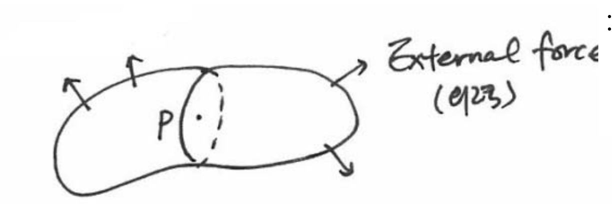
2장 응력(Stress)

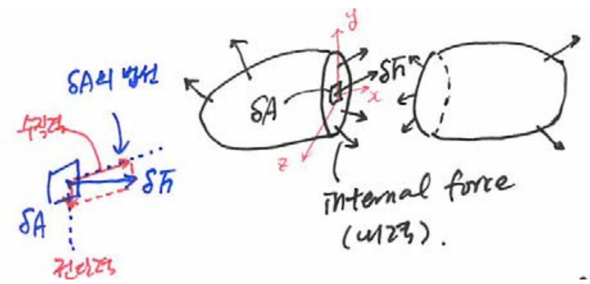
2.1 응력



: The force acting on the entire body in equilibrium (힘의 평형 상태)

└ if not, moving

└ dividing the body into two parts using an imaginary plane



: The force acting on each part alone must be in equilibrium

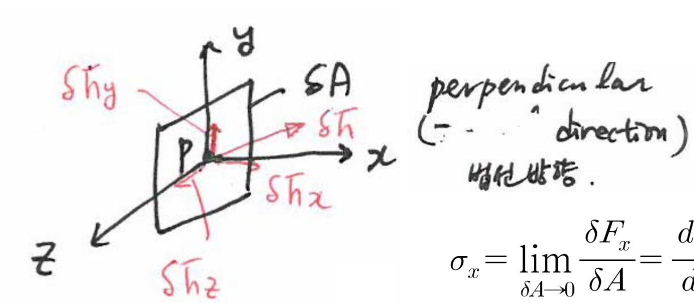
면적 δA에서 평균응력(average stress) 개념

- Definition of stress : $\lim_{\delta A \to 0}(\frac{\delta F}{\delta A})$ 가 어떤 값(Stress, 응력)으로 결정됨

→ P점에서의 미소면적(δA)을 지나고 P점에서 δF의 방향을 가지는 상태

⇒ 따라서 응력은 특정한 평면과 그 평면에서의 특정 방향의 힘(응력 벡터)이 동시에 정의됨 (2nd order tensor)

└ matrix



cf) Pressure(압력)

$$\sigma_x = \lim_{\delta A \to 0} \frac{\delta F_x}{\delta A} = \frac{dF_x}{dA}$$: normal stress (인장:+ 압축:-)

$$\left.\begin{aligned}\tau_{xy} &= \lim_{\delta A \to 0} \frac{\delta F_y}{\delta A} = \frac{dF_y}{dA} \\ \tau_{xz} &= \lim_{\delta A \to 0} \frac{\delta F_z}{\delta A} = \frac{dF_z}{dA}\end{aligned}\right)$$ shear stress

⁎ Tensor(multi - dimensional array of numerical values)

Tensors, defined mathematically, are simply arrays of numbers, that transform according to certain rules under a change of coordinates. In physics, tensors characterize the properties of a physical system.

(ex, dot product, cross product, linear mapping

⇒ 물리 현상을 기술하기 위하여 동비한 좌표계로 무한공간 또는 물리적 특성을 명확히 나타내기 위해 만들어진 일반화된 좌표계

rank, degree : dimensionality of the array needed to represent in

- Tensor of order zero (Scalar)

  ex) mass of a particle → 작용점에서 하나의 값으로 표현

- Tensor of order one (Vector)

  ex) force → 방향, 힘이 작용하는 점(위치) 모두 필요

- Tensor of order two (tensor) → matrix

  ex) stress → 힘이 작용하는 점(면적), 그 면의 방향, 힘의 방향

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix}$$

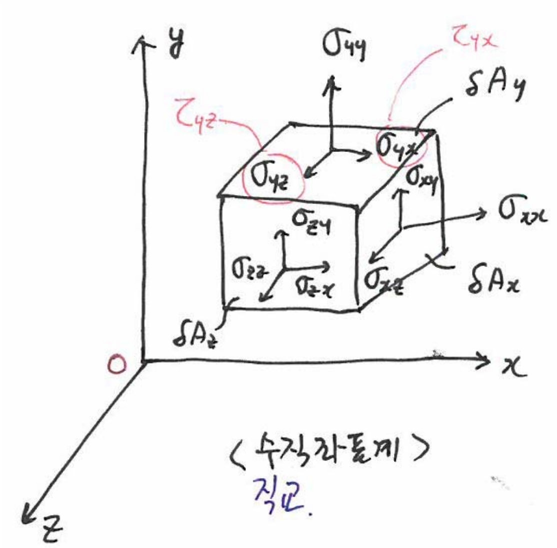
- Tensor of high order

Q. E-value는 어떤 tensor인가?

- 응력 성분

: 3차원 공간에서 한 점에 대한 완전한 응력 상태 표현 → 9개의 응력성분

└ 한점을 지나며 서로 직교하는 3개의 평면에 관한 응력 성분을 모두 표기



$$\Rightarrow \sigma_{ij} = \begin{bmatrix} \sigma_{xx}(=\sigma_x) & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy}(=\sigma_y) & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz}(=\sigma_z) \end{bmatrix}$$

↓ Stress tensor

cf).원통좌표계(r, θ, z)인 경우 응력성분 → (P.21, 그림 2.5 참조)

- Unit(units of force per unit area)

• $\frac{N}{m^2} = Pa$ (Pascal) (SI) ⇒ 1Pa = 0.000145 psi

• Pounds per squal inch : psi (U.S)

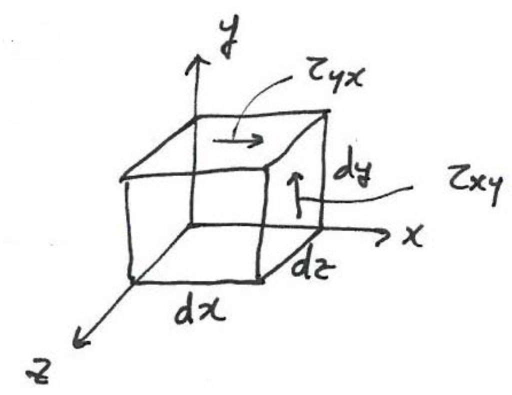
- Equality of shear stress

  ⁎ Equilibrium in all directions,

$$\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma F_z = 0$$

  ⁎ Rotational equilibrium,

$$\Sigma M_x = 0,\ \Sigma M_y = 0,\ \Sigma M_z = 0$$



ex) $\Sigma M_z = 0,$

$$(-\tau_{xy} dydz)dx + (\tau_{yx} dxdz)dy = 0$$

$$\therefore \tau_{xy} = \tau_{yx}$$

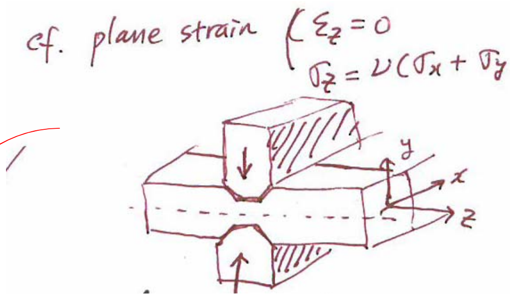
likewise, from $\Sigma M_x = 0,\ \Sigma M_y = 0$

$$\therefore \tau_{xz} = \tau_{zx}\ ,\ \tau_{yz} = \tau_{zy}$$

→ This means that shearing stresses on mutually perpendicular planes of the element are equal.

$$\begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$ ⟶ 6개의 성분만 알면 된다

- Various types of stress condition



a. Triaxial stress

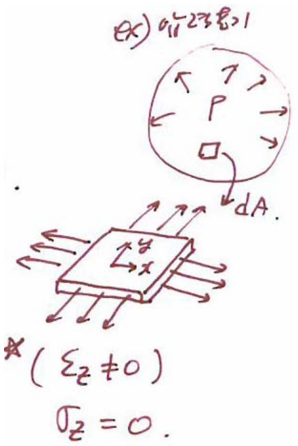
$$\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

: 주응력 상태, principal stress state

b. Two-dimensional or Plane stress

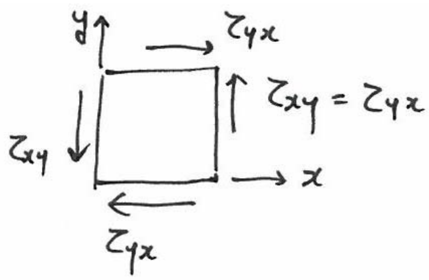
: 응력이 x, y 평면에만 작용하는 경우, (z 성분 = 0)

$$\begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \rightarrow \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{bmatrix}$$



c. Pure shear

: In this case, the element is subjected to plane shearing stress only,

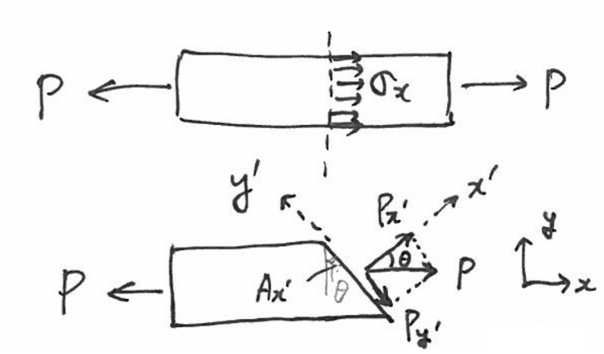


d. Uniaxial Stress

: normal stress act along one direction only,

ex) uniaxial tension or compression

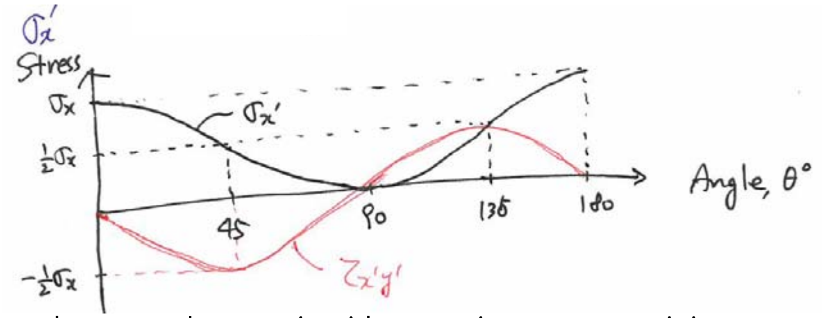
- Stresses on inclined planes in an axially loaded member



Axial Loading : $\sigma_x = \dfrac{P}{A}$

$$\sigma_x{}' = \frac{Pcos\theta(=P_x{}')}{A_{x'}} = \sigma_x \cos^2\theta \;\; (\leftarrow A_{x'}\cos\theta = \frac{A_x}{\cos\theta} \bullet \cos\theta = A_x) \qquad \text{— } (\ast)$$

$$\tau_{x'y'} = -\frac{\Psi n\theta}{A_{x'}} = -\sigma_x \mathrm{sin}\theta\cos\theta \qquad \text{———— } (\ast\ast)$$

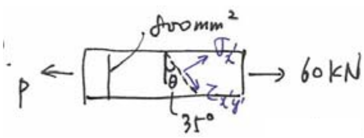


⇒ the normal stress is either maximum or a minimum on planes for which the shear stress is zero.

⇒ Shear stress in the range of $\underline{-\frac{1}{2}\sigma_x \sim \frac{1}{2}\sigma_x}$

⇒ When $\theta > 90^\circ$, the sign of $\tau_{x'y'}$ is changed. (from - → to +)

Example)



Sol) $\sigma_x = \dfrac{P}{A} = \dfrac{60(10^3)}{800(10^{-6})} = 75MPa$

35° 평면에서 응력 상태는?

$\sigma_{x'}, \tau_{x'y'}$?

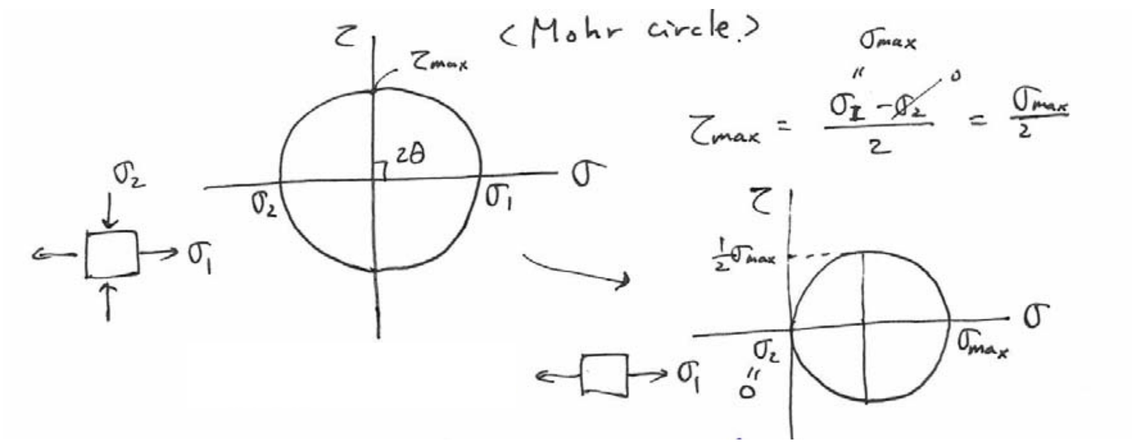
from (∗), (∗∗)

$\therefore \sigma_{x'} = \sigma_x \cos^2\theta = 50.33\,MPa,\ \tau_{x'y'} = -35.24\,MPa$

- From Eq(∗) and (∗∗),

As expected, $\sigma_{x'}$ is a maximum ($\sigma_{\max}$) when θ is 0° or 180°,

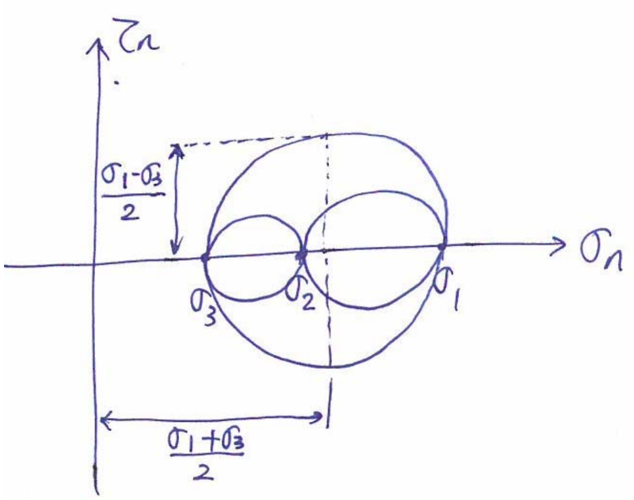
and $\tau_{x'y'}$ is maximum ($\tau_{\max}$) when θ is 45° or 135°, $\tau_{\max} = \pm\dfrac{1}{2}\sigma_{\max}$



<Mohr's circle>

∗ Mohr's circle is a two-dimensional
graphical representation of the state of stress at a point.

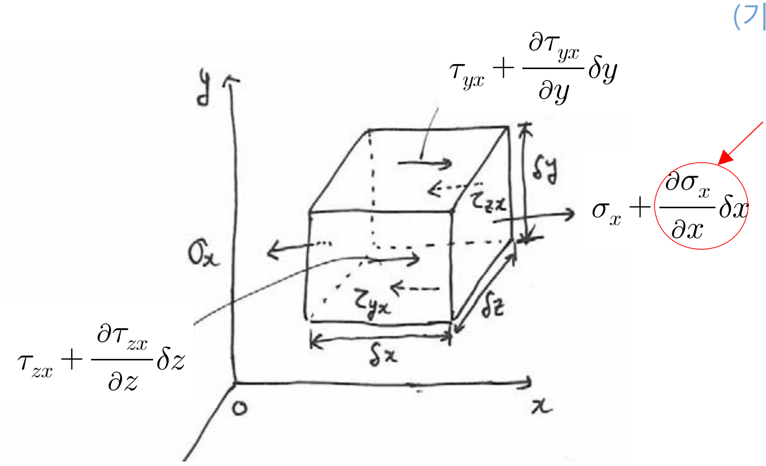
cf). 3D State of stress



2.2 평형방정식

외력을 받고 있는 상태의 물체 내부의 미소체적에 걸리는 힘의 평형

(기본 가정: body force 무시)



δx 사이의 σ_x의 응력변화성분

(y, z의 변화는 일정하게 유지

: $Taylor's\ expansion$

→ 6면체의 5개 면 위에서 ⇒ 3X6 → <u>18개의 힘이 작용</u>

- x축 방향의 힘의 평형,

$$(\sigma_x + \frac{\partial \sigma_x}{\partial x}\delta x)\delta y \delta z + (\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z}\delta z)\delta x \delta y + (\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}\delta y)\delta x \delta z$$

$$- \sigma_x \delta y \delta z - \tau_{zx}\delta x \delta y - \tau_{yx}\delta z \delta x = 0 \qquad —— ①$$

$$\frac{\partial \sigma_x}{x}\delta x \delta y \delta z + \frac{\partial \tau_{zx}}{\partial z}\delta x \delta y \delta z + \frac{\partial \tau_{yx}}{\partial y}\delta x \delta y \delta z = 0 \qquad —— ②$$

x방향의 힘의 평형이 미소체적에서 만족함

$$\therefore \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0 \qquad —— ③$$

- y와 z 방향에서도 힘의 평형 적용,

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} = 0 \qquad —— ④$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0 \qquad —— ⑤$$

$$\Rightarrow \frac{\partial \sigma_{ij}}{\partial x_i} = 0 \qquad —— ⑥$$

$(x, y, z \rightarrow x_1, x_2, x_3)$

$(\sigma_{xx} \rightarrow \sigma_{11}$

$\sigma_{xy} \rightarrow \sigma_{12}$

- ⑥ 식에서 단위체적 당 질량 f_i(체적력)을 포함하면,

$$\frac{\partial \sigma_{ij}}{\partial x_i} + \rho f_i = 0 \quad (\rho\text{: 밀도}) \qquad \text{—— ⑦}$$

2.3 모멘트의 평형

＊ see the section of Equality of shear stress

$$\sigma_{ij} = \sigma_{ji}$$