Angle Modulation

KEEE343 Communication Theory

Lecture #14, April 19, 2011

Prof. Young-Chai Ko
koyc@korea.ac.kr

Summary

- Frequency Division Multiplexing (FDM)
- Angle Modulation

Angle Modulation

- Basic Definition of Angle Modulation

$$s(t) = A_c \cos[\theta_i(t)] = A_c \cos[2\pi f_c t + \phi_c]$$

  - Phase modulation (PM) if

$$\theta_i(t) = 2\pi f_c t + k_p m(t)$$

  - Frequency modulation (FM) if

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau$$

Angle Modulation

- The angle of the carrier wave is varied according to the information-bearing signal.
- Lesson 1: Angle modulation is a nonlinear process
    - In analytic terms, the spectral analysis of angle modulation is complicated
    - In practical terms, the implementation of angle modulation is demanding
- Lesson 2: Whereas the transmission bandwidth of an amplitude-modulated wave is of limited extent, the transmission bandwidth of an angle-modulated wave may an infinite extent, at least in theory.
- Lesson 3: Given that the amplitude of the carrier wave is maintained constant, we would intuitively expect that additive noise would affect the performance of angle modulation to a lesser extent than amplitude modulation.

Basic Definitions

- Angle-modulated wave

$$s(t) = A_c \cos[\theta_i(t))]$$

  - Average frequency in hertz

$$f_{\Delta t} = \frac{\theta(t + \Delta t) - \theta(t)}{2\pi \Delta t}$$

  - Instantaneous frequency of the angle-modulated signal

$$f_i(t) = \lim_{\Delta t \to 0} f_{\Delta t}(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$$

$$\theta_i(t) = 2\pi f_c t + \phi_c, \qquad \text{for } m(t) = 0$$

- Phase modulation (PM) is that form of angle modulation in which instantaneous angle is varied linearly with the message signal

$$\theta_i(t) = 2\pi f_c t + k_p m(t)$$

$$s(t) = A_c \cos\left[2\pi f_c t + k_p m(t)\right]$$

- Frequency modulation (FM) is that form of angle modulation in which the instantaneous frequency is varied linearly with the message signal

$$f_i(t) = f_c + k_f m(t)$$

$$\theta_i(t) = 2\pi \int_0^t f_i(t)\, d\tau = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau$$

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right]$$

Table 4.1 *Summary of Basic Definitions in Angle Modulation*

	Phase modulation	*Frequency modulation*	*Comments*
Instantaneous phase $\theta_i(t)$	$2\pi f_c t + k_p m(t)$	$2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau$	A_c: carrier amplitude f_c: carrier frequency $m(t)$: message signal k_p: phase-sensitivity factor k_f: frequency-sensitivity factor
Instantaneous frequency $f_i(t)$	$f_c + \frac{k_p}{2\pi}\frac{d}{dt}m(t)$	$f_c + k_f m(t)$	
Modulated wave $s(t)$	$A_c \cos[2\pi f_c t + k_p m(t)]$	$A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right]$	

[Ref: Haykin & Moher, Textbook]

Properties of Angle-Modulated Wave

- Property 1: Constancy of transmitted wave
  - The amplitude of PM and FM waves is maintained at a constant value equal to the carrier amplitude for all time.
  - The average transmitted power of angle-modulated wave is a constant

$$P_{av} = \frac{1}{2} A_c^2$$

- Property 2: Nonlinearity of the modulated process

$$m(t) = m_1(t) + m_2(t)$$

$$s(t) = A_c \cos\left[2\pi f_c t + k_p(m_1(t) + m_2(t))\right]$$

$$s_1(t) = A_c \cos(2\pi f_c t + k_p m_1(t)), \qquad s_2(t) = A_c \cos(2\pi f_c t + k_p m_2(t))$$

$$s(t) \neq s_1(t) + s_2(t)$$

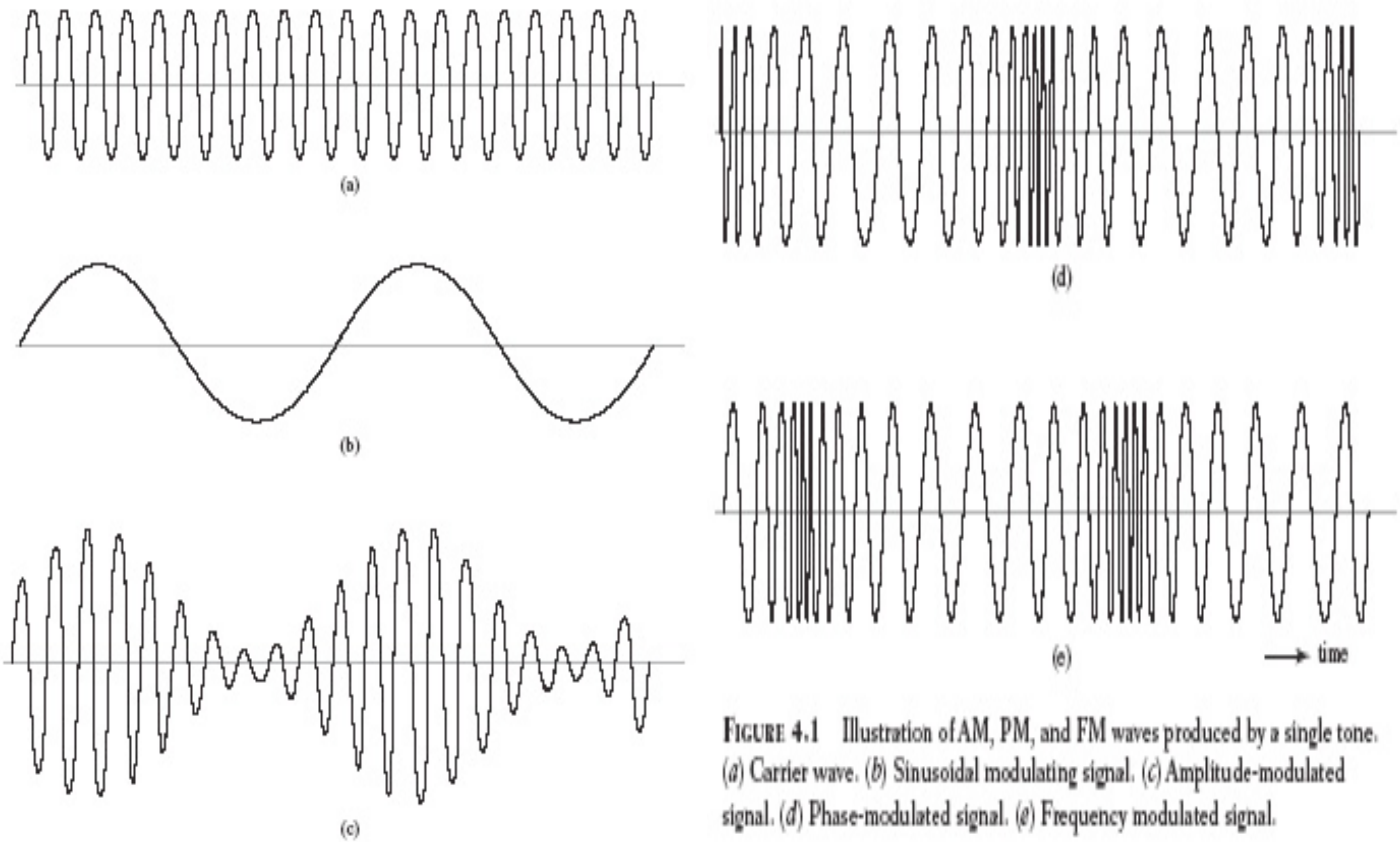

FIGURE 4.1 Illustration of AM, PM, and FM waves produced by a single tone. (*a*) Carrier wave. (*b*) Sinusoidal modulating signal. (*c*) Amplitude-modulated signal. (*d*) Phase-modulated signal. (*e*) Frequency modulated signal.

[Ref: Haykin & Moher, Textbook]

- Property 3: Irregularity of zero-crossings

  - Zero-crossings are defined as the instants of time at which a waveform changes its amplitude from a positive to negative value or the other way around

  - The irregularity of zero-crossings in angle-modulation wave is attributed to the nonlinear character of the modulation process.

    - The message signal m(t) increases or decreases linearly with time t, in which case the instantaneous frequency $f_i(t)$ of the PM wave changes form the unmodulated carrier frequency f_c to a new constant value dependent on the constant value of $m(t)$

- Property 4: Visualization difficulty of message waveform

  - The difficulty in visualizing the message waveform in angle-modulated waves is also attributed to the nonlinear character of angle-modulated waves.

- Property 5: Tradeoff of increased transmission bandwidth for improved noise performance

  - The transmission of a message signal by modulating the angle of a sinusoidal carrier wave is less sensitive to the presence of additive noise

Example of Zero-Crossing

- Consider a modulating wave $m(t)$ given as

$$m(t) = \begin{cases} at, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

where a is the slope parameter. In what follows we study the zero-crossing of PM and FM waves for the following set of parameters

$$f_c = \frac{1}{4} \text{ [Hz]}$$

$$a = 1 \text{ volt/s}$$

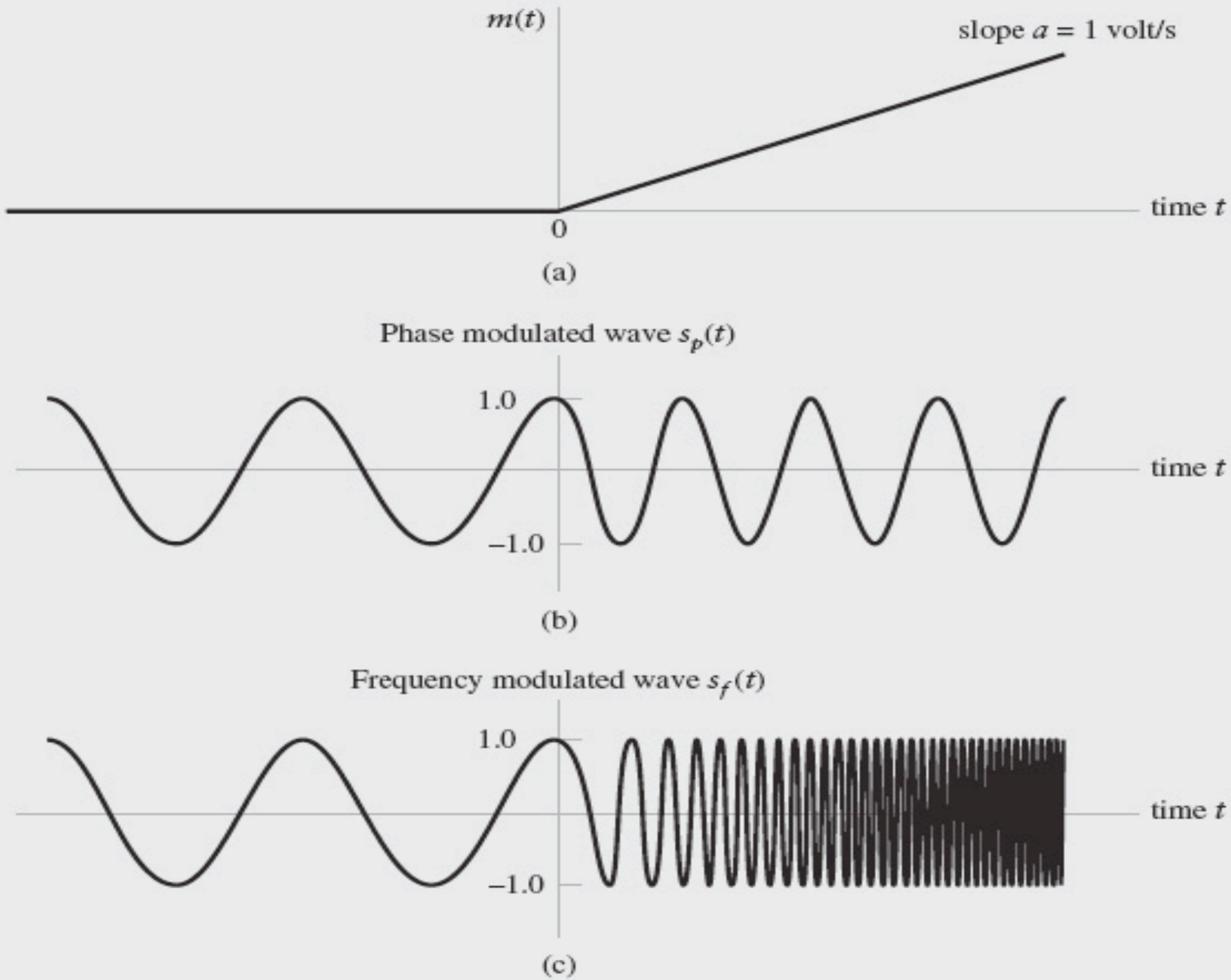

FIGURE 4.2 Starting at time $t = 0$, the figure displays (*a*) linearly increasing message signal $m(t)$, (*b*) phase-modulated wave, and (*c*) frequency-modulated wave.

[Ref: Haykin & Moher, Textbook]

- Phase modulation: phase-sensitivity factor $k_p = \frac{\pi}{2}$ radians/volt. Then, the PM wave is

$$s(t) = \begin{cases} A_c \cos(2\pi f_c t + k_p a t), & t \geq 0 \\ A_c \cos(2\pi f_c t), & t < 0 \end{cases}$$

Let t_n denote the instance of time at which the PM wave experiences a zero-crossing; this occurs whenever the angle of the PM wave is an odd multiple of $\pi/2$. Then we may set up

$$2\pi f_c t_n + k_p a t_n = \frac{\pi}{2} + n\pi, \qquad n = 0, 1, 2, ...$$

as the linear equation for t_n. Solving this equation for t_n, we get the linear formula

$$t_n = \frac{\frac{1}{2} + n}{2f_c + \frac{k_p}{\pi} a} = \frac{1}{2} + n, \qquad n = 0, 1, 2, ...$$

$$f_c = 1/4 \, [\mathrm{Hz}] \text{ and } a = 1 \, \mathrm{volt/s}$$

- Frequency modulation

  - Let $k_f = 1$. Then the FM wave is

$$s(t) = \begin{cases} A_c \cos(2\pi f_c t + \pi k_f a t^2), & t \geq 0 \\ A_c \cos(2\pi f_c t), & t < 0 \end{cases}$$

  - Invoking the definition of a zero-crossing, we may set up

$$2\pi f_c t_n + \pi k_f a t_n^2 = \frac{\pi}{2} + n\pi, \qquad n = 0, 1, 2, ...$$

$$t_n = \frac{1}{a k_f}\left(-f_c + \sqrt{f_c^2 + a k_f \left(\frac{1}{2} + n\right)}\right), \qquad n = 0, 1, 2, ...$$

$$t_n = \frac{1}{4}\left(-1 + \sqrt{9 + 16n}\right), \qquad n = 0, 1, 2, ...$$

$$f_c = 1/4\ [\mathrm{Hz}] \ \text{ and } \ a = 1\ \mathrm{volt/s}$$

- Comparing the zero-crossing results derived for PM and FM waves, we may make the following observations once the linear modulating wave begins to act on the sinusoidal carrier wave:

  - For PM, regularity of the zero-crossing is maintained; the instantaneous frequency changes from the unmodulated value of $f_c = 1/4$ Hz to the new constant value of
$$f_c + k_p(a/2\pi) = \frac{1}{2} \text{ Hz}.$$

  - For FM, the zero-crossings assume an irregular form; as expected, the instantaneous frequency increases linearly with time t

Relationship between PM and FM

- An FM wave can be generated by first integrating the message signal m(t) with respect to time t and thus using the resulting signal as the input to a phase modulation.
- A PM wave can be generated by first differentiating m(t) with respect to time t and then using the resulting signal as the input to a frequency modulator.
  - We may deduce the properties of phase modulation from those frequency modulation and vice versa.

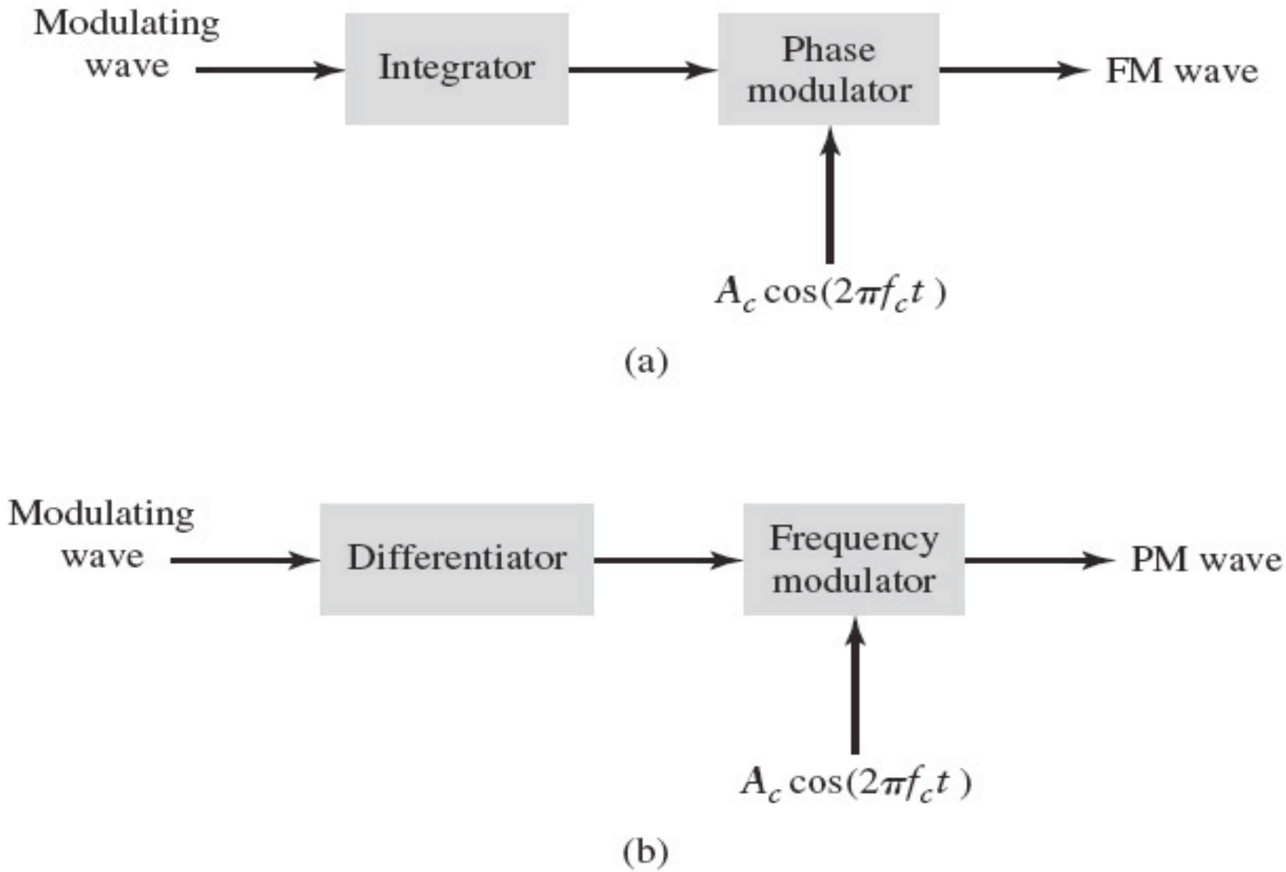

FIGURE 4.3 Illustration of the relationship between frequency modulation and phase modulation. (*a*) Scheme for generating an FM wave by using a phase modulator. (*b*) Scheme for generating a PM wave by using a frequency modulator.

[Ref: Haykin & Moher, Textbook]

Narrow-Band Frequency Modulation

- Narrow-Band FM means that the message signal has narrow bandwidth.
- Consider the single-tone wave as a message signal, which is extremely narrow banded:

$$m(t) = A_m \cos(2\pi f_m t)$$

- FM signal
  - Instantaneous frequency

$$\begin{aligned} f_i(t) &= f_c + k_f A_m \cos(2\pi f_m t) \\ &= f_c + \Delta f \cos(2\pi f_m t) \end{aligned} \qquad \Delta f = k_f A_m$$

  - Phase

$$\begin{aligned} \theta_i(t) &= 2\pi \int_0^t f_i(\tau)\, d\tau = 2\pi \left[f_c t + \frac{\Delta f}{2\pi f_m} \sin(2\pi f_m t) \right] \\ &= 2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t) \end{aligned}$$

- Definitions
  - Phase deviation of the FM wave $\beta = \dfrac{\Delta f}{f_m}$
  - Modulation index of the FM wave: f_m
- Then, FM wave is

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$$

  - For small f_m

$$\cos[\beta(2\pi f_m t)] \approx 1, \qquad \sin[\beta \sin(2\pi f_m t)] \approx \beta \sin(2\pi f_m t)$$

$$s(t) \approx A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t) \sin(2\pi f_m t)$$

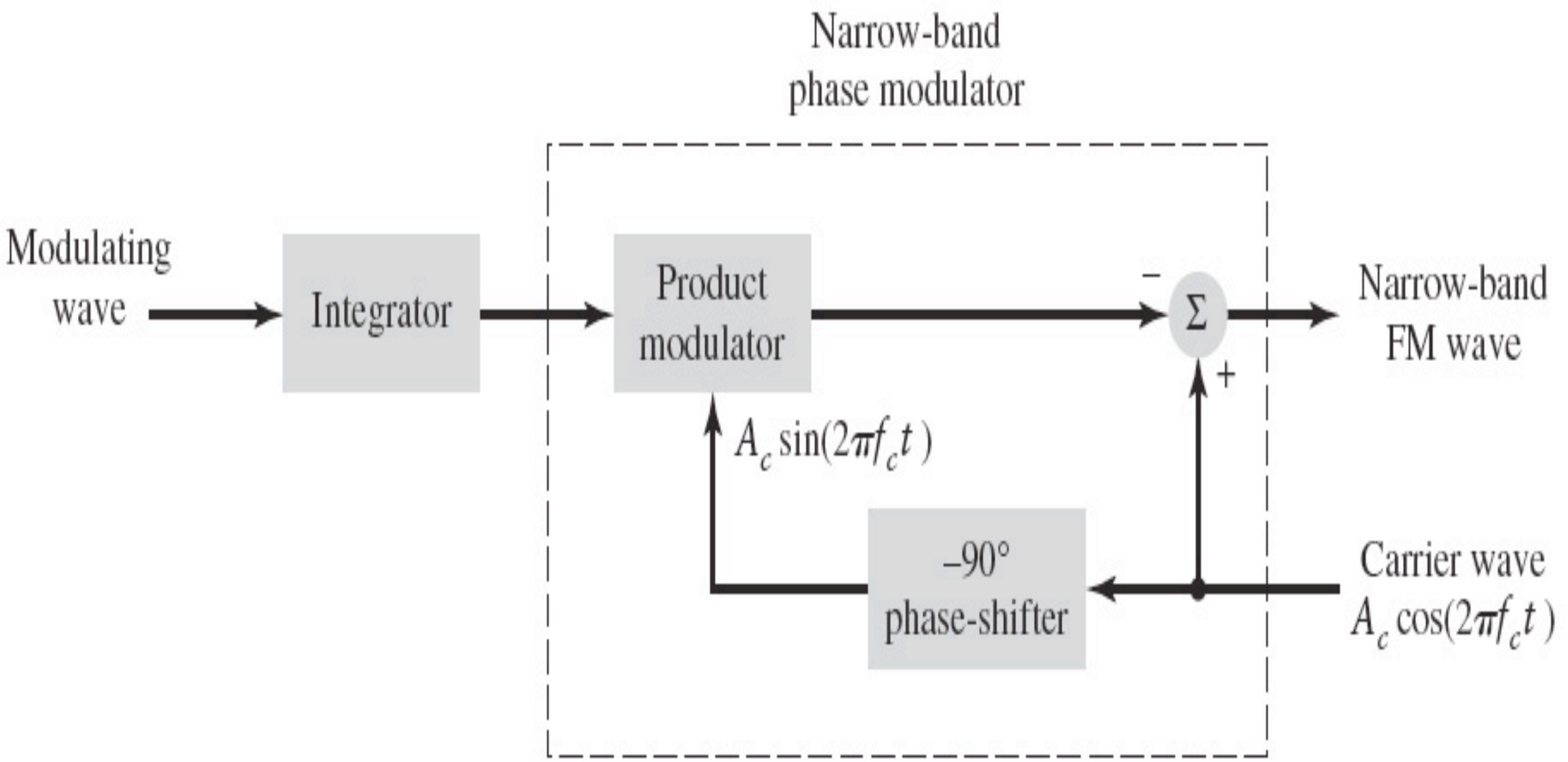

FIGURE 4.4 Block diagram of an indirect method for generating a narrow-band FM wave.

[Ref: Haykin & Moher, Textbook]