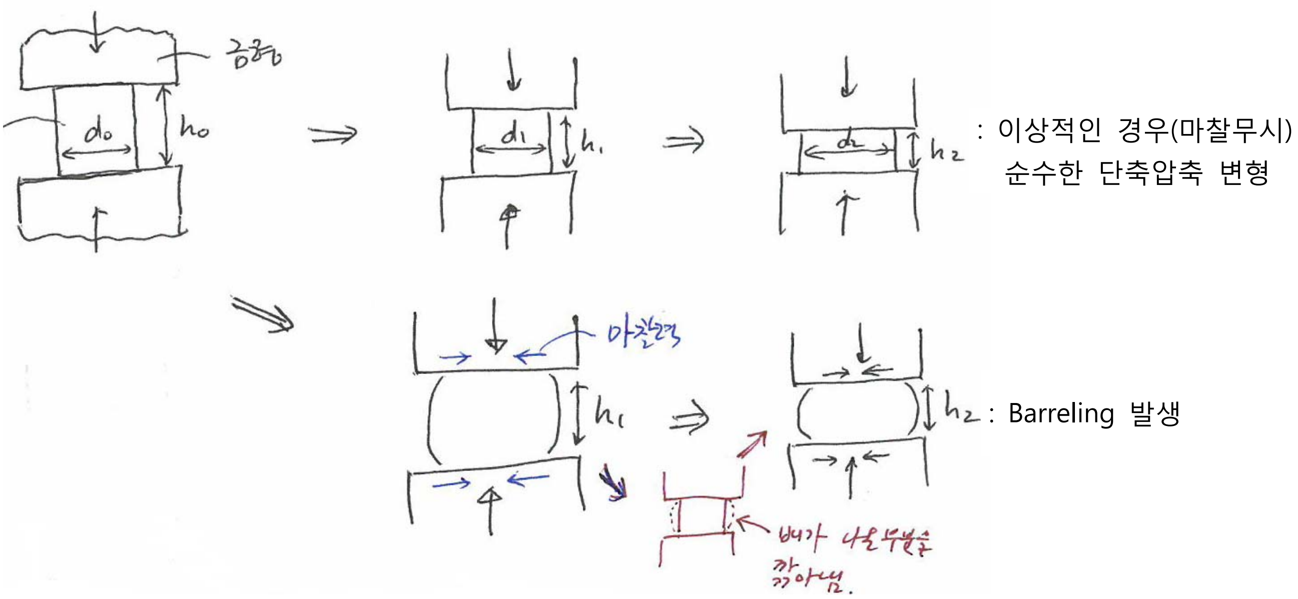
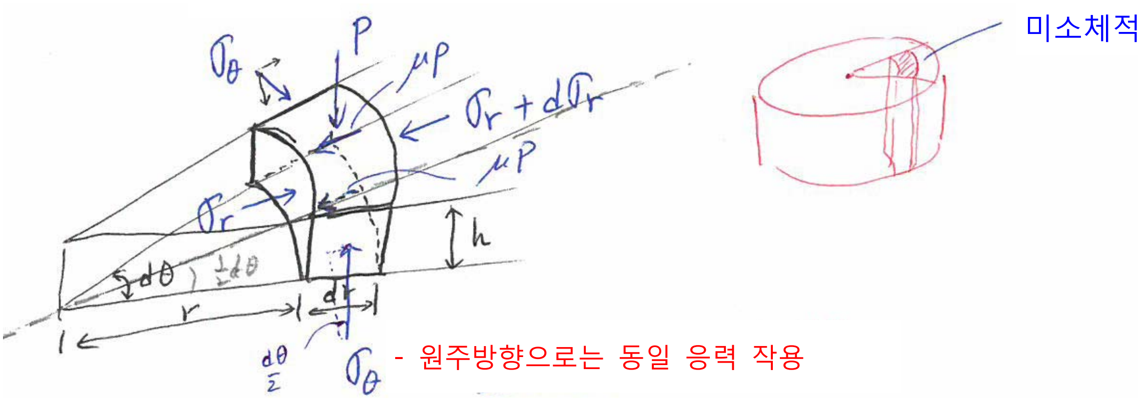
8.2 원주의 압축



- 미소요소에 작용하는 모든 힘을 표시



- 평형조건을 적용(반지름 방향-힘의 평형)

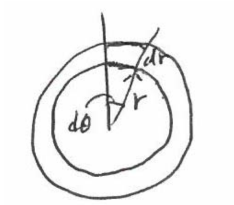
$$- (\sigma_r + d\sigma_r)h(r+dr)d\theta + \sigma_r hrd\theta + 2\sigma_\theta hdr\sin\frac{d\theta}{2} - 2\mu Prd\theta dr = 0$$

여기서 $d\sigma_r,\ dr,\ d\theta$는 매우 작은 값 → 0

$$\therefore \frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = -\frac{2\mu P}{h} \qquad \text{—— ①}$$

- 항복조건(Tresca or Von Mises)를 적용하여 응력변수를 1개로 통일

  • 원기둥의 단축 압축시 원주방향(θ-방향) 변형률증분;



$$d\epsilon_\theta : \frac{(r+dr)d\theta - rd\theta}{rd\theta} = \frac{drd\theta}{rd\theta} = \frac{dr}{r} \quad \text{— ②}$$

  • 반경방향(r-방향) 변형률증분;

$$d\epsilon_r = \frac{dr}{r} \qquad \text{—— ③}$$

②, ③에서 $\therefore\ d\epsilon_\theta = d\epsilon_r$ —— ④

- Levy-Mises 식을 이용

$$\frac{d\epsilon_r}{\sigma_r{}'} = \frac{d\epsilon_\theta}{\sigma_\theta{}'} = \frac{d\epsilon_z}{\sigma_z{}'} \qquad \text{—— ⑤}$$

⑤ 식에서, $\sigma_r{}' = \sigma_\theta{}' \rightarrow \sigma_r - \sigma_m = \sigma_\theta - \sigma_m \quad \therefore\ \sigma_r = \sigma_\theta$ —— ⑥

- 따라서 ① 식에서(⑥을 대입)

$$\frac{d\sigma_r}{dr} = -\frac{2\mu P}{h} \qquad \text{—— ⑦}$$

- Tresca 항복조건 사용, 주응력($\sigma_r = \sigma_\theta , P$)에서

$\tau_{\max} = \frac{1}{2}(\sigma_1 - \sigma_3) = k$ 일 때 항복

단축 인장: $\sigma_3 = 0, \sigma_1 = Y \rightarrow \frac{1}{2}Y = k \qquad \therefore \sigma_1 - \sigma_3 = Y$

$$P - \sigma_r = Y \qquad \text{—— ⑧}$$ Y : constant

$$dP - d\sigma_r = 0 \qquad \therefore dP = d\sigma_r \qquad \text{—— ⑨}$$

- 식 ⑨ → 식 ⑦ 대입

$$\therefore \ \frac{dP}{dr} = -\frac{2\mu P}{h} \ \rightarrow \ \frac{dP}{P} = -\frac{2\mu}{h}dr$$

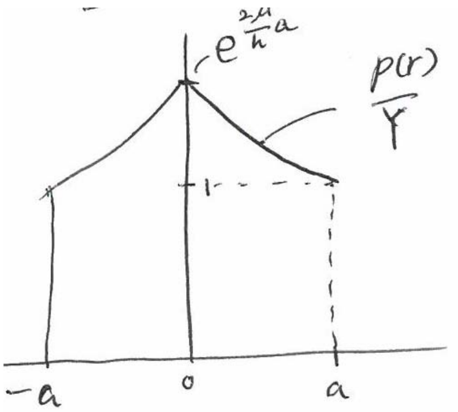
- 양변을 적분(적분구간 $P: P \rightarrow Y, \ \ r: r \rightarrow a$)

$$\int_{P(r)}^{Y} \frac{dP}{P}\left(= \ln P|_{P(r)}^{Y}\right) = \int_{r}^{a}\left(-\frac{2\mu}{h}\right)dr = -\frac{2\mu}{h}(a-r)$$

$$\because \ \int_{P(r)}^{Y} \frac{dP}{P} = \ln P|_{P(r)}^{Y}$$

$$\therefore \ -\frac{2\mu}{h}(a-r) = \ln\frac{Y}{P(r)}$$

$$\therefore \ P(r) = Ye^{\frac{2\mu}{h}(a-r)} \qquad \text{—— ⑩}$$



⇒ • 마찰계수 ↑ → P ↑

• h(초기시편) ↓ → P ↑ (P : 압력)

• Y값 ↑ → P ↑

- 단조하중(up-setting force)

$$F = \int_{r=0}^{r=a} P 2\pi r dr = \int_{0}^{a} Y e^{\frac{2\mu}{h}(a-r)} 2\pi r dr$$

$$= \frac{\pi h^2}{2\mu^2} Y \left[e^{\frac{2\mu a}{h}} - \frac{2\mu a}{h} - 1 \right] \cong \pi a^2 Y \left(1 + \frac{2\mu a}{3h} \right)$$

- 평균하중(압력)

$$P_{avg} = \frac{F}{\pi a^2} \cong Y \left(1 + \frac{2\mu a}{3h} \right)$$

$\ast$ $a \uparrow \rightarrow F \uparrow$

$Y \uparrow \rightarrow F \uparrow$

$h \uparrow \rightarrow F \uparrow$