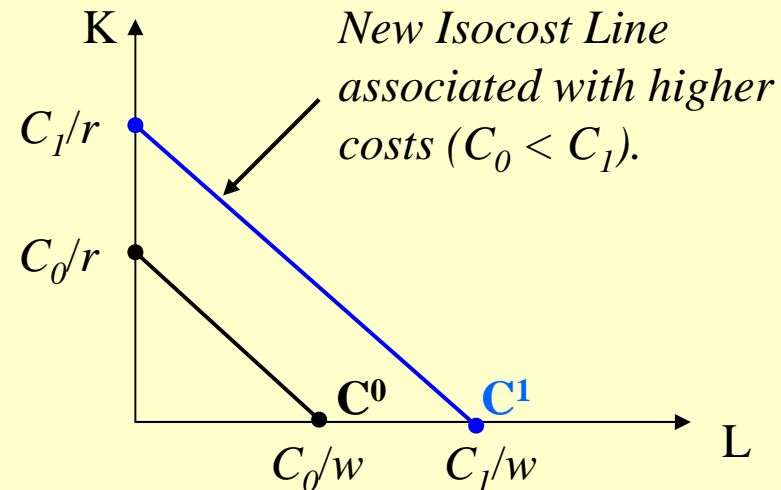
- The combinations of inputs that produce a given level of output at the same cost:

$$wL + rK = C$$

- Rearranging,

$$K = (1/r)C - (w/r)L$$

- For given input prices, isocosts farther from the origin are associated with higher costs.
- Changes in input prices change the slope of the isocost line.



비용최소화 Cost Minimization

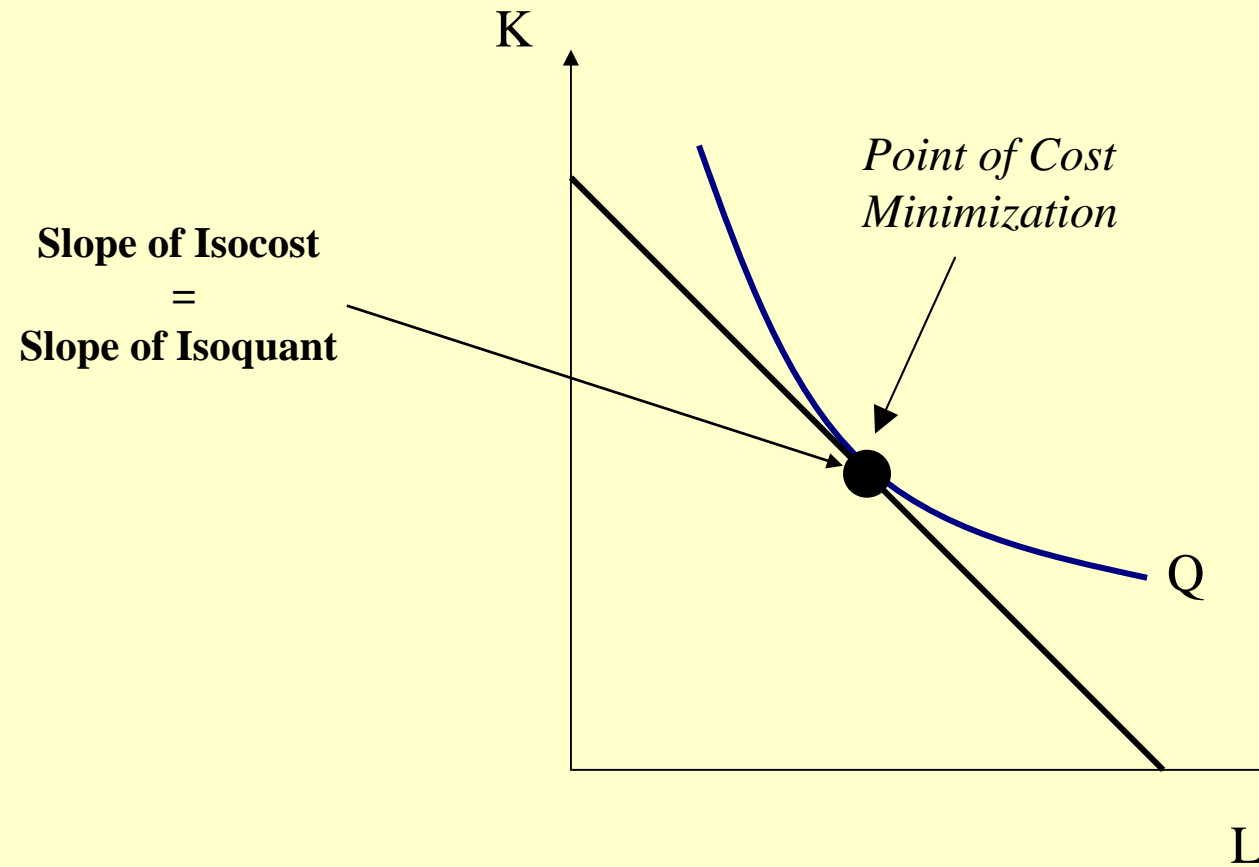
- Marginal product per dollar spent should be equal for all inputs:

$$\frac{MP_L}{w} = \frac{MP_K}{r} \Leftrightarrow \frac{MP_L}{MP_K} = \frac{w}{r}$$

- But, this is just

$$MRTS_{KL} = \frac{w}{r}$$

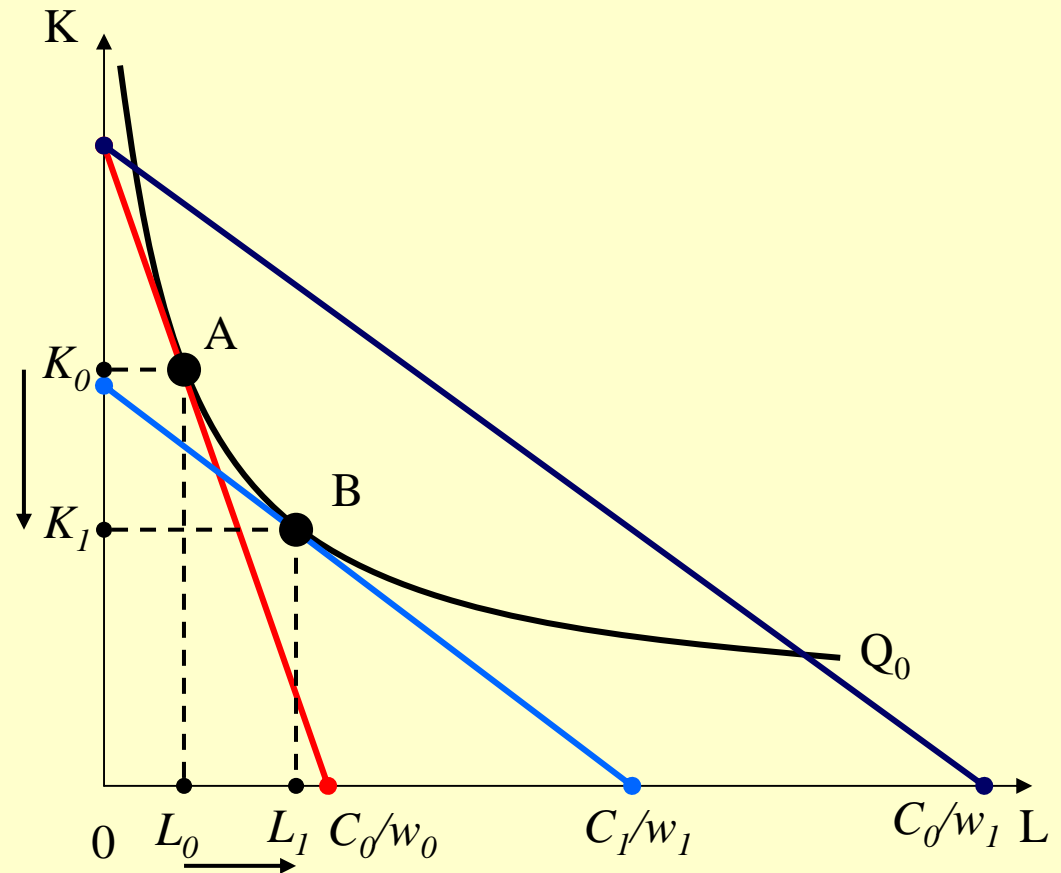
Cost Minimization



최적요소대체

Optimal Input Substitution

- A firm initially produces Q_0 by employing the combination of inputs represented by point A at a cost of C_0 .
- Suppose w_0 falls to w_1 .
 - α The isocost curve rotates counterclockwise; which represents the same cost level prior to the wage change.
 - α To produce the same level of output, Q_0 , the firm will produce on a lower isocost line (C_1) at a point B.
 - α The slope of the new isocost line represents the lower wage relative to the rental rate of capital.



비용분석

Cost Analysis

- Types of Costs
 - ☐ Fixed costs (FC)
 - ☐ Variable costs (VC)
 - ☐ Total costs (TC)
 - ☐ Sunk costs



총비용과 가변비용

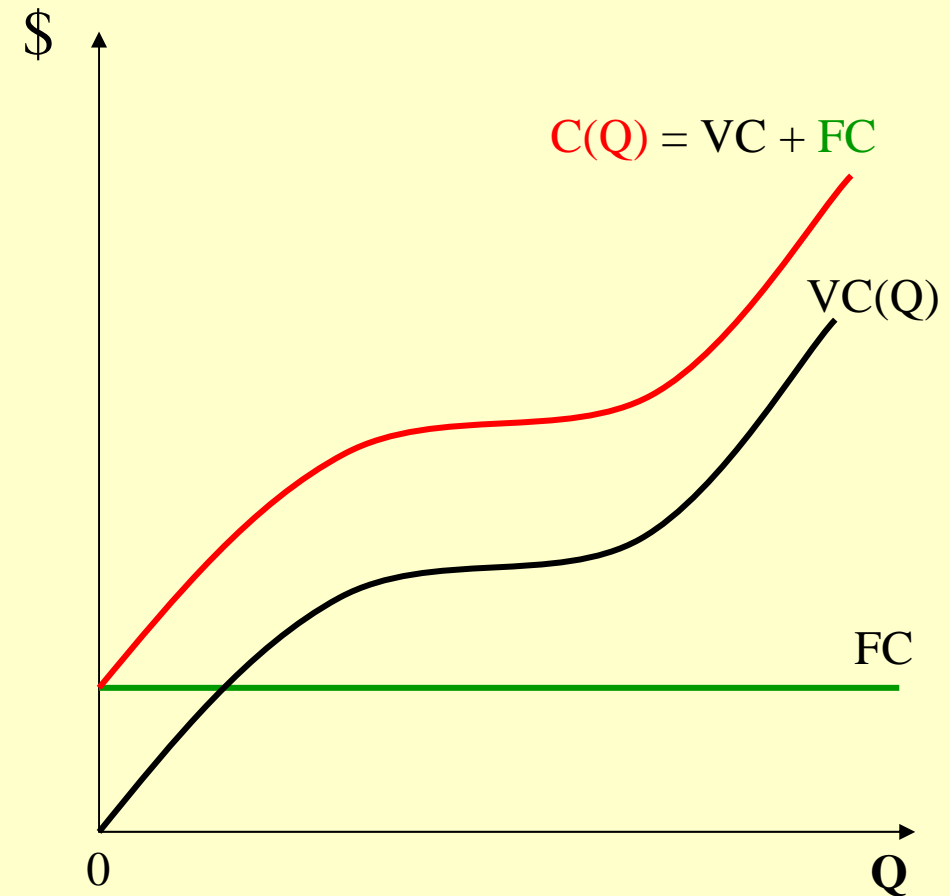
Total and Variable Costs

$C(Q)$: Minimum total cost of producing alternative levels of output:

$$C(Q) = VC(Q) + FC$$

$VC(Q)$: Costs that vary with output. 가변비용

FC : Costs that do not vary with output. 고정투입비용

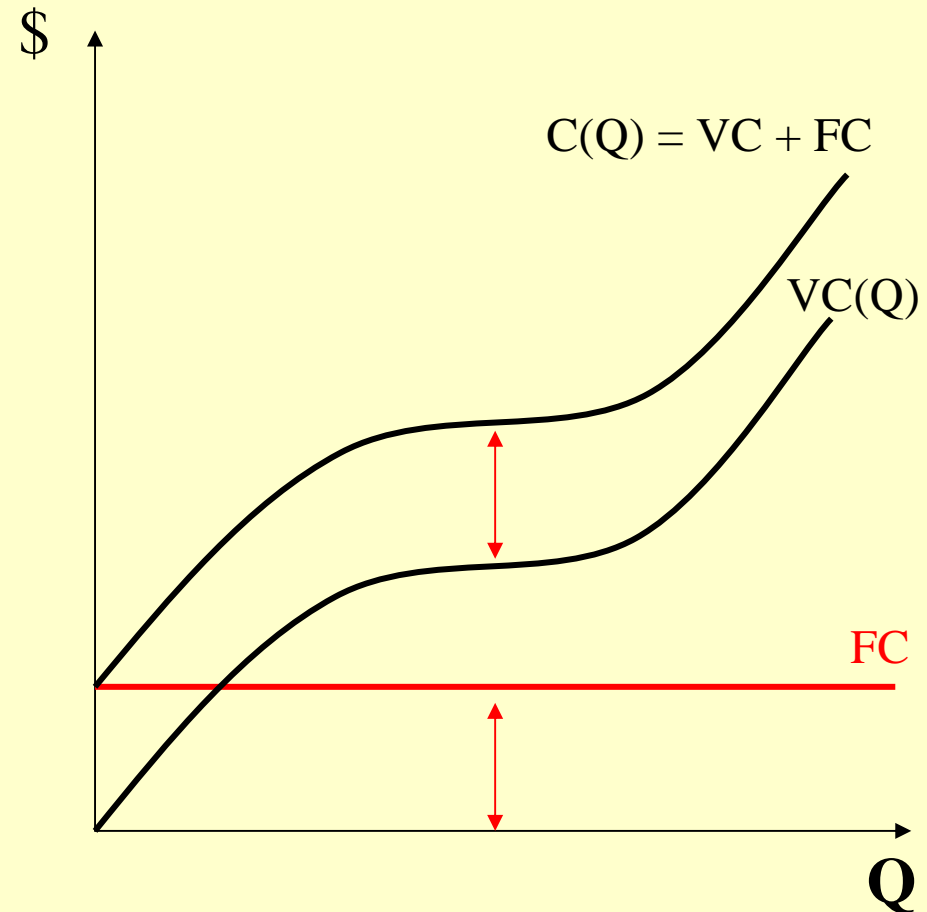


고정비용 및 매출비용

Fixed and Sunk Costs

FC: Costs that do not change as output changes.

Sunk Cost: A cost that is forever lost after it has been paid.



Some Definitions

Average Total Cost

$$ATC = AVC + AFC$$

$$ATC = C(Q)/Q$$

Average Variable Cost

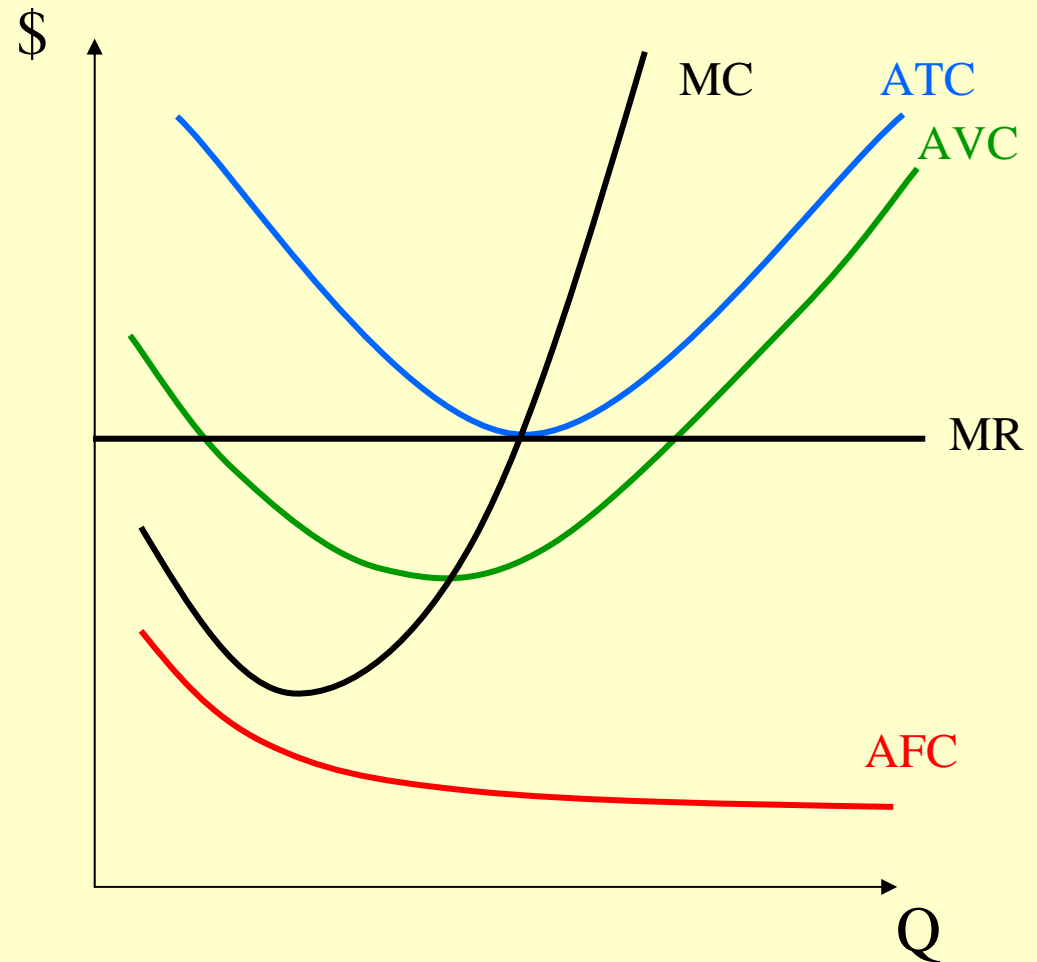
$$AVC = VC(Q)/Q$$

Average Fixed Cost

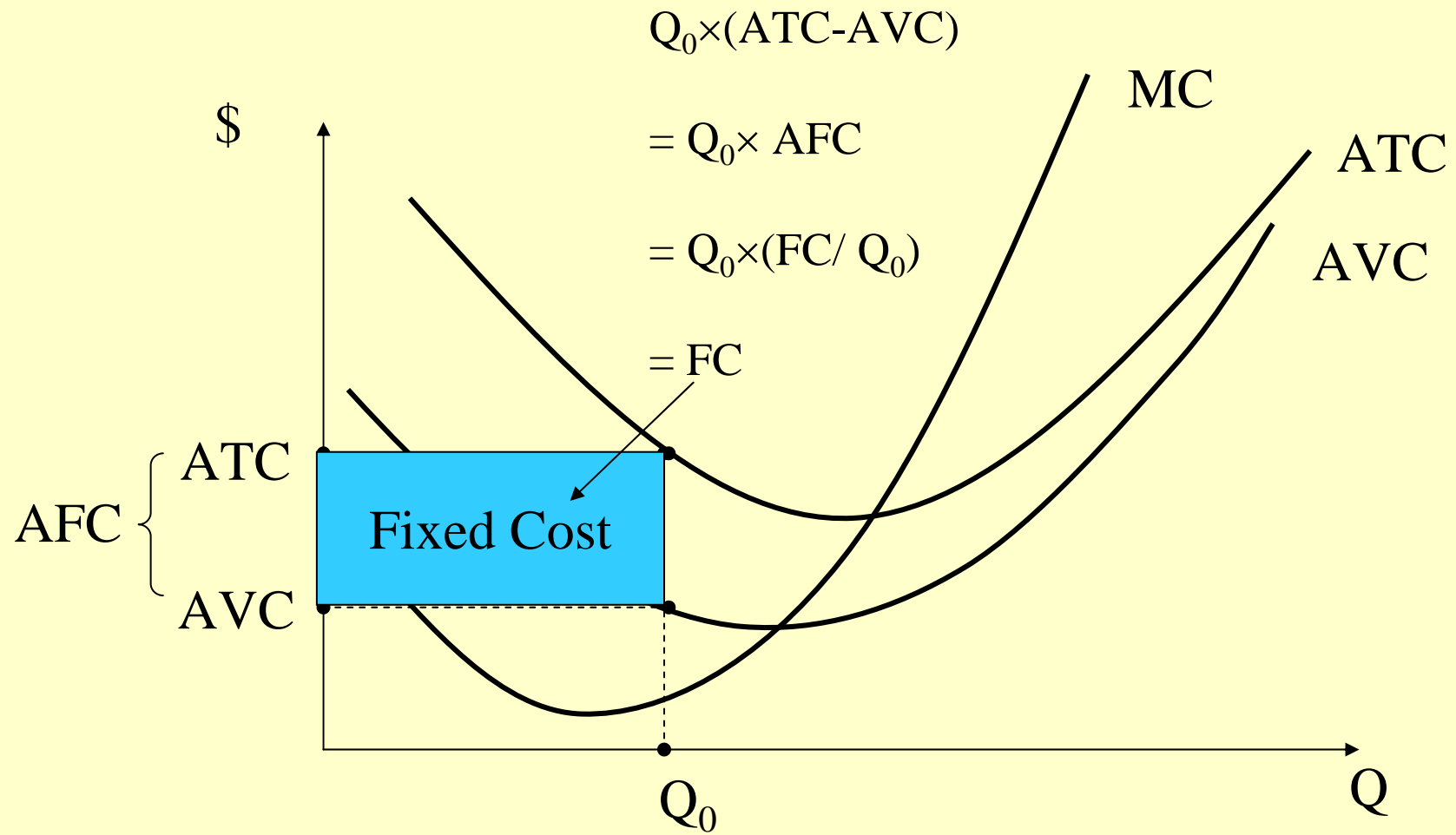
$$AFC = FC/Q$$

Marginal Cost

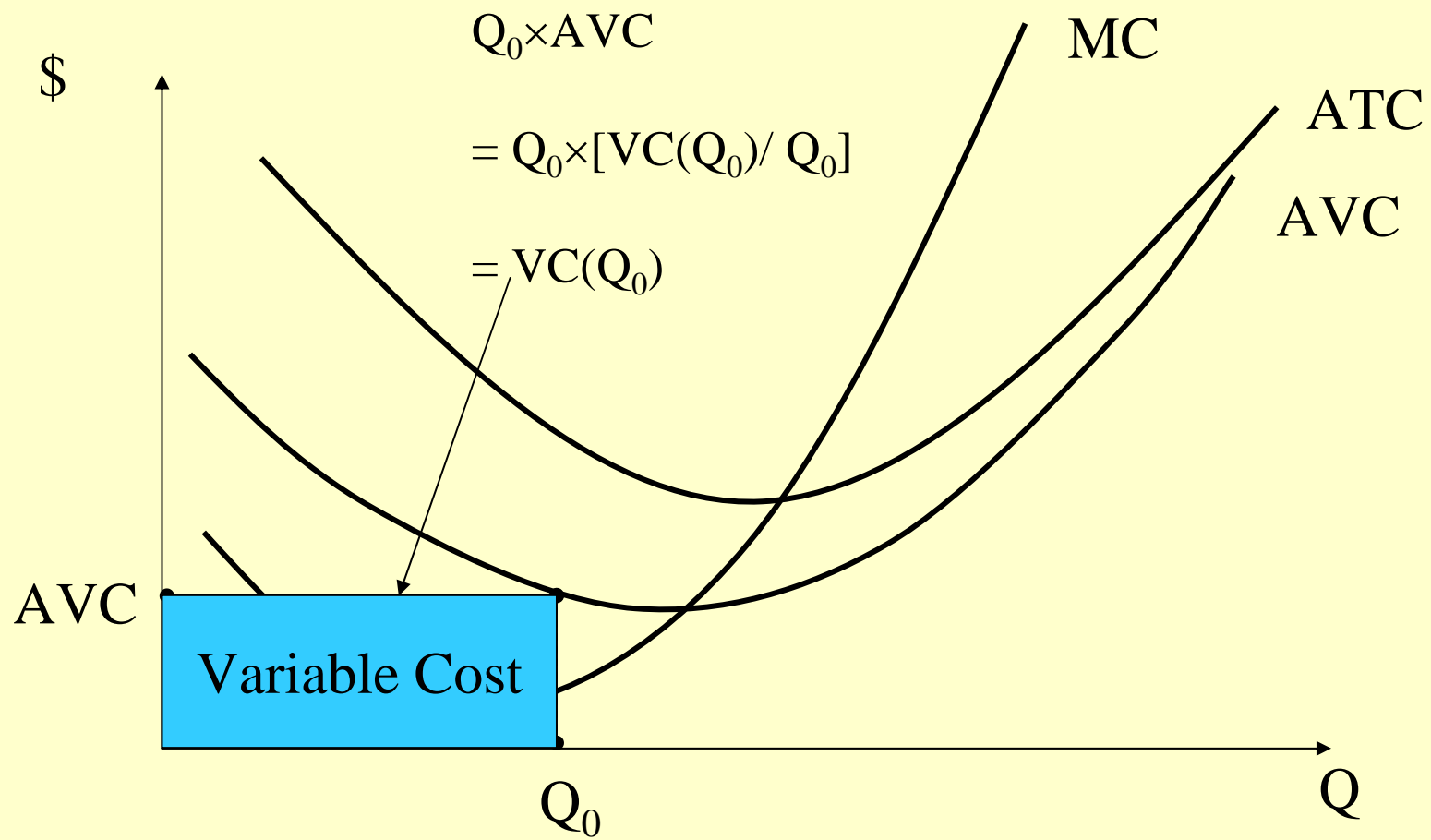
$$MC = \Delta C/\Delta Q$$



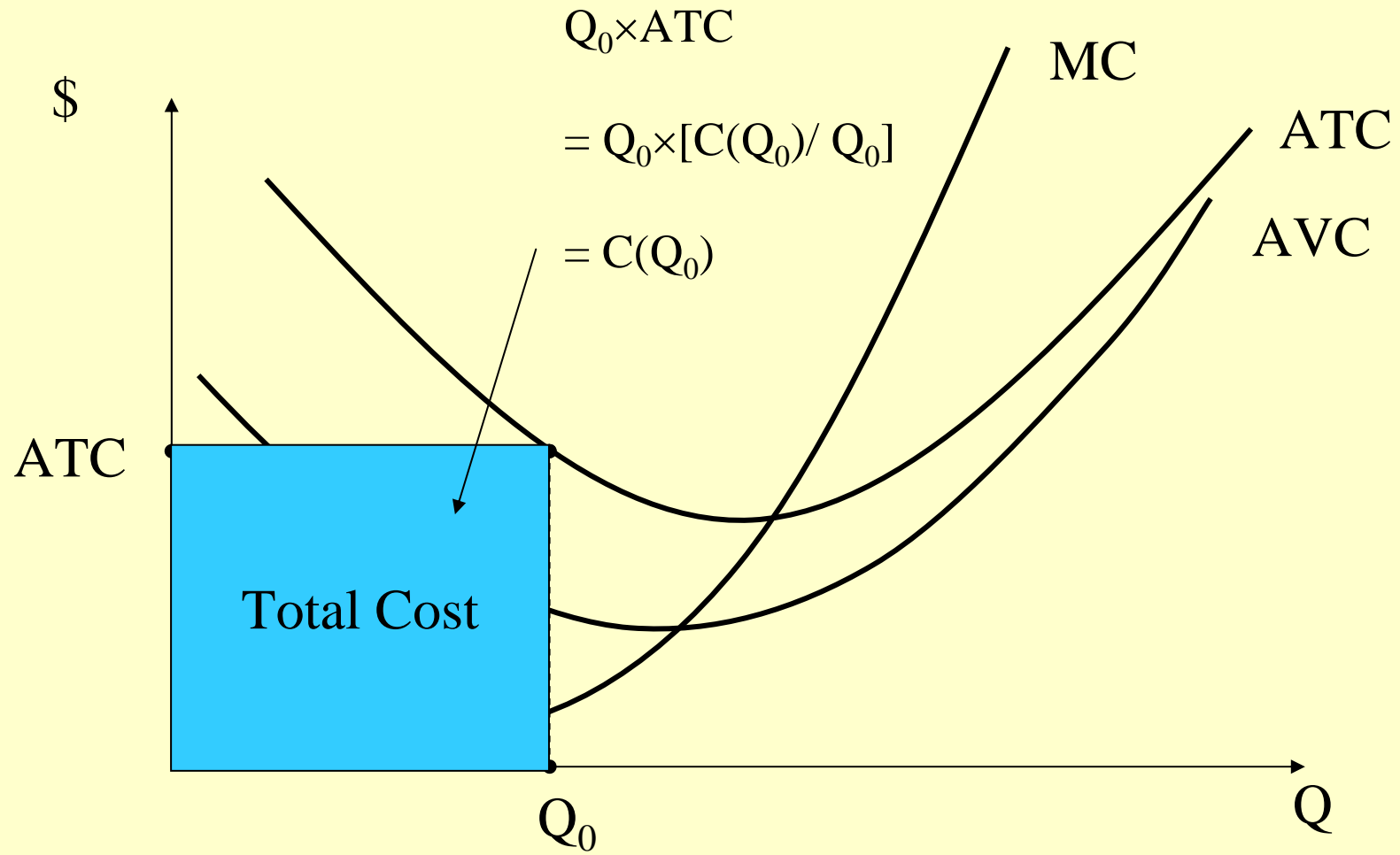
Fixed Cost



Variable Cost



Total Cost



Cubic Cost Function

- $C(Q) = f + aQ + bQ^2 + cQ^3$
- Marginal Cost?

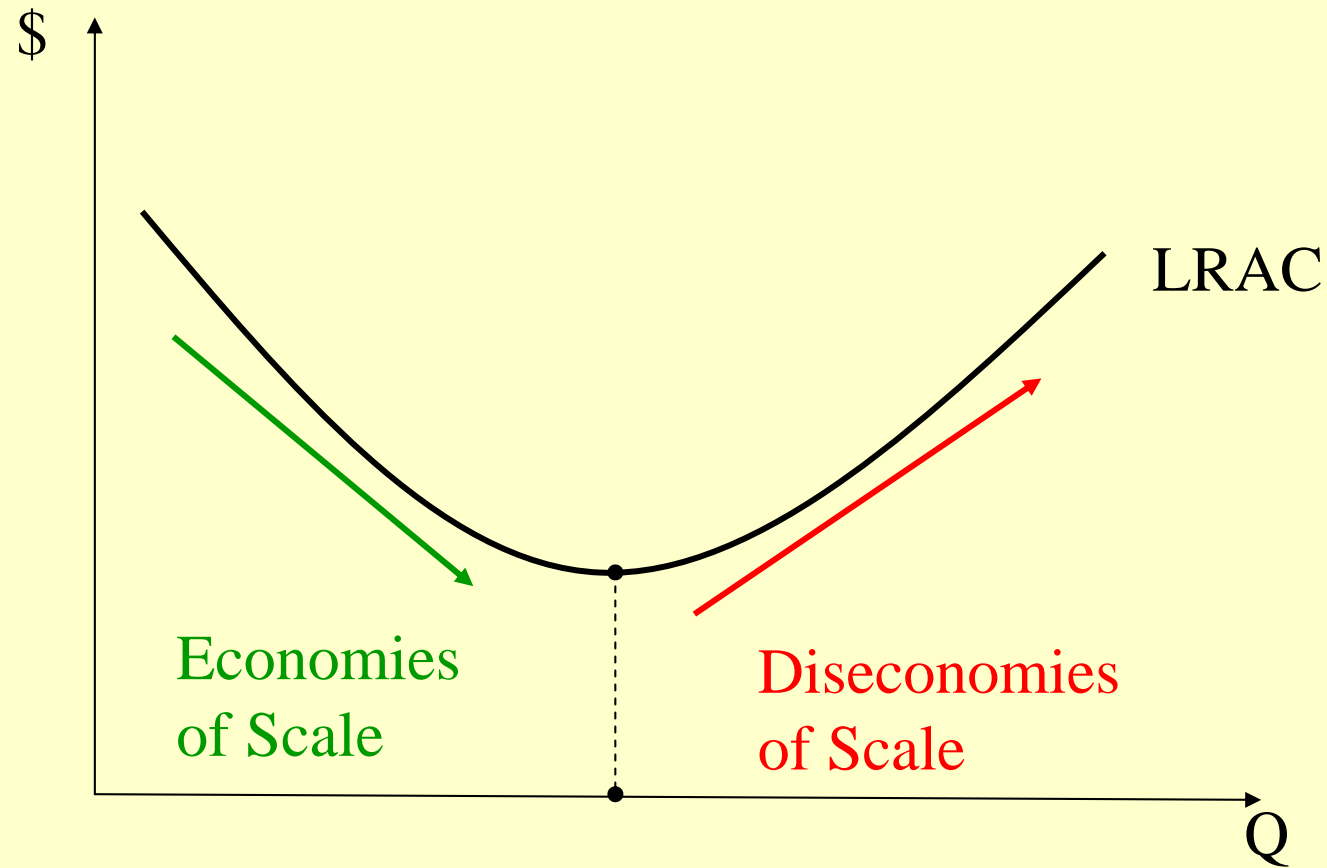
□ Memorize:

$$MC(Q) = a + 2bQ + 3cQ^2$$

□ Calculus:

$$dC/dQ = a + 2bQ + 3cQ^2$$

규모의 경제 Economies of Scale



범위의 경제 Economies of Scope

- $C(Q_1, 0) + C(0, Q_2) > C(Q_1, Q_2)$.
 - ◻ It is cheaper to produce the two outputs jointly instead of separately.
- Example:
 - ◻ It is cheaper for Time-Warner to produce Internet connections and Instant Messaging services jointly than separately.

Cost Complementarity

- The marginal cost of producing good 1 declines as more of good two is produced:

$$\Delta MC_1(Q_1, Q_2) / \Delta Q_2 < 0.$$

- Example:
 - Cow hides and steaks.

결론

Conclusion

- To maximize profits (minimize costs) managers must use inputs such that the value of marginal of each input reflects price the firm must pay to employ the input.
- The optimal mix of inputs is achieved when the $MRTS_{KL} = (w/r)$.
- Cost functions are the foundation for helping to determine profit-maximizing behavior in future chapters.