경제수학 제 2 장

경제학에 쓰이는 방정식들

수요와 공급

- We plot supply and demand with P on the vertical axis
- Before plotting a supply or demand function, write it so that P is on the left, Q is on the right

시장균형 (Market Equilibrium)

- Market equilibrium occurs when the quantity supplied equals the quantity demanded of a good
- The supply and demand curves cross at the equilibrium price and quantity
- You can read off approximate equilibrium values from the graph
- Solving algebraically for the point where the demand and supply equations are equal gives exact values

Cost–Volume–Profit (CVP) Analysis

- Two simplifying assumptions are made: namely that price and average variable costs are both fixed

$$\pi = P.Q - (\text{FC} + \text{VC}) = P.Q - \text{FC} - \text{VC}$$

- Multiplying both sides of the expression for AVC by Q we obtain

$$\text{AVC}.Q = \text{VC} \quad \text{and substituting this}$$

$$\pi = P.Q - \text{FC} - \text{AVC}.Q$$

Special Assumptions of CVP Analysis

- P is fixed
- AVC is fixed
- π is a function of Q but P, FC, and AVC are not
- We can write the inverse function expressing Q as a function of π
- Adding FC to both sides gives

$$\pi + \text{FC} = P.Q - \text{AVC}.Q$$

- Interchanging the sides we obtain

$$P.Q - \text{AVC}.Q = \pi + \text{FC}$$

Solving for Desired Sales Level

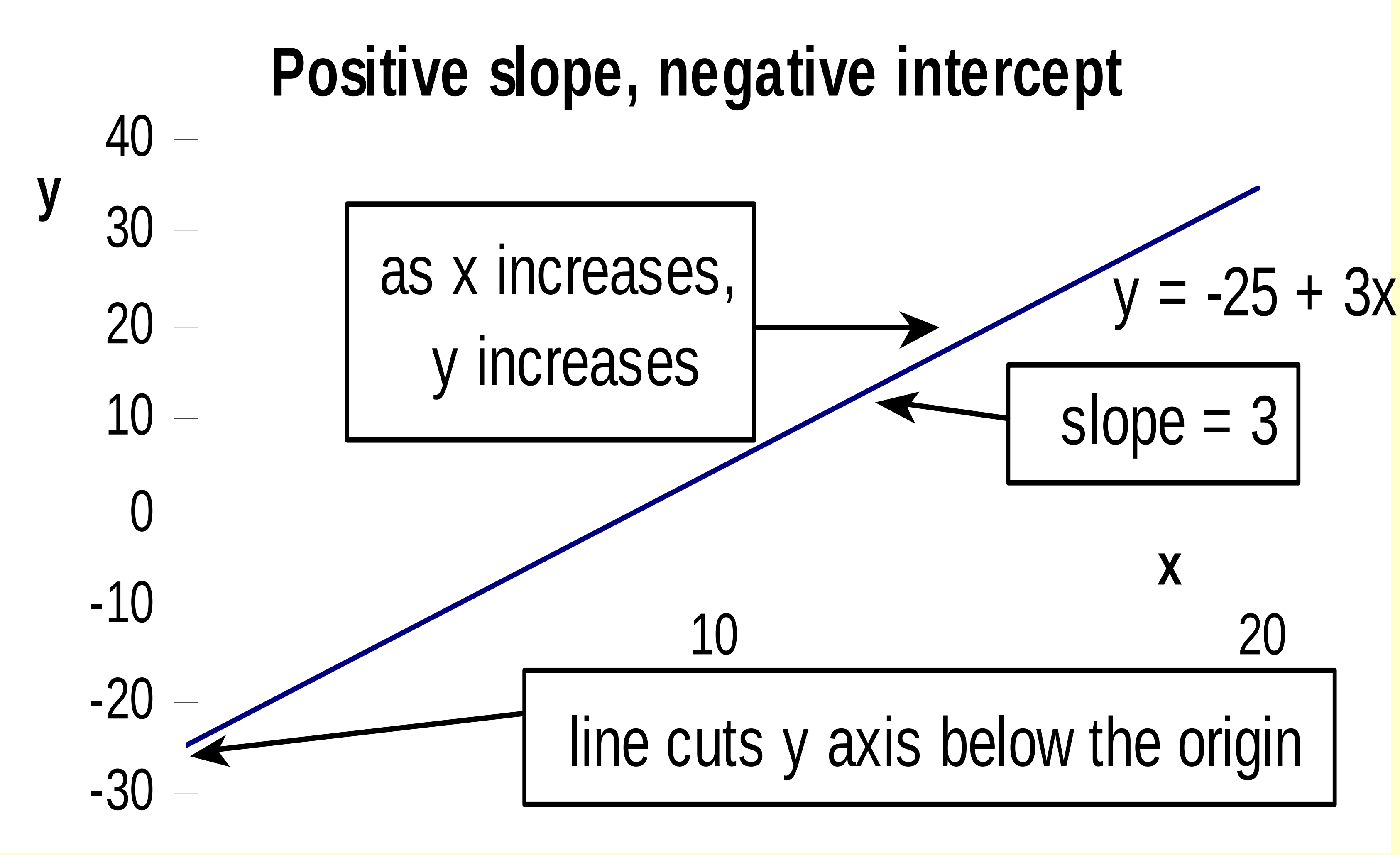
- Q is a factor of both terms on the left so we may write
- $Q(P - \text{AVC}) = \pi + \text{FC}$
- Dividing through by $(P - \text{AVC})$ gives
- $Q = (\pi + \text{FC})/(P - \text{AVC})$
- If the firm's accountant can estimate FC, P and AVC, substituting these together with the target level of profit, π, gives the desired sales level

Linear Equations

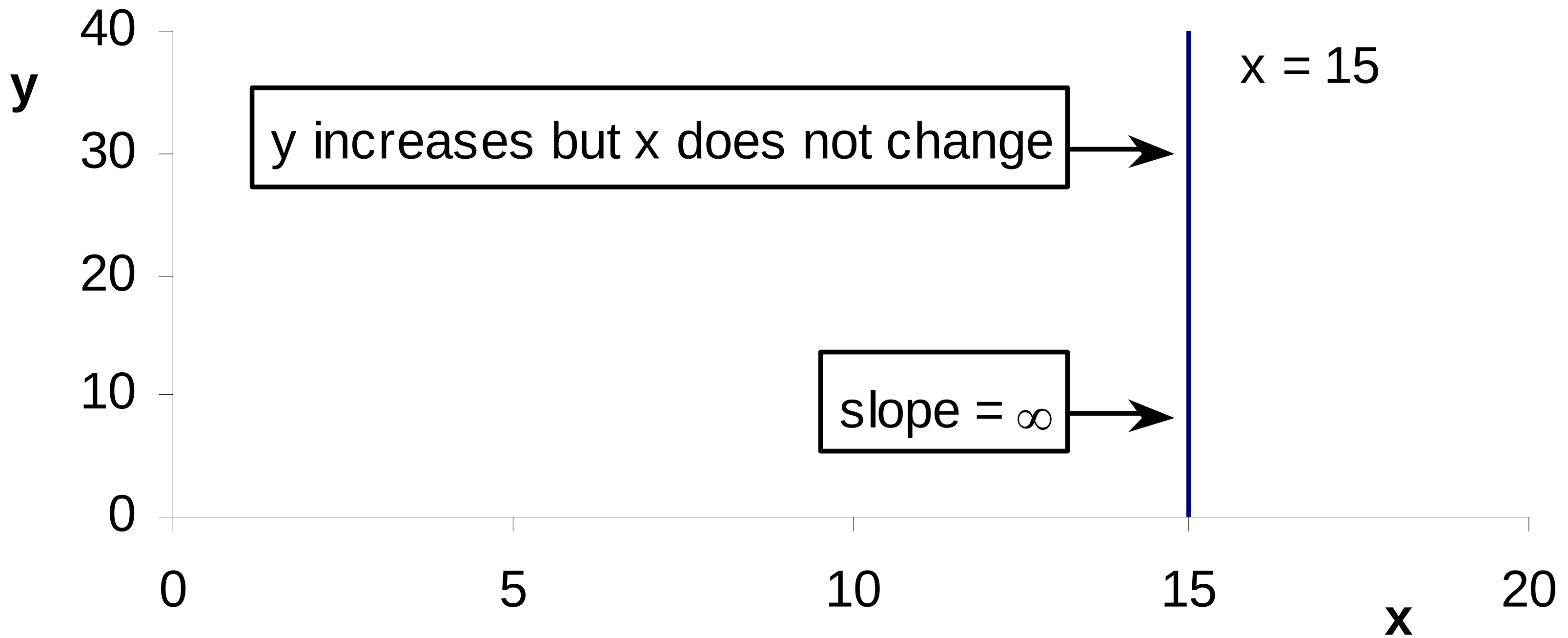
- Slope of a line: distance up divided by distance moved to the right between any two points on the line
- Coefficient: a value that is multiplied by a variable
- Intercept: the value at which a function cuts the y axis

Representing a Line as $y = mx + b$

- The constant term, b, gives the y intercept
- The slope of the line is m, the coefficient of x
- Slope = $\Delta y/\Delta x$ = (distance up)/(distance to right)
- Lines with positive slope go up from left to right
- Lines with negative slope go down from left to right
- Parameter: a value that is constant for a specific function but that changes to give other functions of the same type; m and b are parameters


Positive slope, negative intercept
y
40
30
20
10
0
-10
-20
-30
as x increases,
y increases
y = -25 + 3x
slope = 3
x
10
20
line cuts y axis below the origin

A vertical line has infinite slope



예산집합 (Budget Line)

- If two goods x and y are bought
  the budget line equation is $x.P_x + y.P_y = M$
- To plot the line, rewrite as
  $$\text{y} = M/P_y - (P_x/P_y)x$$
- Slope = $-P_x/P_y$
  the negative of the ratio of the prices of the goods
- Intercept = M/P_y
  the constant term in the equation

The Parameters of a Budget Line

- Changing P_x rotates the line about the point where it cuts the *y* axis
- If P_y alters, both the slope and the *y* intercept change
  - the line rotates about the point where it cuts the *x* axis
- An increase or decrease in income *M* alters the intercept but does not change the slope
  - the line shifts outwards or inwards

2차 방정식 (Quadratic Equations)

- A quadratic equation takes the form
  $$ax^2 + bx + c = 0$$
- You can solve it graphically
- or sometimes by factorizing it
- or by using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where a is the coefficient of x^2, b is the coefficient of x and c is the constant term

연립방정식 (Simultaneous Equations)

- Simultaneous equations can usually (but not always) be solved if

  number of equations = number of unknowns

연립방정식 풀이하기

- Solution methods for two simultaneous equations include
  - Finding where functions cross on a graph
  - Eliminating a variable by substitution
  - Eliminating a variable by subtracting (or adding) equations
- Once you know the value of one variable, substitute it in the other equation

Simultaneous Equilibrium in Related Markets

- Demand in each market depends both on the price of the good itself and on the price of the related good
- To solve the model use the equilibrium condition for each market

  demand = supply
- This gives two equations (one from each market) in two unknowns which we then solve

지수함수 (Exponential Functions)

- Exponential function: has the form a^x where the base, a, is a positive constant and is not equal to 1
- The exponential function most used in economics is $y = e^x$
- The independent variable is in the power and the base is the mathematical constant

$$e = 2.71828\ldots$$

- Use your calculator or computer to evaluate e^x

로그함수 (Logarithmic Functions)

- Logarithm: the power to which you must raise the base to obtain the number whose logarithm it is
- Common logarithms denoted log or $\log_{10}$ are to base 10
- Natural logarithms denoted ln or $\log_e$ are to base e and are more useful in analytical work
- Equal differences between logarithms correspond to equal proportional changes in the original variables

The doubling time of an exponential function $f(t) = Aa^t$ was defined as the time it takes for $f(t)$ to become twice as large. In order to find the doubling time t^*, we must solve the equation $a^{t^*} = 2$ for t^*. In economics, we often need to solve similar problems:

A. At the present rate of inflation, how long will it take the price level to triple?

B. If the world's population grows at 2% annually, how long does it take to double the size?

C. If $1,000is invested in a savings account bearing interest at the annual rate of 8%, how long does it take for the account to reach $10,000?

Ex) From $a^{t^*} = 2$, take natural logarithm of both sides to yield $\ln a^{t^*} = t^* \ln a = \ln 2$. So, $t^* = \dfrac{\ln 2}{\ln a}$.

Ex) Express $\ln 4$, $\ln \sqrt[3]{2^5}$, and $\ln(1/16)$ in terms of $\ln 2$.

Ex) Solve the following equations for x:

(a) $5e^{-3x} = 16$ (b) $A\alpha e^{-\alpha x} = k$ (c) $(1.08)^x = 10$ (d) $e^x + 4e^{-x} = 4$

로그함수의 성질

- $\log(xy) = \log(x) + \log(y)$
- $\log(x/y) = \log(x) - \log(y)$
- $\log(x^n) = n \log(x)$
- $\ln(e^x) = x$
- The reverse process to taking the natural logarithm is to exponentiate